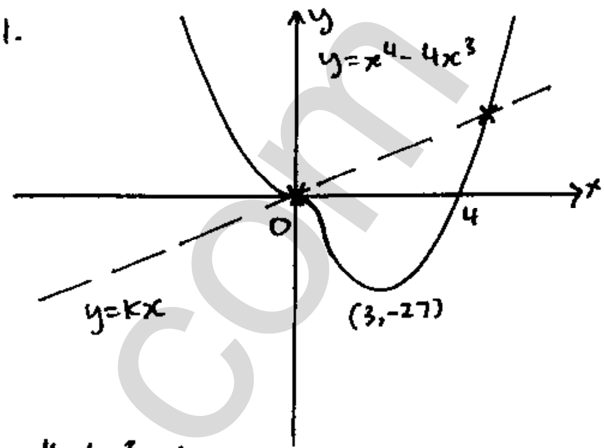
EXERCISE: 1.10

1.

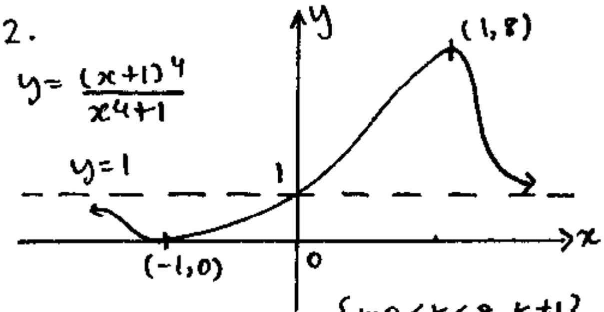


$x^4-4x^3=kx$

The roots of this equation are the points of intersection of $y=kx$ and $y=x^4-4x^3$.

$\therefore$ From the graph we can see that there are 2 real roots for the equation, one of them $(0,0)$ and the other has co-ordinates $x>4, y>0$.

2.



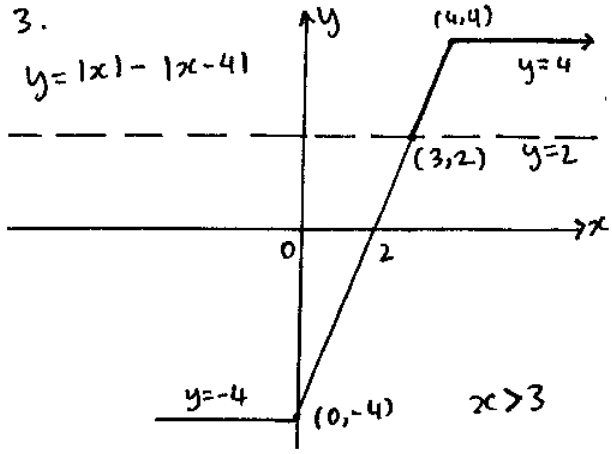
$\{k: 0<k<8, k\neq 1\}$

$\frac{(x+1)^4}{x^4+1}=k$

The roots of this equation are the points of intersection of $y=k$ and $y=\frac{(x+1)^4}{x^4+1}$.

$\therefore$ From the graph we can see that there are 2 real roots for the equation when $0<k<8$ except $k=1$.

3.



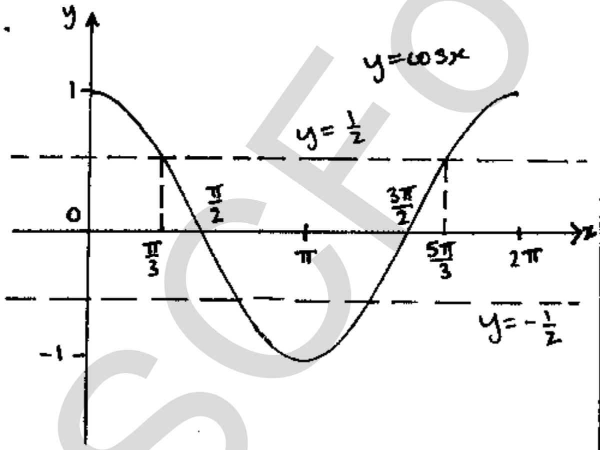
$y=|x|-|x-4|$

$$\therefore y=\begin{cases} -4; & x\le 0 \\ 2x-4; & 0<x<4 \\ 4; & x\ge 4 \end{cases}$$

$|x|-|x-4|>2$

From the graph the solution is the shaded part i.e. $x>3$

4.



(a) $\cos x \le \frac{1}{2}$ $\quad 0\le x\le 2\pi$

$\cos x = \cos\frac{\pi}{3}$

$\therefore x=\frac{\pi}{3}+2k\pi$ or $x=-\frac{\pi}{3}+2k\pi$

for $k=0$, $x=\frac{\pi}{3}$ $\quad$ for $k=1$, $x=\frac{5\pi}{3}$

$\therefore$ The solutions are $x=\frac{\pi}{3}, \frac{5\pi}{3}$ in the domain $0\le x\le 2\pi$.

$\therefore$ From the graph the solution is $\frac{\pi}{3}\le x\le\frac{5\pi}{3}$.

