

COMPLEX NUMBERS – WORKSHEET #4**COURSE/LEVEL**

NSW Secondary High School Year 12 HSC Mathematics Extension 2.

TOPIC

Complex Numbers: Curves and regions. (Syllabus Ref: 2.5)

1. Sketch the locus of the point P representing the complex number z , on an Argand diagram, for each of the following:

(a) $\operatorname{Re}(z) = 2$

(f) $\arg\left(\frac{z-i}{z-2}\right) = \frac{\pi}{3}$

(b) $\operatorname{Re}(z) = \operatorname{Im}(z)$

(g) $|z-2| = \operatorname{Re}(z)$

(c) $|z-1-3i| = 2$

(h) $|z-2| + |z+2| = 6$

(d) $|z-1| = |z+2-i|$

(i) $|z-2| - |z+2| = 2$

(e) $\arg(z-i) = \frac{\pi}{4}$

(j) $|z-3| = 2|z|$

2. Sketch the locus of z if $|\operatorname{Re}(z)| + |\operatorname{Im}(z)| = 1$.

3. If $(z-\bar{z})^2 + 8(z+\bar{z}) = 16$ show that the locus of z is a parabola.

- (i) Sketch the parabola

- (ii) Show that $-\frac{\pi}{4} \leq \arg z \leq \frac{\pi}{4}$

4. Sketch the locus of the points P representing z in each case:

(a) $1 < \operatorname{Re}(z) < 2$

(f) $(1-i)\bar{z} = (1+i)z$

(b) $\left|\frac{z-1}{z+1}\right| \leq 1$

(g) $|z-1| = |z+2| = |z-i|$

(c) $1 \leq |z+2+i| \leq 2$

(h) $|z+2i| = \operatorname{Re}(z) + 2i$

(d) $|z-1| < |z-i|$

(i) $\frac{\pi}{3} < \arg(z-i) \leq \frac{\pi}{2}$

(e) $\operatorname{Im}\left(\frac{1}{z}\right) < -\frac{1}{2}$

(j) $\frac{1}{4} < \operatorname{Re}\left(\frac{1}{z}\right) + \operatorname{Im}\left(\frac{1}{z}\right) < \frac{1}{2}$

5. If z satisfies $|z+3i|^2 - |z-3i|^2 = 12$, prove that the locus is the line $y = 1$.

6. If z satisfies $|z+ik|^2 + |z-ik|^2 = 10k^2$, $k > 0$, prove that the locus of z is a circle, centre 0, radius $2k$.

7. P_1, P_2, P_3 represent the complex numbers z_1, z_2, z_3 respectively where $z_1 z_3 = z_2^2$. Show geometrically that OP_2 bisects the angle P_1OP_3 .