Further Questions 1

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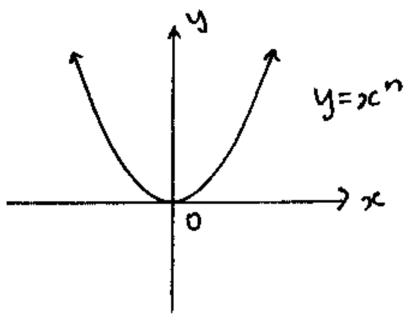

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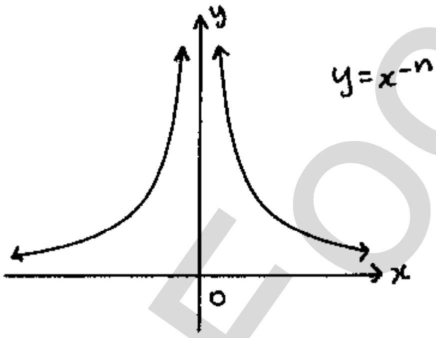

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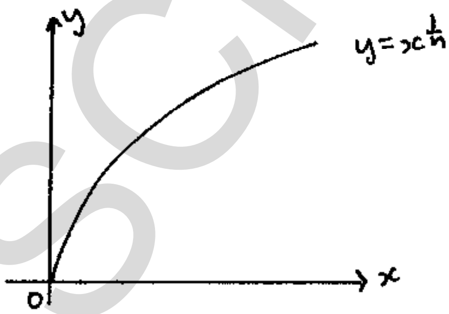

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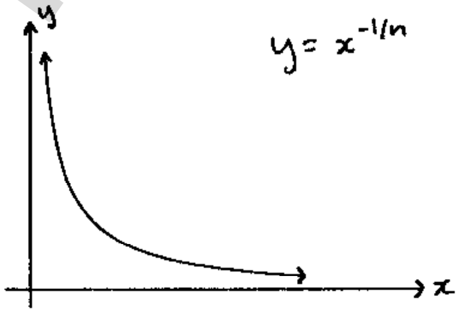

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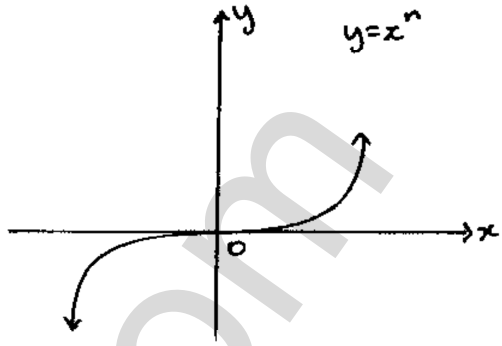

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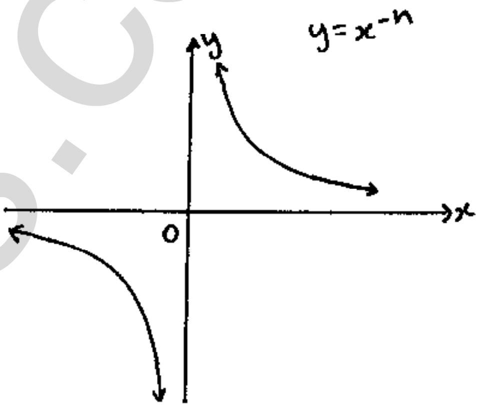

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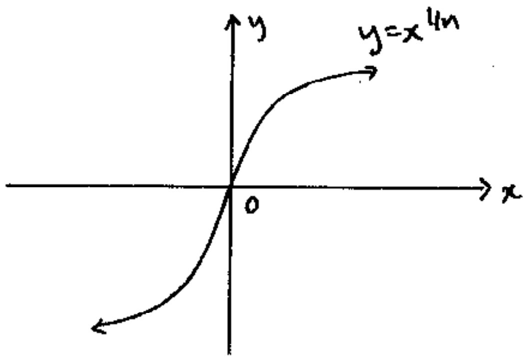

