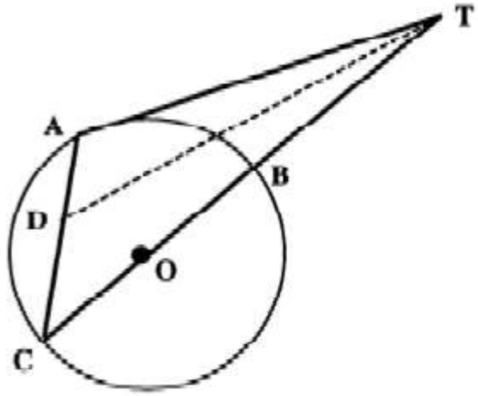
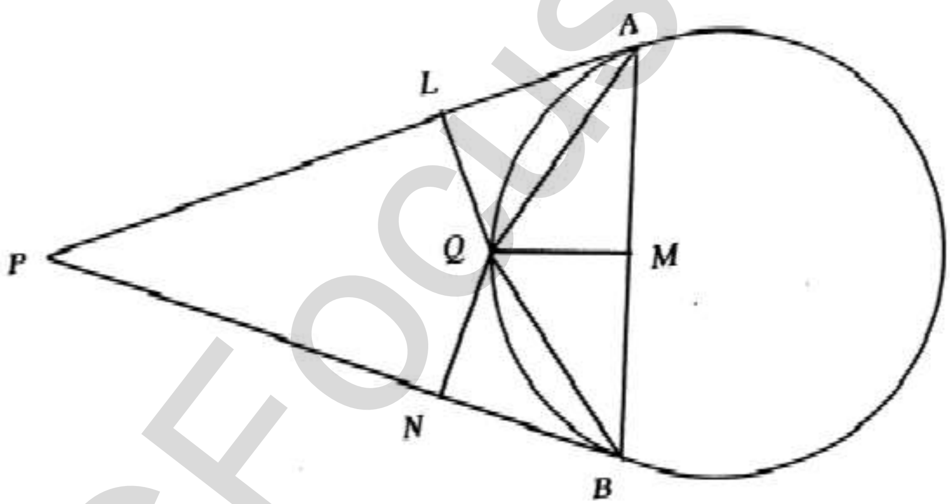
8. A circle with centre O has tangent TA touching the circle at A. TO produced cuts the circle at B and C. D is a point on AC such that TD bisects angle ATC. This is shown in the diagram below.



(a) Prove that $\angle TDC = 135°$

8. (a)



In the diagram PA and PB ate tangents from P to the circle. Q is a point on the minor arc AB. QL, QM and QN are the perpendiculars from Q to PA, AB and PB respectively.

(i) Copy the diagram showing thye above information.

(ii) Show that $\triangle ALQ ||| \triangle BMQ$ and $\triangle BNQ ||| \triangle AMQ$

(iii) Hence show that QL, QM and QN are consecutive terms in a geometric sequence.

(a) M lies on the chord AB of a circle centre O. Prove that if the diameter OM bisects AB then it is perpendicular to it.

AB and CD are two parallel chords. The tangent to the circle at D meets AB produced at T. The other tangent from T touches the circle at E. EC cuts AB at F.

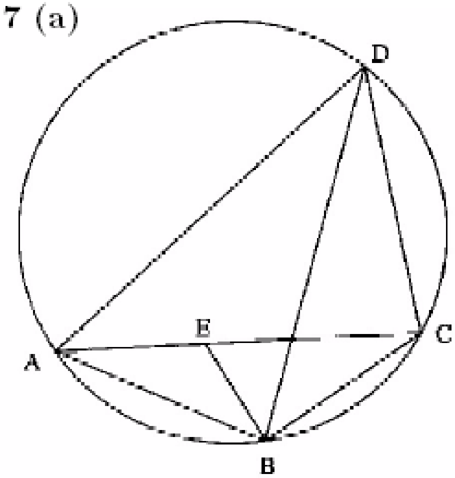
(b) Prove E, F, D and T are concyclic.

(c) Prove FC = FD.

(d) Hence prove that AF = FB.

(e) Prove that OFET and OFTD are cyclic quadrilaterals, where O is the centre of the circle. (It may be helpful to draw a new diagram for the sake of clarity.)

7 (a)



In the figure, $ABCD$ is a cyclic quadrilateral. E is the point on AC such that $A\widehat{B}E = D\widehat{B}C$.

(**i**) Show that $\triangle ABC \| \triangle DBC$ and $\triangle ABD \| \triangle EBC$

(**ii**) Hence show that $AB.DC + AD.BC = AC.DB$

(**b**) ABC is an equilateral triangle inscribed in a circle. P is a point on the minor arc AB of the circle. Show that $PC = PA + PB$.

