

N.S.W. DEPARTMENT OF EDUCATION  
HIGHER SCHOOL CERTIFICATE EXAMINATION 1977  
MATHEMATICS - 2 UNIT COURSE

TIME 3 HOURS

**Instructions.** All questions may be attempted. All questions are of equal value. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

**QUESTION 1**

(i) (a) Express  $\sqrt{5} + \sqrt{80}$  in the form  $a\sqrt{5}$ , and find  $a$

(b) Express  $\frac{4 + \sqrt{5}}{2 + \sqrt{3}}$  in the form  $b - 2\sqrt{3}$ , and find  $b$

(ii) Find the discriminant of the following equation and state whether the roots are real:  $2x^2 + 3x + 5 = 0$

(iii) In the Cartesian plane, indicate, by shading, the region whose points simultaneously satisfy the inequalities:  $x + y \leq 3$ ,  $x - y \geq 1$ .

**QUESTION 2**

(i) Find the axis of symmetry and the vertex of the parabola  $y = 5x^2 + 10x + 2$ .

(ii) Find the acute angle between the lines  $2x + 3y = 13$ ,  $2x - 5y = 5$ .

(iii) Find the perpendicular distance from the point  $(2, -6)$  to the line  $4x - 3y + 9 = 0$ .

**QUESTION 3**

(i) Differentiate with respect to  $x$ : (a)  $2x^2 - 6x + 7$  (b)  $\sqrt{x^2 + 5}$  (c)  $\sin\left(\frac{x}{2}\right)$

(ii) Evaluate  $\lim_{x \rightarrow 0} \frac{\sin 3x}{x}$

(iii) Find the minimum value of  $x \log_e x$  (where  $x > 0$ ) correct to 3 decimal places.

**QUESTION 4**

(i) If  $\log_e 3 = 1.0986123$  and  $\log_e 2 = 0.6931472$  evaluate  $\log_e 12$  correct to 5 decimal places.

(ii) Solve the inequality  $|x - 5| > 9$

(iii) If  $f(x) = ax^2 + bx + c$  find the value of  $f(x) - f(-x)$

(iv) The area under the curve  $y = 2x - x^2$  between  $x = 0$  and  $x = 2$  is rotated about the  $x$ -axis through one complete revolution. Find the volume of the solid so formed.

**QUESTION 5**

(i) Find primitives (i.e. indefinite integrals) of: (a)  $x^{1/3}$  (b)  $x^{-1/3}$  (c)  $e^{2x}$

(ii) Evaluate (a)  $\int_2^7 \frac{4}{x+3} dx$  (b)  $\int_0^{\pi/3} \sin 2x dx$

§ (iii) Use the mid-ordinate rule with one strip to find an approximate value of  $\int_2^4 \frac{\log x^2}{x} dx$

#### QUESTION 6

- § (i) If  $\sin A + \sin 40^\circ = 2 \sin 50^\circ \cos 10^\circ$  and  $A$  is acute, find  $A$ .
- (ii) A triangular piece of land is enclosed by three sections of straight road. Two of these sections are 2.1 km and 3.4 km, and meet at an angle of  $67^\circ$ . Find the length of the third side to two significant figures.
- (iii) Draw separate sketches (not on graph paper) of:
- (a)  $y = 2 \cos x$  for  $-2\pi \leq x \leq 2\pi$  (b)  $y = 2 + 2 \cos x$  for  $-2\pi \leq x \leq 2\pi$

#### QUESTION 7

- (i) Show that the points  $A(3, -1)$ ,  $B(7, 2)$  and  $C(1, 10)$  are the vertices of a right angled triangle. Also find the area of the triangle  $ABC$ .
- (ii) The tangent at the point  $L(3, 4)$  on the curve  $y = \frac{12}{x}$  cuts the  $x$ -axis at  $P$  and the  $y$ -axis at  $Q$ . Find the coordinates of  $P$  and  $Q$ . In what ratio does  $L$  divide the interval  $PQ$ ?

#### QUESTION 8

- (i) A particle moves along the  $x$ -axis with acceleration at time  $t$  equal to  $12(t+3)^2$ . If the particle is initially at rest at the origin, (i.e.  $x = \frac{dx}{dt} = 0$  at  $t = 0$ ), find its position when  $t = 2$ .
- (ii) Show that of all rectangles with perimeter 36 cm, the one which has maximum area is a square.

#### QUESTION 9

- (i) Comment on the following reasoning:
- “When two coins are tossed they can either fall as two heads or two tails or as a head and a tail. As there are three possibilities, the probability of two heads is  $\frac{1}{3}$ .”
- (ii) If three coins are tossed, what is the probability of their falling as two heads and a tail?
- (iii) In a certain school the senior students are divided into two classes  $A$  and  $B$ . Class  $A$  contains 11 female and 9 male students and class  $B$  contains 8 female and 12 male students. A certain occasion requires two speakers, one from each class. These are selected at random. Find the probabilities that:
- (a) both speakers are male;
- (b) both speakers are female;
- (c) one speaker is male and one is female.

#### QUESTION 10

- (i) (a) Sum the arithmetic series  $1 + 3 + 5 + \dots$  to  $n$  terms.
- (b) Sum the arithmetic series  $2 + 4 + 6 + \dots$  to  $n$  terms.
- (c) Find the sum of the first 2000 terms of the series  $1 - 2 + 3 - 4 + 5 - 6 + \dots + (-1)^{n+1} n + \dots$
- (ii) If  $\frac{dy}{dt} = -4y$  and  $y = 10$  when  $t = 0$
- (a) express  $y$  as a function of  $t$ ;
- (b) find the value of  $t$  for which  $y = 0.1$ ;
- (c) if  $y = \frac{dx}{dt}$  and  $x = 10$  when  $t = 0$ , find and expression for  $x$  in terms of  $t$ .