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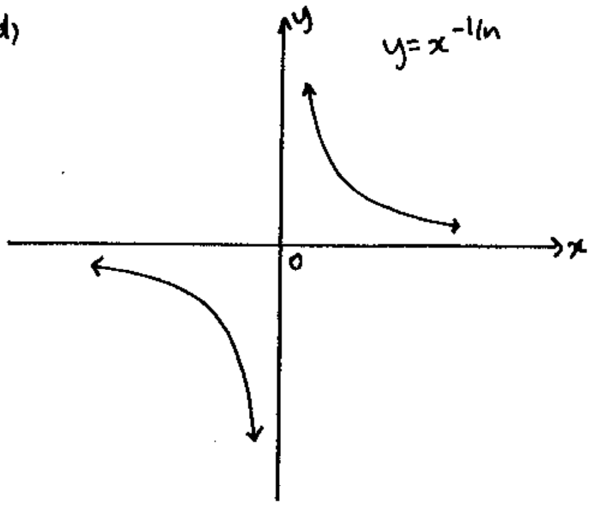

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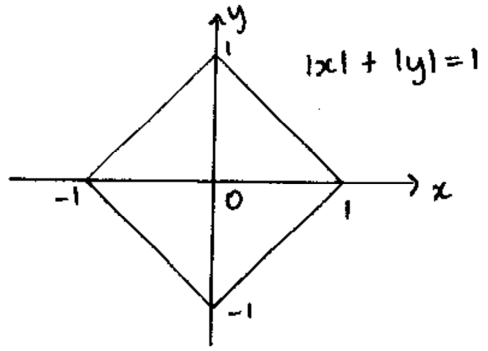

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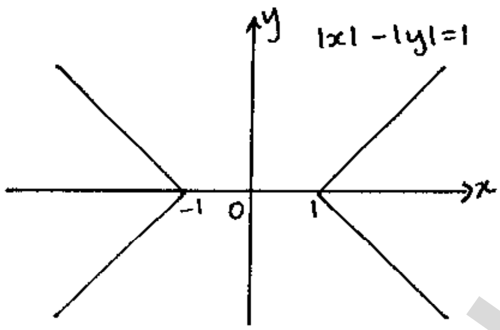

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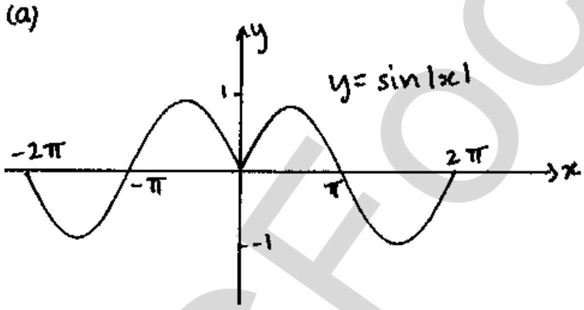

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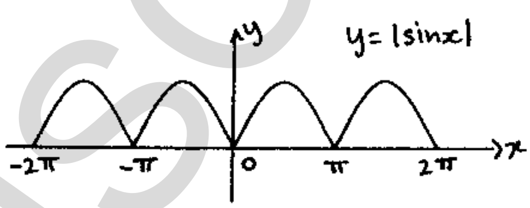

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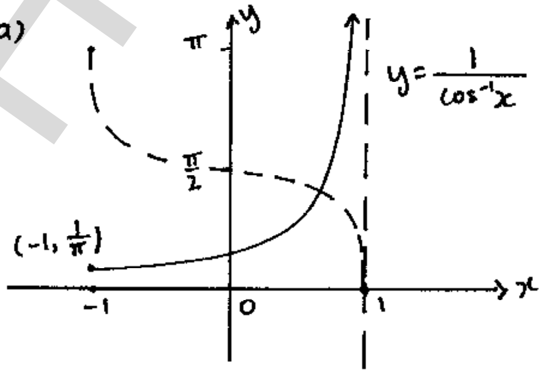