In the diagram $ABCD$ is a cyclic quadrilateral. E is the point on AC such that $\angle ABE = \angle DBC$.
(α) Show that $\triangle ABE|||\triangle DBC$ and $\triangle ABD|||\triangle EBC$.
(β) Hence show that $AB.DC + AD.BC = AC.DB$.

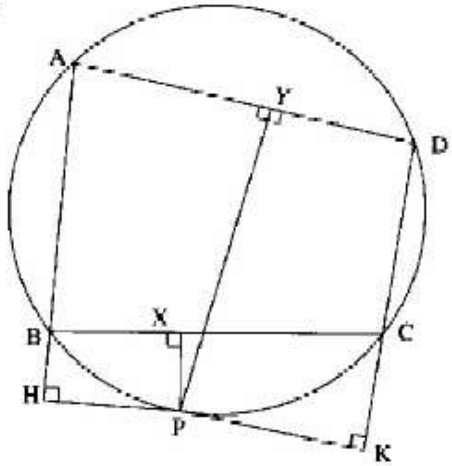
Circles *APD* and *BPC* touch at *P*. *D*, *P* and *C* are collinear. *TP* is the common tangent at *P*. *TC* cuts circle *BPC* in *B* while *TD* cuts circle *APD* in *A*.

(i) Copy the diagram.

(ii) Show that *ATBP* is a cyclic quadrilateral.

(iii) Show that *ABCD* is a cyclic quadrilateral.

7

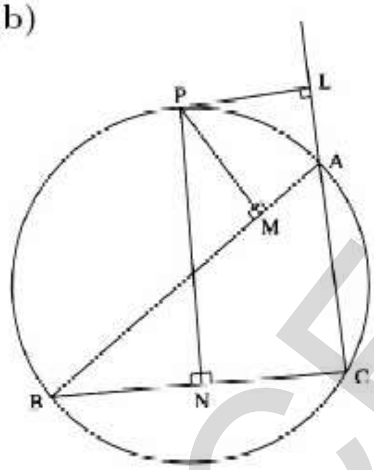


$ABCD$ is a cyclic quadrilateral. P is a point on the circle through A, B, C and D. PH, PX, PK and PY are the perpendiculars from P to AB produced, BC, DC produced and DA, respectively.

(i) Show that $\triangle XPK \,|||\, \triangle HPY$

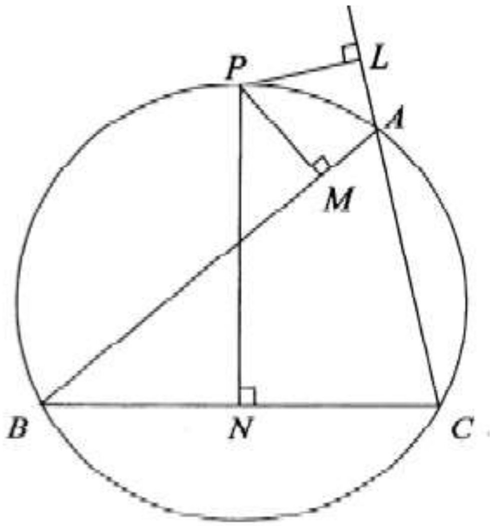
(ii) Hence show that $PX.PY = PH.PK$ and $\frac{PX.PK}{PH.PY} = \frac{XK^2}{HY^2}$.

(b)



ABC is a triangle inscribed in a circle. P is a point on the minor arc AB. L, M and N are the feet of the perpendiculars from P to CA produced, AB, and BC respectively.

Show that L, M and N are collinear.



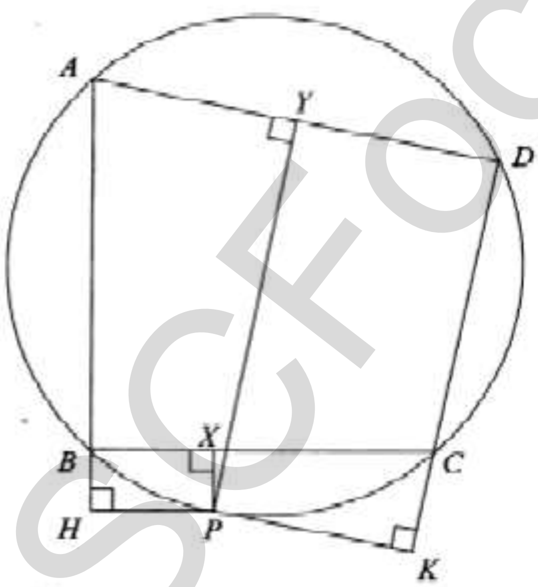
In the diagram above, ABC is a triangle inscribed in a circle. P is a point on the minor arc AB. L, M and N are the feet of the perpendiculars from P to CA (produced), AB, and BC respectively.