(b) $|\cos x|\le\frac{1}{2}$ $\quad 0\le x\le 2\pi$

$\therefore \cos x=\frac{1}{2}$ or $\cos x=-\frac{1}{2}$

for $\cos x=\frac{1}{2}$, $\therefore \cos x=\cos\frac{\pi}{3}$

$\therefore x=\frac{\pi}{3}+2k\pi$ or $x=-\frac{\pi}{3}+2k\pi$

for $k=0$, $x=\frac{\pi}{3}$ $\quad$ for $k=1$, $x=\frac{5\pi}{3}$

for $\cos x=-\frac{1}{2}$, $\therefore \cos x=\cos\frac{2\pi}{3}$

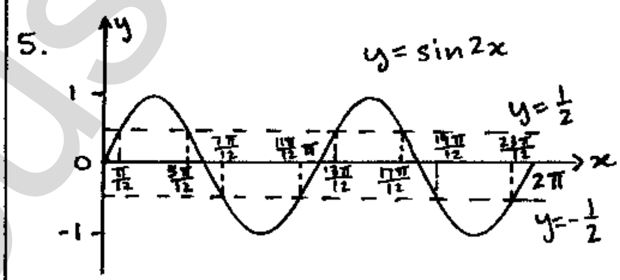
$\therefore x=\frac{2\pi}{3}+2k\pi$ or $x=-\frac{2\pi}{3}+2k\pi$

for $k=0$, $x=\frac{2\pi}{3}$ $\quad$ for $k=1$, $x=\frac{4\pi}{3}$

$\therefore$ The solutions are $x=\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ in the domain $0\le x\le 2\pi$.

$\therefore$ From the graph, the solutions are $\frac{\pi}{3}\le x\le\frac{2\pi}{3}$ and $\frac{4\pi}{3}\le x\le\frac{5\pi}{3}$.

5.



(a) $\sin 2x\ge\frac{1}{2}$ $\quad 0\le x\le 2\pi$

$\therefore \sin 2x=\sin\frac{\pi}{6}$

$\therefore 2x=\frac{\pi}{6}+2k\pi$ or $2x=\frac{5\pi}{6}+2k\pi$

$\therefore x=\frac{\pi}{12}+k\pi$ $\quad$ $x=\frac{5\pi}{12}+k\pi$

for $k=0$, $x=\frac{\pi}{12}$ $\quad$ for $k=0$, $x=\frac{5\pi}{12}$

$k=1$, $x=\frac{13\pi}{12}$ $\quad$ $k=1$, $x=\frac{17\pi}{12}$

$\therefore$ The solutions are $x=\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ in the domain $0\le x\le 2\pi$.

$\therefore$ From the graph, the solutions are $\frac{\pi}{12}\le x\le\frac{5\pi}{12}$ and $\frac{13\pi}{12}\le x\le\frac{17\pi}{12}$.

(b) $|\sin 2x|\ge\frac{1}{2}$ $\quad 0\le x\le 2\pi$

$\therefore \sin 2x=\frac{1}{2}$ or $\sin 2x=-\frac{1}{2}$

for $\sin 2x=\frac{1}{2}$, $\therefore \sin 2x=\sin\frac{\pi}{6}$

$\therefore 2x=\frac{\pi}{6}+2k\pi$ or $2x=\frac{5\pi}{6}+2k\pi$

$\therefore x=\frac{\pi}{12}+k\pi$ $\quad$ $x=\frac{5\pi}{12}+k\pi$

for $k=0$, $x=\frac{\pi}{12}$ $\quad$ for $k=0$, $x=\frac{5\pi}{12}$

for $\sin 2x=-\frac{1}{2}$, $\therefore \sin 2x=\sin(-\frac{\pi}{6})$

$\therefore 2x=-\frac{\pi}{6}+2k\pi$ or $2x=\frac{7\pi}{6}+2k\pi$

$\therefore x = -\frac{\pi}{12} + k\pi$ or $x = \frac{7\pi}{12} + k\pi$

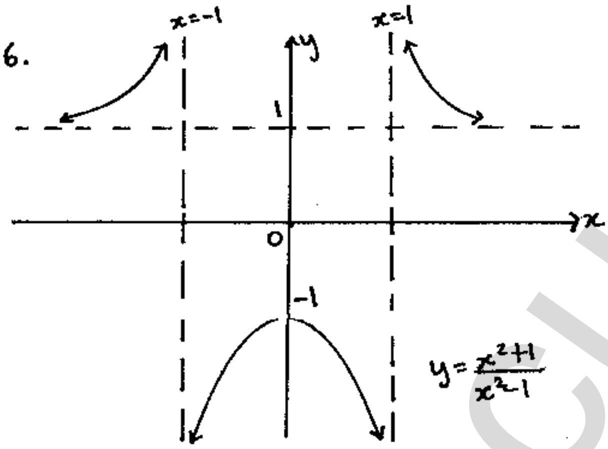
for $k=1$, $x = \frac{11\pi}{12}$ for $k=0$, $x = \frac{7\pi}{12}$

$k=2$, $x = \frac{23\pi}{12}$ $k=1$, $x = \frac{19\pi}{12}$

$\therefore$ The solutions are $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$.