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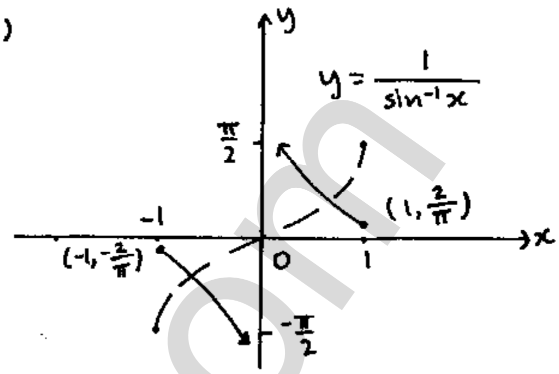

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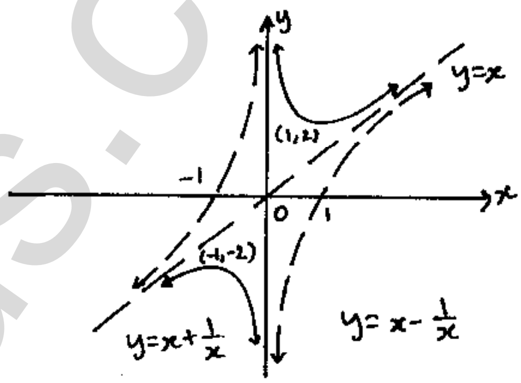

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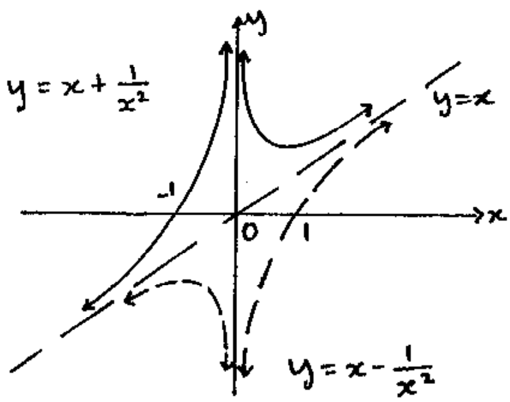

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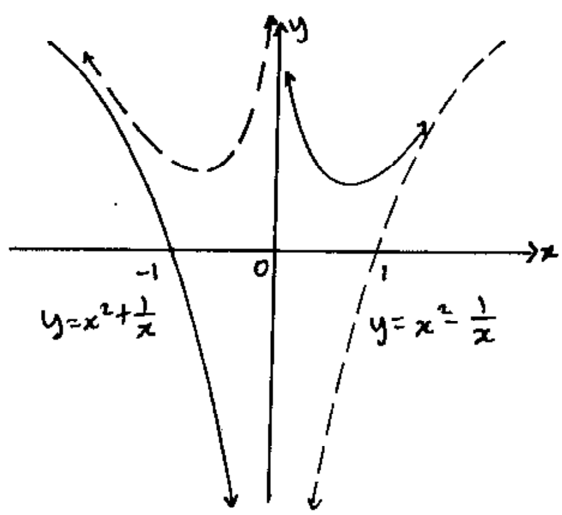

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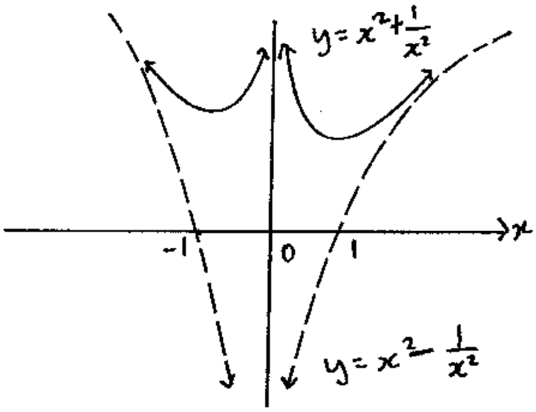