(i) Copy the diagram.

(ii) State a reason why P, M, A and L are concyclic points.

(iii) State a reason why P, B, N and M are concyclic points.

(iv) Show that L, M and N are collinear.



$ABCD$ is a cyclic quadrilateral. P is a point on the circle through A, B, C and D. PH, PX, PK and PY are the perpendiculars from P to AB produced, BC, DC produced and DA respectively.

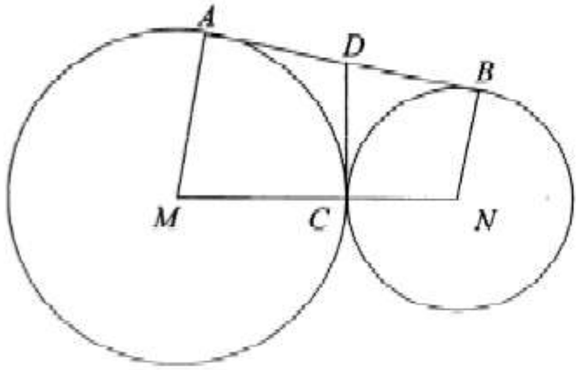
(a) Copy the diagram.

(b) (i) Explain why $XPKC$ an d $AHPY$ are cyclic quadrilaterals.

(ii) Hence show that $\angle XPK = \angle HPY$ and $\angle PKX = \angle PYH$.

(c) (i) Deduce that $\triangle XPK \,|||\, \triangle HPY$.

(ii) Hence show that $PX.PY = PH.PK$ and $\frac{PX.PK}{PH.PY} = \frac{XK^2}{HY^2}$.



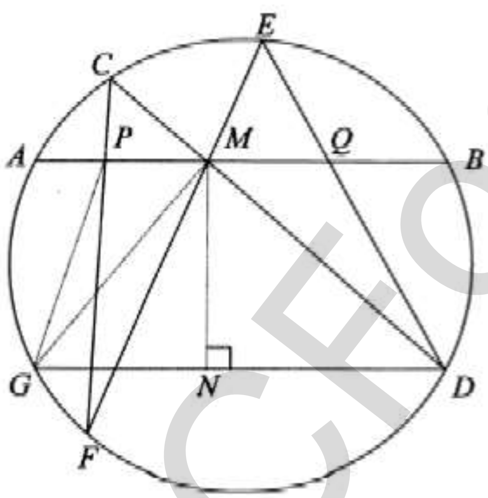
In the diagram MCN is a staight line. Circles are drawn with centre M, radius MC and centre N, radius NC. AB is a common tangent to the two circles with points of contact at A and B respectively. CD is the common tangent at C, and meets AB at D.

(i) Copy the diagram.

(ii) Explain why $AMCD$ and $BNCD$ are cyclic quadrilaterals.

(iii) Show that $\triangle ACD ||| \triangle CBN$.

(iv) Show that $MD || CB$.



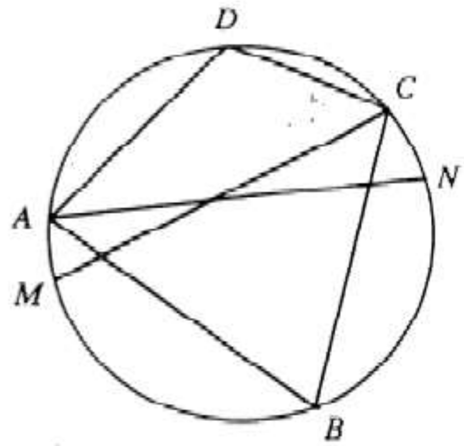
Chords AB, CD, EF are concurrent in M where M is the midpoint of AB. CF, ED meet AB in P, Q respectively. Chord DG is constructed parallel to AB, and N is the foot of the perpendicular from M to DG.

(i) Copy the diagram.

(ii) Show that $\triangle MGD$ is isosceles.

(iii) Show that $PMFG$ is a cyclic quadrilateral.

