Exercise 6

Extracts from

Taylors College COMPLEX NUMBERS Study Guide

+ ANSWERS

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Complex Roots of Unity (1)

Solve $z^n = 1$.
z^n has _______ solution.
If n is odd, $z^n = 1$ has _______ real solutions and _______ complex nonreal solutions in _______ pairs because the coefficients of the polynomial are _______.

Example: Solve $z^3 = 1$, i.e., find the cube roots of unity.

Method 1: Factorise $z^3 - 1$ and write the solutions in the form $a + ib$. If $\omega = \frac{-1+i\sqrt{3}}{2}$ then $\overline{\omega} =$ _______. Since ω is a solution to $z^3 = 1$ then $\omega^2 + \omega + 1 =$ _______.

Method 2: Let $z = rcis\theta$ and write the solutions in mod-arg form. The roots are equally spaced around _______.

If n is even, $z^n = -1$ has _______ real solutions and _______ nonreal complex solutions in _______ pairs because the coefficients of the polynomial are _______. The roots are equally spaced around the _______. Check if $\pm$_______ is a possible conjugate pair.

Examples: Solve $z^6 = -1$; $z^4 = -1$; $z^5 = -4 + 4i$; $z^5 = 1$

If n is even, $z^n = 1$ has _______ real solutions and _______ nonreal complex solutions in _______ pairs because the coefficients of the polynomial are _______. The roots are equally spaced around the _______. When joined, the roots form a _______ with _______ sides centred at _______ and with vertex at _______.

Example: Solve $z^6 = 1$.

If n is odd, $z^n = -1$ has _______ real solutions of _______ and _______ nonreal complex solutions in _______ pairs because the coefficients of the polynomial are _______. The roots are equally spaced around the _______.

Example: Solve $z^7 = -1$.

ANSWERS

Complex Roots of Unity (1)

n; 1; $n-1$; conjugate; real; $1, -\frac{1}{2}(1 \pm i\sqrt{3})$; $\text{c}i\text{s}\frac{2k\pi}{3}, k=-1,0,1$; a unit circle; 0, n; conjugate; real; unit circle; i; $\text{c}i\text{s}\frac{(2k+1)\pi}{6}, k=-3,-2,-1,0,1,2$; $\text{c}i\text{s}\frac{(2k+1)\pi}{4}, k=-2,-1,0,1$; $\sqrt{2}\text{c}i\text{s}\frac{(8k+3)\pi}{20}, k=-2,-1,0,1,2$; $\text{c}i\text{s}\frac{2k\pi}{5}, k=-2,-1,0,1,2$; 2; $(n-2)$; conjugate; real; unit circle; regular polygon; n; 0; the roots;
$\text{c}i\text{s}\frac{k\pi}{3}$, $k=-3,-2,-1,0,1,2$; 1; -1; $(n-1)$; conjugate; real; unit circle;
$\text{c}i\text{s}\frac{(2k+1)\pi}{7}, k=-3,-2,-1,0,1,2,3$