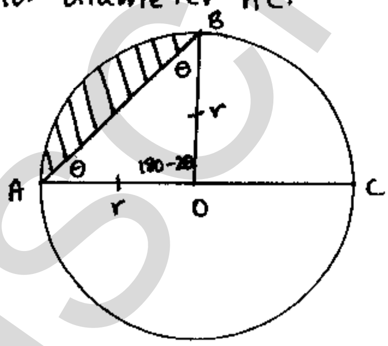
$\therefore$ From the graph the solutions are $\frac{\pi}{12} \le x \le \frac{5\pi}{12}$, $\frac{13\pi}{12} \le x \le \frac{17\pi}{12}$, $\frac{7\pi}{12} \le x \le \frac{11\pi}{12}$, $\frac{19\pi}{12} \le x \le \frac{23\pi}{12}$.

6.



From the graph, the solution is $-1 < x < 1$.

7. Considering a circle O of radius r and diameter AC.



$AO = OB$ (equal radii of circle)

$\therefore \Delta ABO$ is isosceles.

$\angle BAO = \theta$ (data)

$\therefore \angle ABO = \theta$ (base angles of isosceles ΔABO are equal.)

$\therefore \angle AOB = 180 - 2\theta$ (angle sum ΔABO)

A(minor segment) $= A$(Sector) $- A(\Delta ABO)$

$= \frac{1}{2}r^2(\pi - 2\theta) - \frac{1}{2}r^2\sin(\pi - 2\theta)$

$= \frac{1}{2}\pi r^2 - r^2\theta - \frac{1}{2}r^2\sin 2\theta$

(Note: $\sin(\pi - 2\theta) = \sin 2\theta$)

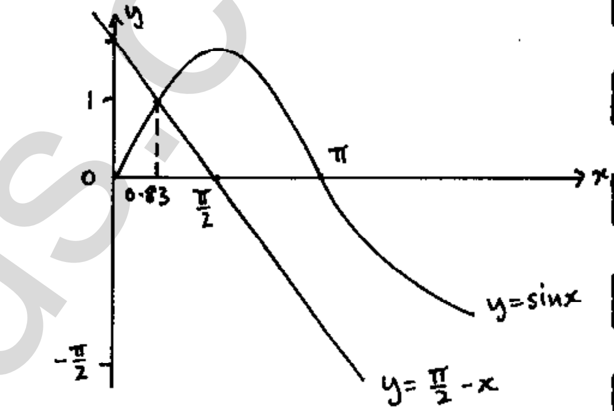
$\therefore \frac{1}{2}\pi r^2 - r^2\theta - \frac{1}{2}r^2\sin 2\theta = \frac{1}{4}\pi r^2$

(dividing by r^2)

$\therefore \frac{\pi}{2} - \theta - \frac{1}{2}\sin 2\theta = \frac{\pi}{4}$

$\frac{\pi}{4} - \theta - \frac{1}{2}\sin 2\theta = 0$ $\therefore \sin 2\theta = \frac{\pi}{2} - 2\theta$

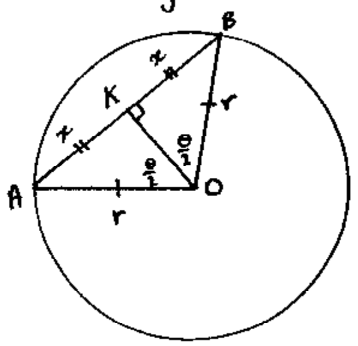
Let $2\theta = x$ $\therefore \sin x = \frac{\pi}{2} - x$



From the graph, $x = 0.83$

$\therefore 2\theta \doteqdot 0.83$ $\therefore \theta \doteqdot 0.4$

8. Considering a circle O of radius r.



Construct KO where KO bisects $\angle AOB$ but since $\angle AOB = \theta$ (data)

$\therefore \angle KOA = \frac{\theta}{2} = \angle KOB$ let $KB = x$ $\therefore KA = x$

P(minor segment) = length arc AB + length chord AB

in ΔKOB, $\sin\left(\frac{\theta}{2}\right) = \frac{x}{r}$ $\therefore x = r\sin\frac{\theta}{2}$

$\therefore AB = 2r\sin\left(\frac{\theta}{2}\right)$

$\therefore$ P(minor segment) $= r\theta + 2r\sin\left(\frac{\theta}{2}\right)$

$\therefore r\theta + 2r\sin\frac{\theta}{2} = \pi r$

(dividing by r)

$\therefore \theta + 2\sin\frac{\theta}{2} = \pi \quad \therefore 2\sin(\frac{\theta}{2}) = \pi - \theta$

Let $\frac{\theta}{2} = x \quad \therefore \sin x = \frac{\pi}{2} - 2x$

From the graph $x = 0.83$

$\therefore \quad \frac{\theta}{2} \doteqdot 0.83 \quad \therefore \theta \doteqdot 1.7$