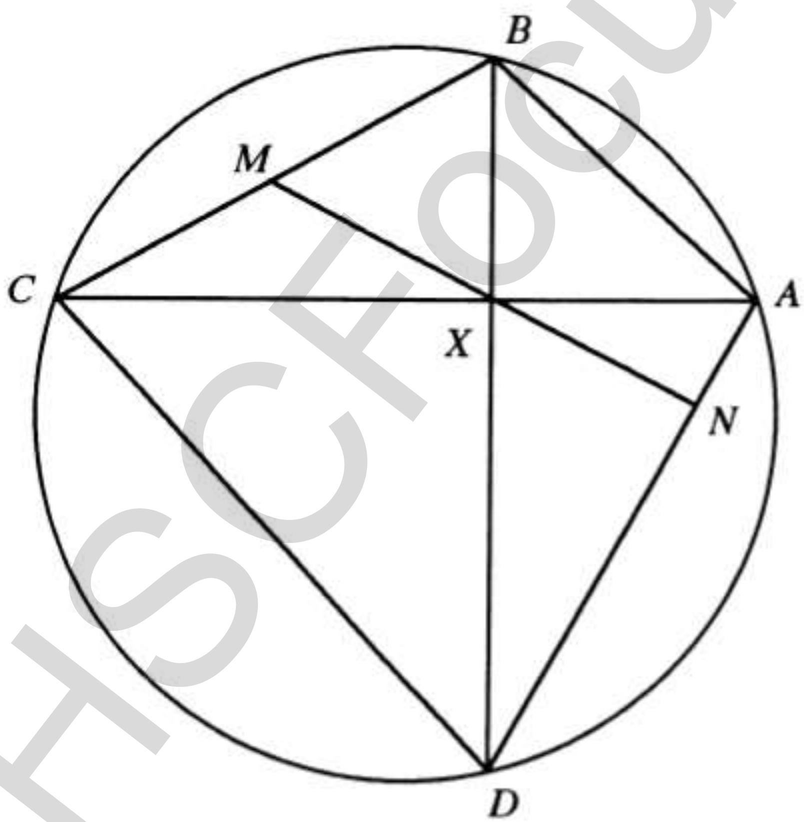
(iv) Show that $MP = MQ$.



In the diagram $ABCD$ is a cyclic quadrilateral. M and N are points on the circle through A, B, C and D such that CM bisects $B\hat{C}D$ and AN bisects $D\hat{A}B$.

(i) Copy the diagram.

(ii) Show that MN is a diameter of the circle.



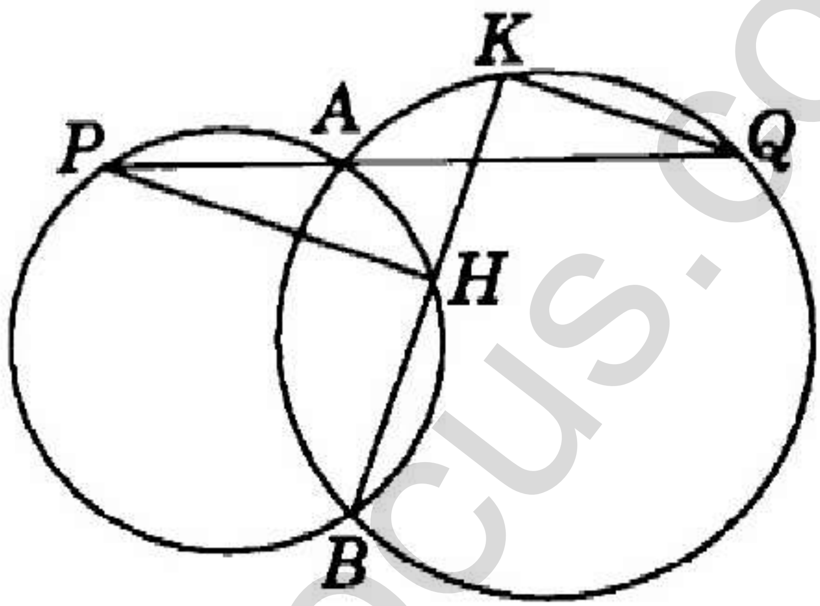
$ABCD$ is a cyclic quadrilateral. The diagonals AC and BD intersect at right angles at X. M is the midpoint of BC. MX produced meets AD at N.

(i) Copy the diagram showing the above information.

(ii) Show that $M\hat{B}X = M\hat{X}B$

(iii) Show that MN is perpendicular to AD.

(c)



In the figure, PAQ and BHK are straight lines. Prove that PH is parallel to KQ.

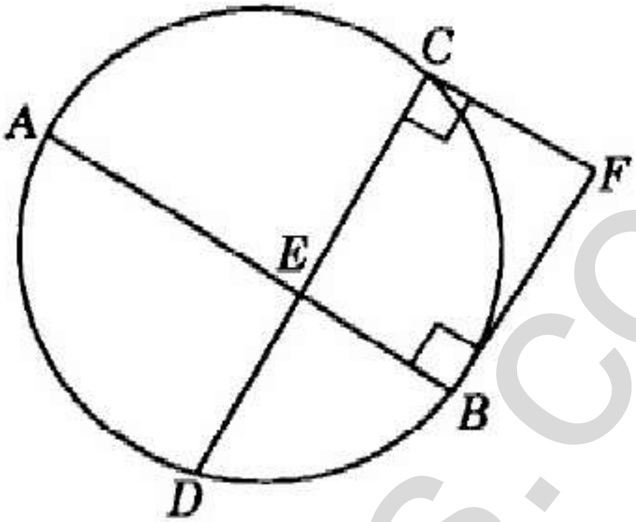
(d) Two circles, centres B and C, touch externally at A. PQ is a direct common tangent touching the circles at P and Q respectively.