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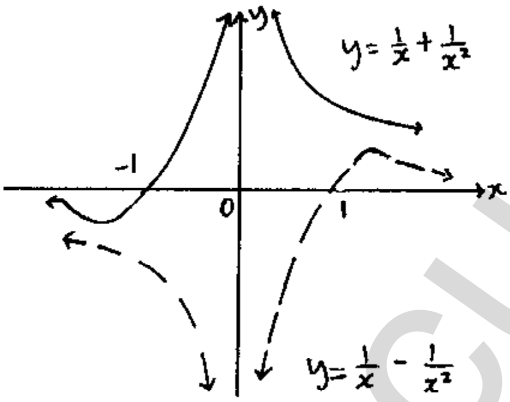

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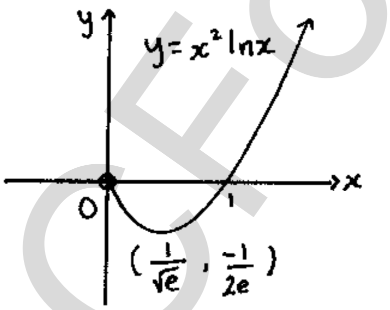

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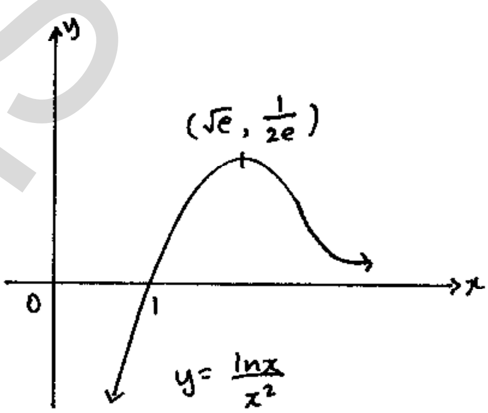

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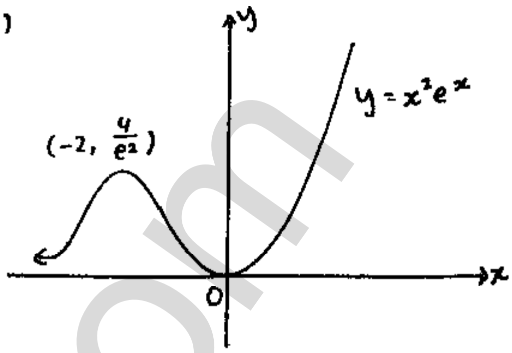

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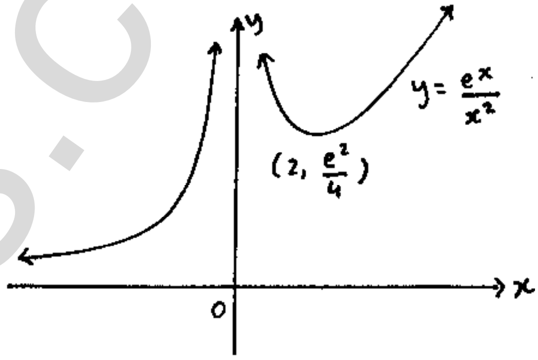

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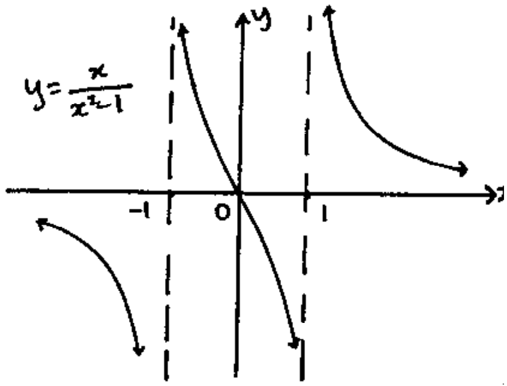

