

**N.S.W. DEPARTMENT OF EDUCATION
HIGHER SCHOOL CERTIFICATE EXAMINATION 1984
MATHEMATICS - 23 UNIT COURSE**

TIME 3 HOURS

Instructions All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

Standard integrals are provided; approved calculators may be used.

QUESTION 1

- (i) Express 357.896 (a) in scientific notation,
(b) rounded off to the nearest whole number (c) rounded off correct to the nearest one-hundredth.
- (ii) Find the exact value of
- (a) $-2 - (-3)$ (b) 2.9181×10^{23} divided by the product of 2.13×10^{11} and 1.37×10^9 ,
(c) $\frac{1}{5}(t^3 - t^2 + 1)$ when $t = -3$ (d) $36^{1/4} \times 36^{1/4}$
- (iii) Solve the equations (a) $t^3 = 4913$, (b) $2w = -7(1000 - w)$.

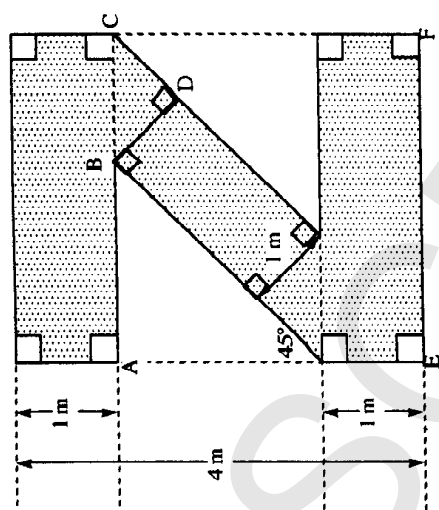
QUESTION 2

- (i) Solve the equation $\frac{3x-1}{5x+1} = \frac{3x-2}{5x+2}$
- (ii) Find the derivative of each of
- (a) $(x+3)(x^2-4)$, (b) $\sin 2x + 3 \tan x$, (c) $\log_e(1+e^x)$.
- (iii) Find (a) $\int_0^{\pi/4} (1 + \tan^2 \theta) d\theta$, (b) the value of k such that $\int_1^k \frac{4}{x^2} dx = 3$.

QUESTION 3

- (i) A car engine is running at 5000 revolutions per minute.
- (a) How many revolutions are made in one hour?
(b) If this engine uses 10 litres of fuel in one hour, how much fuel does it use every revolution?
- (c) This engine has four cylinders and fuel is supplied to two of them each revolution. How much fuel (in millilitres rounded off correct to three decimal places) is supplied to a cylinder when it receives fuel during a revolution?
- (ii) The figure below {next page} show the letter Z which is painted on a wall as part of a sign for a Zoo.
- (a) How long is the segment AB of the letter?
(b) The right angled isosceles triangle BCD has equal sides of length one metre. How long is its hypotenuse?

- (c) Use the results of (a) and (b) to show that the length EF shown on the diagram is $(2 + \sqrt{2})$ metres.
- (d) Find, in square metres, the area painted.
- (e) Find, in metres, the perimeter of this painted area



QUESTION 4

- (i) The lines L_1, L_2 have equations $L_1: x - 2y - 1 = 0, L_2: 2x + y + 18 = 0$.

(a) Find the shortest distance from $P(8, 4)$ to L_1 .

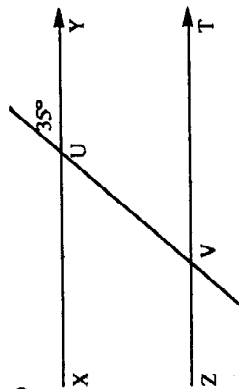
(b) Show that the lines intersect at $Q(-7, -4)$.

(c) Find the distance PQ.

- (iii) Given that $Q(x) = ax^2 + bx + c$, for all x , and that $Q(0) = 4, Q(1) = 23, Q(-1) = 1$, determine the constants a, b, c . Show that the equation $Q(x) = 0$ has no (real) solutions.

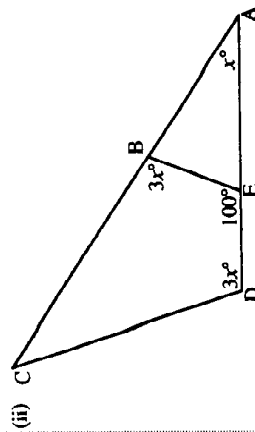
QUESTION 5

(i)



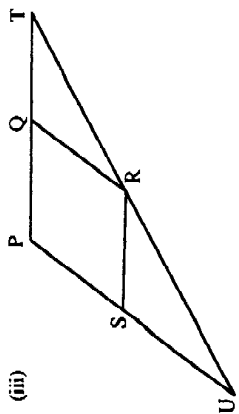
- (i) In the diagram, $XY \parallel ZT$. Find the size of $\angle UVT$ and give a reason for your answer

(ii)



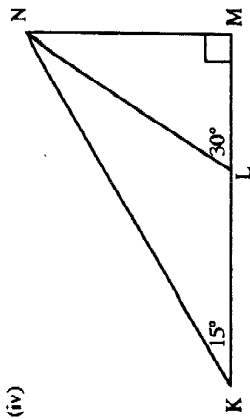
- (ii) As shown in the figure (which is not to scale), B, E lie on the sides AC, DA respectively of $\triangle ACD$. Use the information shown on the figure to find the value of x and hence give $\angle ACD$ in degrees. Give reasons for your answer.