(i) Draw a neat diagram depicting the given information.
(ii) Prove that the circle with BC as its diameter touches the line PQ.

(b) Two circles intersect in A and B. C and D are points on the respective circles such that $\angle CAB = \angle DAB$. CB and DB are produced to cut the circles again at E and F.

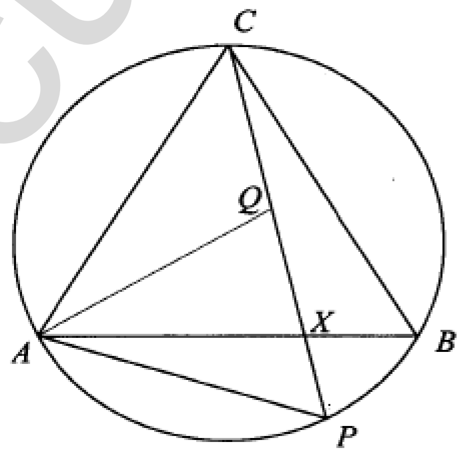
(i) Draw a neat diagram depicting the given information.
(ii) Show that $BC.BE = BD.BF$.

(a) In the figure, AB and CD are two chords of the circle. AB and CD intersect at E. F is a point such that $A\hat{B}F$ and $D\hat{C}F$ are right angles.



Prove that FE produced is perpendicular to AD.

(b) ABC is an equilateral triangle inscribed in a circle and P is another point on the circumference. PC crosses AB at X.



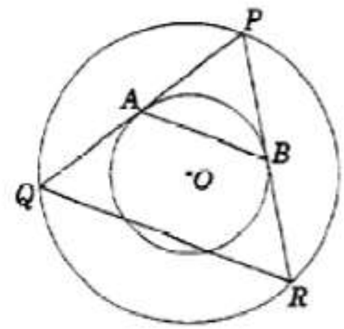
(i) Prove that $\Delta CXB \parallel\!| \Delta CBP$. **2**

(ii) Find the size of $\angle APB$, giving reasons. **2**

(iii) Q is a point which lies on PC such that $AP = QP$. Find $\angle AQC$, giving reasons. **2**

(iv) Show that $CP = AP + PB$. **2**

(a)



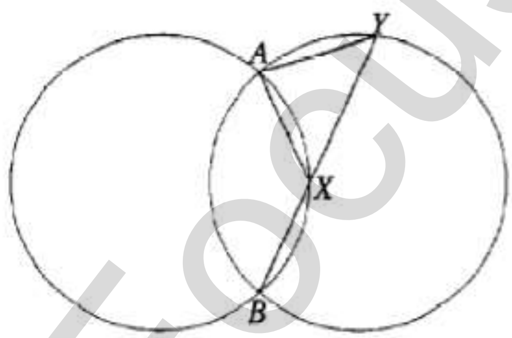
PQ and PR are chords of a circle, centre O, which touch a concentric circle at A and B.

(i) Prove that $PA = AQ$. **2**

(ii) Prove that $QR = 2AB$. **1**

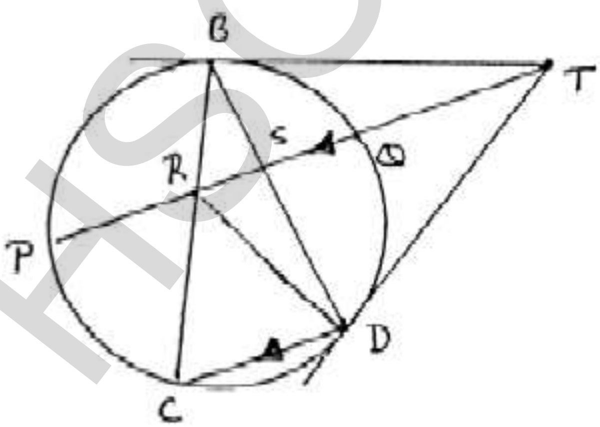
(iii) Prove that OP is the perpendicular bisector of AB. **2**

(b)