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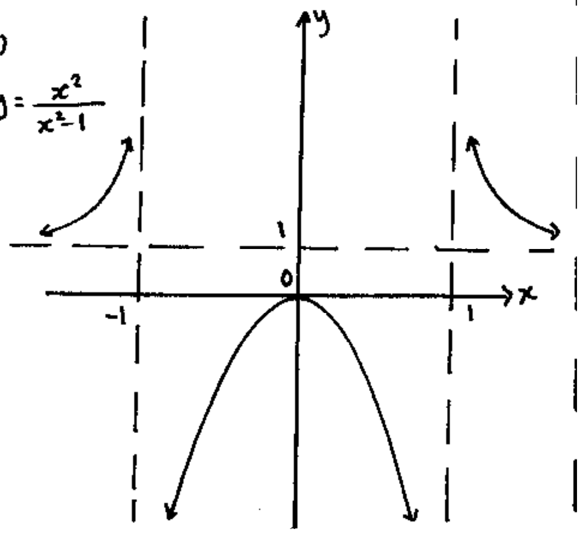

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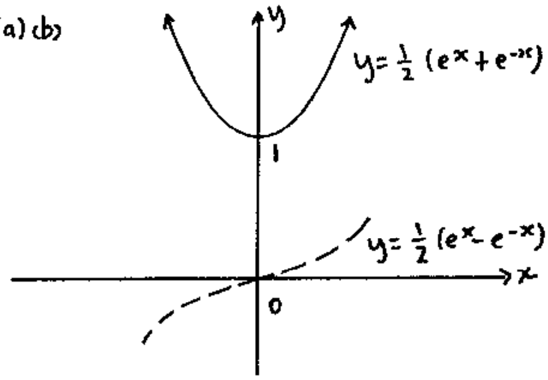

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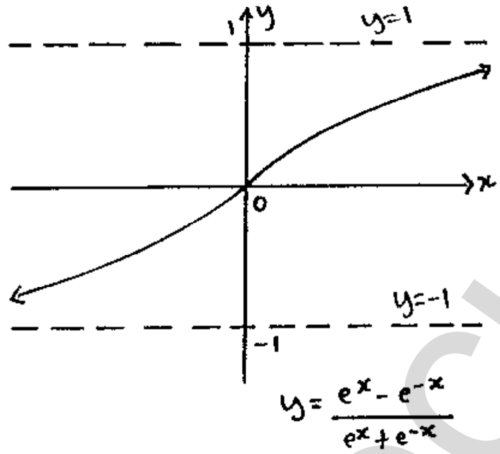

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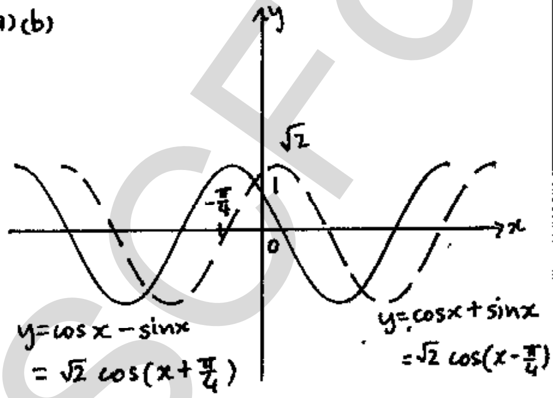

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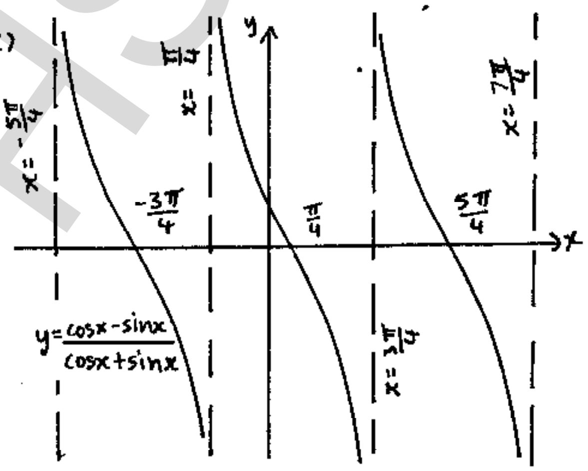

