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NEW SOUTH WALES

Higher School Certificate

Mathematics Extension 2

Exercise 54/67

by James Coroneos*

1. $P(x_1, y_1)$ is a point 'inside' the hyperbola $x^2/9 - y^2/4 = 1$. By drawing a line through P parallel to the x-axis, show that $x_1^2/9 - y_1^2/4 > 1$. On separate sketches, graph the solution sets of

(a) $4x^2 - 9y^2 \geq 36,\ 9x^2 + 16y^2 \leq 144$ **(b)** $4x^2 - 9y^2 \leq 36,\ xy \leq 24$.

2. **(a)** P is the point $(4\cos\theta, 3\sin\theta)$. Show clearly the positions of P and the angles θ when $\theta = \pi/3$ and $\theta = 3\pi/4$. What is the cartesian equation of the locus of P? On the sketch of the positions of P draw this locus.

(b) Q is the point $(4\sec\phi, 3\tan\phi)$. Show clearly the positions of Q and the angles ϕ when $\phi = \pi/3$ and $\phi = 3\pi/4$. What is the cartesian equation of the locus of Q? On the sketch of the positions of Q, draw this locus.

3. For the hyperbola $3x^2 - y^2 = 12$ find the eccentricity, the coordinates of the foci, the equations of the directrices and the asymptotes, and the length of the semi-latus rectum. Sketch the hyperbola.

(a) Find the angle between the asymptotes and the common points of these asymptotes and the auxiliary circle.

(b) Give a parametric representation for the point P on the curve, and taking 4 points on it (one in each quadrant of the plane) give an approximate value of the parameter corresponding to each point.

(c) What would the coefficient of y^2 need to be for the hyperbola to be rectangular? Determine the eccentricity then.

*Other resources by James Coroneos are available. Write to P.O. Box 25, Rose Bay, NSW, 2029, Australia, for a catalogue. TYPESET BY $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX.

4. **(i)** Find the equation of the hyperbola with the same foci as the ellipse $3x^2+4y^2=192$, and whose eccentricity is the reciprocal of that of the ellipse. Find the equations of the asymptotes of the hyperbola.

(ii) For the hyperbola $H:\ x^2/9-y^2/4=1$, determine the eccentricity e and the coordinates of the foci. Find the equation of the ellipse E which has the same foci as H, but with eccentricity $1/e$. What is the ratio of the lengths of the latus recta of H and E?

5. **(i)** Find the equation of the tangent and normal to the hyperbola $x^2-2y^2=1$ at the point $(-3,-2)$ on it.

(ii) The tangent at the point $(2,1)$ on the hyperbola $9x^2-4y^2=32$ meets the asymptotes in A,B. Find the length of AB.

(iii) The normal at the point $P(5/4,3/4)$ on the rectangular hyperbola $x^2-y^2=1$ cuts the transverse, conjugate axes at G,g respectively. Show that $PG=Pg=PO$, where O is the origin.

6. **(i)** Show that the line $x-2y+1=0$ is a tangent to the hyperbola $x^2-6y^2=3$ and find the point of contact.

(ii) Prove that $3x+4y=10$ is a normal to the hyperbola $2x^2-3y^2=5$ and find the foot of the normal.

7. **(i)** Find the equations of the tangents to the hyperbola $2x^2-3y^2=6$ which are parallel to the line $x+y=7$.

(ii) Show that the condition for the line $lx+my+n=0$ to touch the hyperbola $H:\ x^2/a^2-y^2/b^2=1$ is that $n^2=a^2l^2-b^2m^2$. Hence find the equations of the tangents to $x^2-4y^2=12$ which are perpendicular to the line $2x+2y=9$.

8. Find the condition for the line $y=mx+c$ to touch the hyperbola $x^2-2y^2=4$ and the circle $x^2+y^2=1$. Hence determine the equations of the 4 common tangents to the 2 curves.