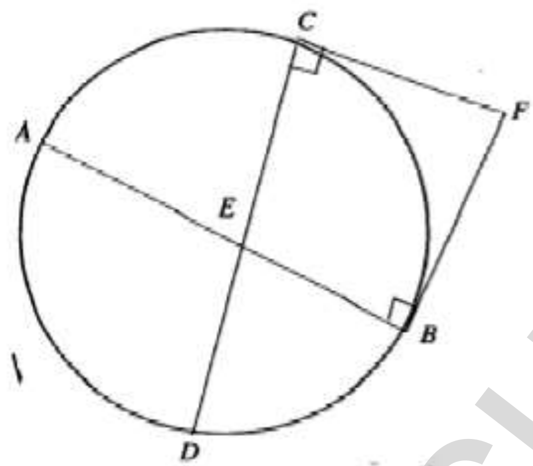
In the figure the circles are equal and BXY is a straight line. **3**

Prove that $AX = AY$.



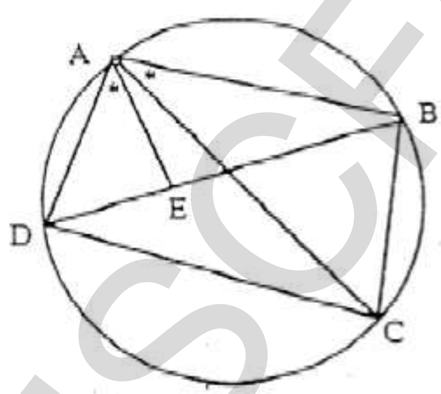
In the diagram, the chords PQ and CD are parallel. The tangent at D cuts the chord PQ at T. The other point of contact from T is B and BC cuts PQ at R.
(i) Copy the diagram.
(ii) Prove that $\angle BDT = \angle BRT$ and state why B, T, D and R are concyclic
(iii) Show that $\triangle RCD$ is isosceles.

(b) In the figure AB and CD are two chords of the circle. AB and CD intersect at E. F is a point such that $\angle ABF$ and $\angle DCF$ are right angles.



Prove that FE produced is perpendicular to AD. 5

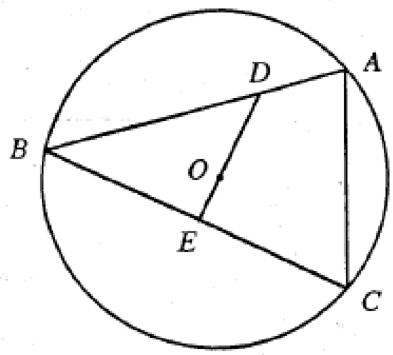
(c)



$ABCD$ is a cyclic quadrilateral. E is a point on diagonal BD, such that $\angle DAE = \angle BAC$. Prove that:
(i) $AB.CD = AC.BE$ **(ii)** $BC.DA = AC.DE$ **(iii)** $AB.CD + BC.DA = AC.BD$

In the diagram, the chords PQ and CD are parallel. The tangent at D cuts the chord PQ at T. The other point of contact from T is B and BC cuts PQ at R.
(i) Copy the diagram.
(ii) Prove that $\angle BDT = \angle BRT$ and state why B, T, D and R are concyclic
(iii) Show that $\triangle RCD$ is isosceles.

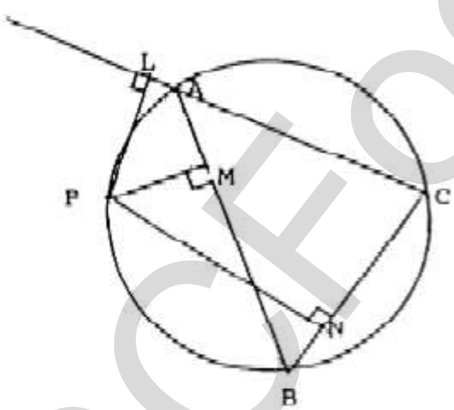
(a)



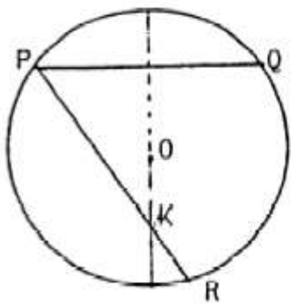
ABC is a triangle inscribed in a circle with centre O. A second circle through the points A, C, O cuts AB at D. DO produced meets BC at E.

(i) Copy the diagram

(ii) Show that $BE = CE$.



ABC is a triangle. P is a point on the minor arc $\widehat{AB}$. PL, PM and PN are perpendicular to AC, AB, BC or these sides produced.
Prove that **(i)** $P\hat{M}L = P\hat{B}C$ **(ii)** L, M, N are collinear.

