

**N.S.W. DEPARTMENT OF EDUCATION  
HIGHER SCHOOL CERTIFICATE EXAMINATION 1970  
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)**

**Instructions:** Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

**QUESTION 1 (12 Marks)**

(i) Express the following numbers as decimals with two figure accuracy:

- (a)  $3^{-3}$  (b)  $27^{1/3}$  (c)  $256^{1/4}$
- (ii) Find the equation of the normal to the parabola  $y = 1 + 3x - 3x^2$  at the point  $(1, 1)$ .

(iii) Find the equation of the line through  $(-1, -1)$  parallel to the line joining  $(-2, -3)$  to the origin.

(iv) Write down the values of (a)  $|7| + |-9|$  (b)  $|-7| - |-9|$

**QUESTION 2 (9 Marks)**

(i) Simplify  $\frac{\cos(180^\circ - A)}{\sin(90^\circ - A)}$ .

(ii) Differentiate  $\sin\left(\frac{\pi}{4} - x\right)$ .

9 (iii) Given that  $\cos 30^\circ = \frac{\sqrt{3}}{2}$ ,  $\cos 45^\circ = \frac{1}{\sqrt{2}}$ ,  $\sin 30^\circ = \frac{1}{2}$ ,  $\sin 45^\circ = \frac{1}{\sqrt{2}}$ , deduct the value of  $\cos 15^\circ$ .

**QUESTION 3 (9 Marks)**

(i) What is the area of the region above the  $x$ -axis and below the curve  $y = x(2 - x)$ ?

(ii) Write down primitives (indefinite integrals) of (a)  $\frac{2}{x^3}$  (b)  $\frac{1}{x+2}$

(iii) A circular sector AOB, centre O, of radius 10 cm, contains an angle of  $\frac{\pi}{4}$  radians. The straight edges OA, OB are joined to form a right circular cone. What is the base radius of the cone in cm?

**QUESTION 4 (10 Marks)**

Consider the function defined for all  $x$  by  $f(x) = 2 \sin(\pi x)$

(i) Sketch the graph of  $y = f(x)$  for  $-2 \leq x \leq 2$

(ii) What is the range of the function?

(iii) What is its period?

(iv) Evaluate  $\int_{-0.25}^{1.75} 2 \sin(\pi x) dx$

**QUESTION 5 (10 Marks)**

For the curve  $y = \frac{1}{2}x^2(x^2 - 1)$

- (i) find the equation of the tangent at  $(1, 0)$ ;  
 (ii) what is the angle between this tangent and the tangent at  $(-1, 0)$ ?  
 (iii) find the point of intersection of the tangents to the curve at  $(-1, 0)$  and  $(1, 0)$ .

**QUESTION 6 (10 Marks)**

(i)  $P_1, P_2$  are points on the curve  $y = x^2$  whose  $x$  coordinates are respectively  $2 - h$  and  $2 + h$ . Show that the equation of the line through  $P_1, P_2$  is  $y = 4x + h^2 - 4$ .

Find the limiting position of this line as  $h \rightarrow 0$ .

(ii) Find the derivatives of (a)  $x \sin x$  (b)  $\log_e\left(\frac{1}{x}\right)$

**QUESTION 7 (10 Marks)**

The position of a particle moving along the  $x$ -axis is given by  $x = 8e^{-2t} - 8 + 16t$ , where  $t$  is the time in seconds and  $x$  is measured in cm.

- (i) Show that the particle is at rest when  $t = 0$ .  
 (ii) What is the velocity after 1 second? (Give two significant figures.)  
 (iii) What is the limiting velocity which the particle approaches as  $t$  increases?  
 (iv) Prove that the acceleration is  $32 - 2v$ , where  $v$  is the velocity.

**QUESTION 8 (10 Marks)**

(i) Find all real roots of the equation  $(x^2 + 2x)^2 - (x^2 + 2x) - 6 = 0$

(ii) What is the natural (i.e. largest possible) domain of the function  $\log_e x$ ? For what values of  $x$  in the interval  $-\pi \leq x \leq \pi$  has the expression  $\log_e(\cos x)$  a real value?

**QUESTION 9 (10 Marks)**

(i) Differentiate (a)  $\frac{1}{\sin x + \cos x}$  (b)  $(x^3 + 3x)^{1/3}$

(ii) Use Simpson's Rule to calculate approximately  $\int_0^4 f(t) dt$  from the following table:

|        |   |    |    |    |    |
|--------|---|----|----|----|----|
| $t$    | 0 | 1  | 2  | 3  | 4  |
| $f(t)$ | 0 | 30 | 50 | 40 | 10 |

If  $t$  represents time in seconds and  $f(t)$  velocity in cm per second, what is a possible physical interpretation of  $\int_0^4 f(t) dt$ ?

**QUESTION 10 (10 Marks)**

Two dice with their faces numbered 1 to 6 are tossed.

(i) By listing the possible outcomes find the probability of

- (a) one die showing a 6 with the other showing at least 4
- (b) one die showing a 6 with the other showing 3 or less

(ii) Deduce from the results in part (i) the probability that neither die shows a 6.

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