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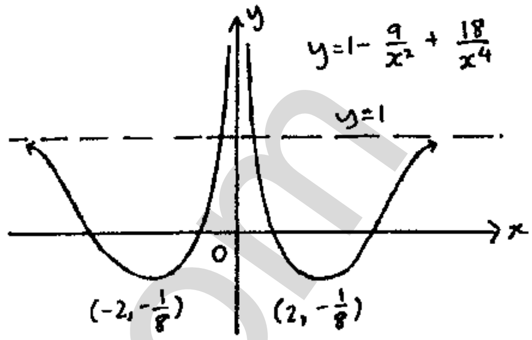
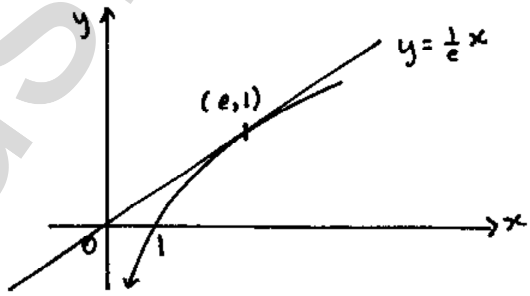


The equation $f(x)=k$ will have 4 roots when the horizontal line $y=k$ crosses the curve 4 times. $\therefore -\frac{1}{8}<k<1$

17.



Equation of a line passing by $(0,0)$ is $y=kx$. Let the line $y=kx$ meet the curve $y=\ln x$ at (x_0, y_0)

$\therefore \ln x_0 = kx_0$ (1)

The gradient of the tangent at (x_0,y_0) on the curve $y=\ln x$ is $\frac{dy}{dx}=\frac{1}{x}$ (grad. at x_0 $\therefore m_{tan}=\frac{1}{x_0}$

and since the line $y=kx$ is also tangent at (x_0,y_0) $\therefore k=\frac{1}{x_0}$ (2)

from (1) and (2):

$\ln x_0 = \frac{1}{x_0}\times x_0$ $\therefore \ln x_0 = 1$

$\therefore x_0 = e$ $\therefore y_0=\ln e = 1$ $\therefore m_{tan}=\frac{1}{e}$ from $(0,0)$

The equation has two real distinct roots when $0<k<\frac{1}{e}$.

18. $xy(x+y)+16=0$ (1)

$x^2y+xy^2+16=0$

Using implicit differentiation,

$2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} = 0$

$\frac{dy}{dx}\left(x^2+2xy\right) = -2xy - y^2$

$\therefore \frac{dy}{dx} = \frac{-2xy-y^2}{x^2+2xy}$ (gradient function)

$\frac{dy}{dx} = -1 \quad \therefore \frac{-2xy-y^2}{x^2+2xy} = -1$

$\therefore 2xy + y^2 = x^2 + 2xy$

$\therefore y = \pm x$ sub $y=x$ into (1)

$\therefore 2x^3 + 16 = 0 \quad \therefore x^3 = -8 \quad \therefore x = -2$

$\therefore y = -2 \quad \therefore$ The point is $(-2,-2)$.