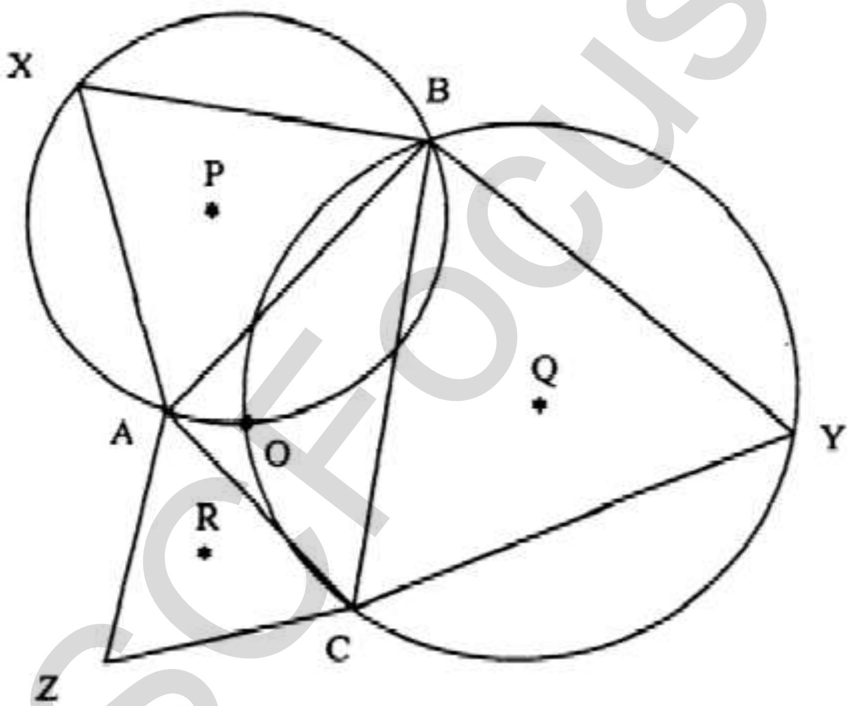


PQ is a chord of a circle. The diameter of the circle perpendicular to PQ meets another chord PR at K such that $OK = KR$.

(i) Prove that quadrilateral $OKRQ$ is cyclic.

(ii) Hence deduce that KQ bisects $O\hat{Q}R$.

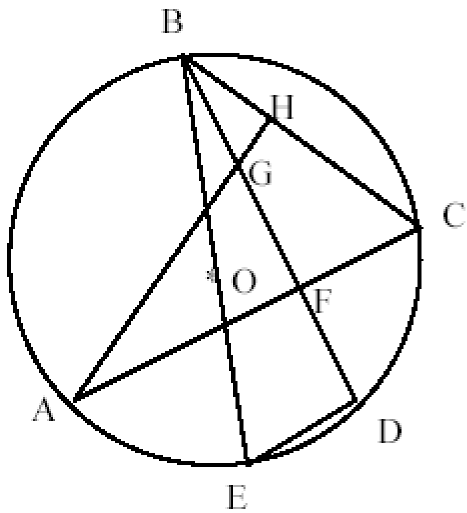
(b) ABC is any triangle. Equilateral triangles ABX, BCY and ACZ are constructed on the sides of $\triangle ABC$. Circles with centres P and Q are drawn to pass through the vertices of $\triangle ABX$ and $\triangle BCY$. The circles meet at B and O. (see diagram)



(i) Find the size of $\angle AOB, \angle BOC$ and $\angle AOC$, giving reasons.

(ii) Prove that $AOCZ$ is a cyclic quadrilateral.

(iii) If R is the centre of the circle $AOCZ$, prove that $\triangle PRQ$ is equilateral.



A , B, C, D, and E are points on a circle centre O with diameter BE and AC || DE

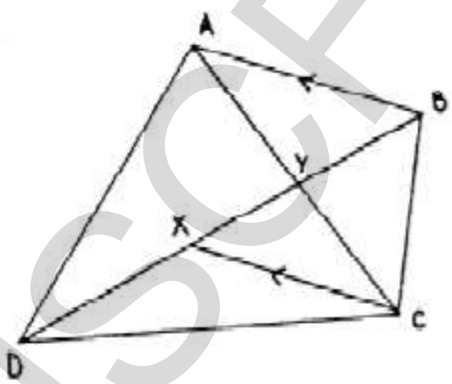
AH $\perp$ BC , and BD intersect AH and AC at G and F respectively .

(i) Prove $\angle$BFC $= 90^0$

(ii) Prove CFGH is a cyclic quadrilateral .