(iii)



- (iii) PQRS is a parallelogram. PQ is produced beyond Q to T so that $QT = QR$ and PS is produced beyond S to U so that $SU = PS$. It is now discovered that T, R, and U are collinear. Prove PQRS is rhombus.

(iv)



- (iv) (a) Find $\angle KNL, \angle LMN$ in degrees and show that $KL = LN$.

(b) Given that the length of NM is 1 unit, write down the lengths of LM, LN and deduce that $\tan 15^\circ = 2 - \sqrt{3}$.

QUESTION 6

- (i) For the domain $-1 \leq x \leq 3$, determine the maximum value and the minimum value of each of the functions defined by $f(x) = 3x^2, g(x) = 4x - x^2$.

Find all values of x in the interval $-1 \leq x \leq 3$ for which $f(x) = g(x)$.

Sketch the curves $y = f(x)$ and $y = g(x)$, for $-1 \leq x \leq 3$, on the same diagram. Indicate clearly the points where maximum and minimum values are attained and the points of intersection of the two curves.

Calculate the area of the region totally enclosed by portions of the two curves.

- (ii) Use the trapezoidal rule with five equal subintervals to estimate $\int_3^8 \log_e(x^2) dx$, rounding your answer to one decimal place.

QUESTION 7

- (i) From a point O the point P bears 120° from the North and is 12.3 km away. The point Q is 15.2 km South-West of O. Mark the relative positions of O, P, Q on a sketch. Calculate the distance PQ in kilometres (rounded off correct to one decimal place).

- (ii) On 1st January 1957, a person joins a superannuation fund by investing \$3000 at 9% p.a. compound interest. A similar amount is invested at the beginning of each subsequent year until the person retires on 31st December, 1984.

(a) Show that the accumulated value of investment at the date of retirement is \$369 406 correct to the nearest dollar.

(b) If this amount is taken as a lump sum payment which is taxable at the rate of 30 cents in every dollar in excess of \$50 000, then how much does the person receive after tax?

QUESTION 8

- (i) The area under the curve $y = \sqrt{9 - x^2}, -3 \leq x \leq 3$ is rotated about the x -axis. Find the volume of the solid of revolution thus obtained. Describe the shape of this solid.

(ii) Write down the formula for:

(a) the n th term of an arithmetic series with first term a and common difference d .

(b) the sum of the first n terms of this series.

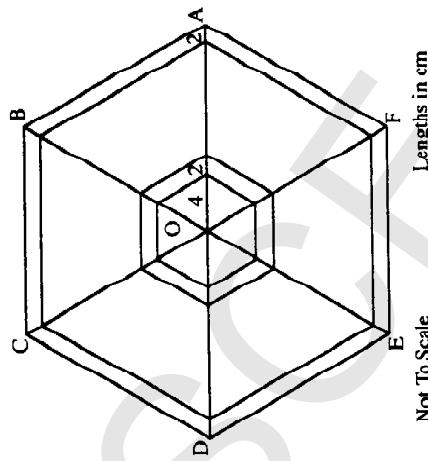
A particular spider's web consists of a series of regular hexagons with a common centre O , held together by rays through O , as in the figure, where only some of the hexagons are shown.

The vertices of the smallest hexagon are 4 cm from O , the vertices of the next are 2 cm further away and they continue at 2 cm intervals along the rays until the vertices of the last hexagon ABCDEF are 60 cm from O .

(c) How many hexagons are there?

(d) What is the length, in cm, of the perimeter of the smallest hexagon?

(e) What is the total length of thread used by the spider in making this web (including the six rays from O)?



Not To Scale Lengths in cm

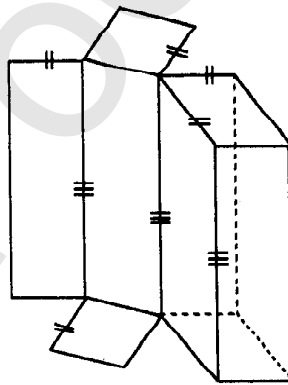
QUESTION 9

(i) Farmer Brown packs her unsalted butter in rectangular cardboard boxes with square ends and over-lapping lids which exactly cover the open top, the square ends, and the front face. Each box uses 0.27 square metres of cardboard. What is the maximum volume of a box?

(ii) A small town has 1000 telephone numbers consisting of the numbers 5000 to 5999 inclusive.

(a) How many of these numbers end in 3?

(b) You remember only that a friend's telephone number ends in 2 and has 4 as one (or both) of its two middle digits. What is the probability that the number is 5142?



QUESTION 10

(i) A particle P moves along a straight line so that at time t ($t \geq 0$) its displacement from a fixed point O on that line is given by $x(t) = 3t^2(4 + t^2)^{-1}$.

(a) Find the velocity of the particle at time t .

(b) Find the times when the particle is momentarily at rest.

(c) Show that P is in (exactly) the same position at both the times $t_1 = 1$, $t_2 = 2 + 2\sqrt{2}$. Give a brief description of the way the displacement changes over the time period from t_1 to t_2 .

(d) Describe the motion of the particle as t increases without bound.

(e) Find the maximum displacement from O .

(ii) The line $y = mx$ is tangent to the curve $y = e^{3x}$. Find m .