$\therefore \frac{y+2}{x+2} = -1 \quad \therefore y+2 = -x-2$

$\therefore x+y+4=0$

Now, for $y=-x$

$-2x^3+16=0 \quad \therefore x^3=8 \quad \therefore x=2 \quad \therefore y=-2$

$\therefore$ The point is $(2,-2)$

$\therefore \frac{y+2}{x-2} = -1 \quad \therefore y+2 = -x+2$

$\therefore x+y=0$

By substituting into (1) we get:

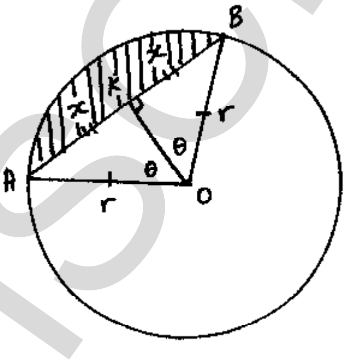
$16=0$ invalid

$\therefore$ There is no double root.

For $x+y=0$

$\therefore$ The only tangent to the curve is $x+y+4=0$.

19.



Perimeter of minor segment = Arc AB + chord AB

using Δ KOB, $\sin\theta = \frac{x}{r} \quad \therefore x = r\sin\theta$

$\therefore AB = 2r\sin\theta$

$\therefore$ Perimeter of minor segment $= r\theta + 2r\sin\theta$

$\therefore$ Perimeter DOAB is $2r + 2r\sin\theta$.

$\therefore 2r\theta + 2r\sin\theta = k(2r + 2r\sin\theta)$

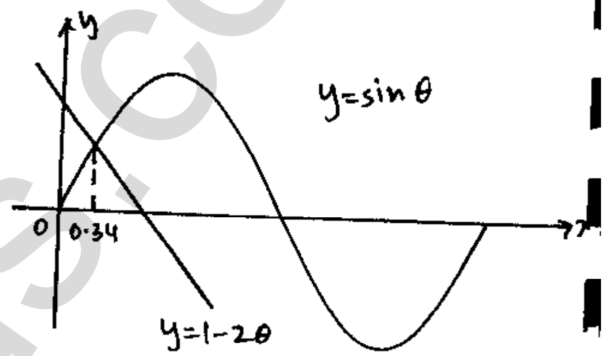
Dividing by $2r$

$\therefore \theta + \sin\theta = k + k\sin\theta$

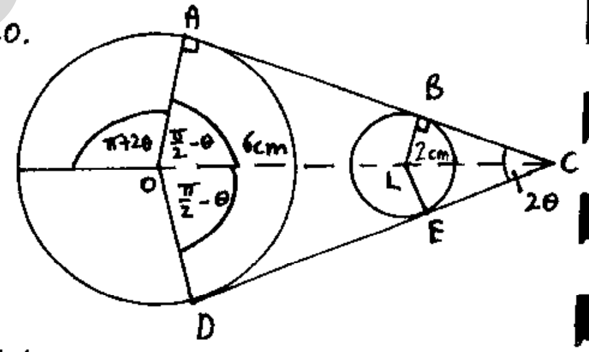
$k\sin\theta - \sin\theta + k = \theta \quad \therefore (k-1)\sin\theta + k = \theta$

for $k=\frac{1}{2} \quad \therefore -\frac{1}{2}\sin\theta + \frac{1}{2} = \theta$

$\therefore \sin\theta - 1 = -2\theta \quad \therefore \sin\theta = 1-2\theta$



20.



Using ΔAOC, $\tan\left(\frac{\pi}{2}-\theta\right) = \frac{AC}{6} \quad \therefore AC = 6\cot\theta$

Using ΔBLC, $\tan\left(\frac{\pi}{2}-\theta\right) = \frac{BC}{2} \quad \therefore BC = 2\cot\theta$

$\therefore AC - BC = 4\cot\theta$ (length of tangent AB)

$\therefore$ Length of both tangents is $8\cot\theta$.

Arc AD $= 6(\pi+2\theta)$; Arc BE $= 2(\pi-2\theta)$

$\therefore$ Total length is $8\cot\theta + 6\pi + 12\theta + 2\pi - 4\theta = 44$c[m]

$\therefore 8\cot\theta + 8\pi + 8\theta = 44\text{cm} \quad \therefore \cot\theta + \pi + \theta = 5.5$