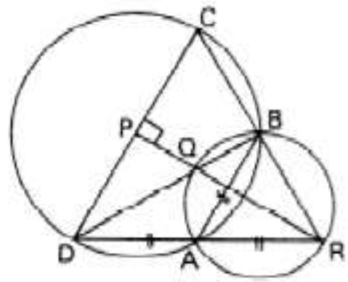
(iii) Prove AB . BG = BE . BH

(c) $ABCD$ is a quadrilateral. $CX || AB$. X lies on diagonal BD. $XC = DX$ and $\angle BXC = 2\angle ACX$.

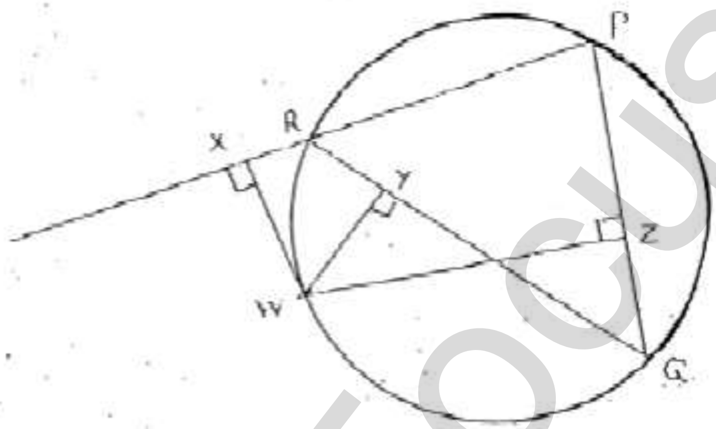


(i) Copy the diagram.
(ii) Prove that $ABCD$ is a cyclic quadrilateral.

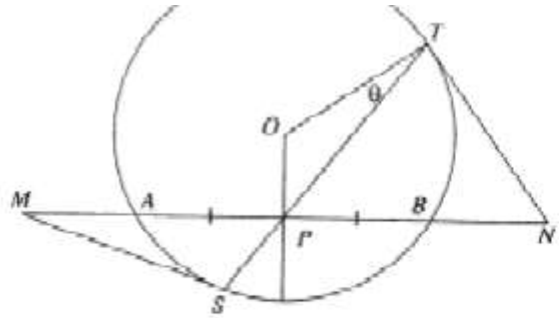
(b) $AB = AD = AR$, $RP \perp DC$.



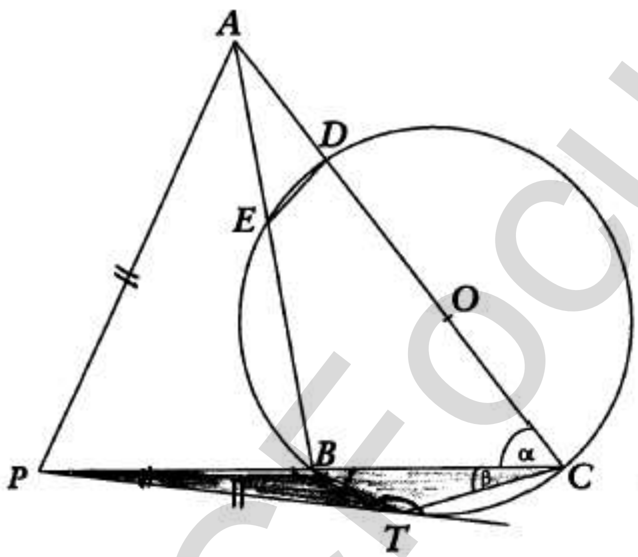
Prove that
(i) $BCPQ$ is a cyclic quadrilateral.
(ii) $\angle CBD = 90°$
(iii) $AB = AP$.
(b)



PQR is a triangle inscribed in a circle with W a point on the arc QR. WX is perpendicular to PR produced, and WZ is perpendicular to PQ.
(i) Copy this diagram.
(ii) Explain why $WXRY$ and $WYZQ$ are cyclic quadrilaterals.
(iii) Show that the points X, Y and Z are collinear.



In the diagram above, P is the midpoint of the chord AB in the circle with centre O. A second chord ST passes through P, and the tangents at the endpoints meet AB produced at M and N respectively.
(i) Explain why $OPNT$ is a cyclic quadrilateral.
(ii) Explain why $OPSM$ is also cyclic.
(iii) Let $\angle OTS = \theta$. Show that $\angle ONP = \angle OMP = \theta$.
(iv) Hence prove that $AM = BN$.



In the diagram above, O is the centre of the circle, PT is a tangent to the circle and $PT = PA$. Let $\angle ACP = \alpha$ and $\angle BCT = \beta$.

(i) Show that $\triangle PBT \;|||\; \triangle PTC$.

(ii) Show that $\triangle APB \;|||\; \triangle CPA$.

(iii) Show that $DE \parallel AP$.