NEW SOUTH WALES

Higher School Certificate

Mathematics Extension 2

Exercise 51/67

by James Coroneos*

1. Find the equation of the tangent to the following circles at the points stated.
(a) $x^2 + y^2 = 34$; $(5, 3)$ **(b)** $4x^2 + 4y^2 = 25$; $(2, -1\frac{1}{2})$
(c) $x^2 + y^2 - 8x - 10y = 0$; $(0, 0)$ **(d)** $x^2 + y^2 + 4x + 10y - 11 = 0$; $(4, -7)$
(e) $x^2 + y^2 - 3x + 2y + 3 = 0$; $(1, -1)$ **(f)** $4x^2 + 4y^2 - 8x + 4y = 5$; $(\frac{1}{2}, 1)$

2. Prove that the line
(a) $4x - 3y + 25 = 0$ is a tangent to the circle $x^2 + y^2 = 25$
(b) $x + 2y = 12$ touches the circle $x^2 + y^2 - 6x - 4y + 8 = 0$
Find the point of contact in each case.

3. Determine the values of k for which the line
(a) $3x + 4y + k = 0$ touches the circle $x^2 + y^2 = 16$
(b) $2x - ky = 3$ is a tangent to $x^2 + y^2 + 4x - 4y - 5 = 0$

4. **(i)** Show that the tangent at the point $(-2, 3)$ on the circle $x^2 + y^2 = 13$ is also a tangent to the circle $x^2 + y^2 - 10x + 2y - 26 = 0$.
(ii) Prove that the tangents at the points $A(0, 1)$, $B(-1, -4)$ on the circle $x^2 + y^2 + 6x + 2y - 3 = 0$ are inclined at $\pi/4$ to the chord of contact AB.

5. Find the radius and centre of the circle $x^2 + y^2 - 6x + 3y + 5 = 0$. The tangents at the point $A(3, 1)$ and $B(5, -3)$ on the circle meet at P. Prove that AP is equal in length to the diameter of the circle.

*Other resources by James Coroneos are available. Write to P.O. Box 25, Rose Bay, NSW, 2029, Australia, for a catalogue. Typeset by $\mathcal{AMS}$-TEX.

6. Prove that, if a circle passes through $A(4,0)$ and $B(0,2)$, its centre lies on the straight line $2x-y-3=0$. Find the equations of the two circles, each of radius $\sqrt{10}$, passing through A and B, and show that the tangents at A are perpendicular to each other.

7. Find the equations of the tangents to the circle
(i) $x^2+y^2=5$ which are parallel to the line $4x-2y=1$
(ii) $x^2+y^2+2x+4y+1=0$ which are perpendicular to $3x-4y=13$.

8. **(i)** Obtain the values of m for which the line $y=mx$ is a tangent to the circle $x^2+y^2-2x-6y+5=0$. Hence determine the equations of the two tangents to this circle which pass through the origin.
(ii) Find the equations of the tangents to the circle $x^2+y^2+x-3y=0$ from the point $(2,4)$.

9. Prove that $y=mx\pm a\sqrt{1+m^2}$ touches the circle $x^2+y^2=a^2$ for all values of m. Hence find the equations of the tangents drawn from the point $(-4,2)$ to the circle $x^2+y^2=10$.

10. Show that the line $y=m(x-a)+a\sqrt{1+m^2}$ touches the circle $(x-a)^2+y^2=a^2$. Write down the equations of the two tangents to this circle that are parallel to $3x=4y$, and the equations of two tangents which are perpendicular to this line. Choose one mutually perpendicular pair of these tangents, and prove that their point of intersection lies on the circle $x^2+y^2-2ax=a^2$.

11. If l is the length of the tangent from the external point (x_1,y_1) to the circle $x^2+y^2+2gx+2fy+c=0$, show that $l^2=x_1^2+y_1^2+2gx_1+2fy_1+c$.
(i) Calculate the length of each of the tangents drawn from the point
(a) $(5,7)$ to the circle $x^2+y^2=16$
(b) $(1,-1)$ to the circle $x^2+y^2+4x-6y+5=0$
(ii) the lengths of the tangents from a variable point P to the circles $x^2+y^2=16$, $x^2+y^2+4x-6y+5=0$ are equal. Show the locus of P is a straight line and find its equation.
(iii) A point moves so that the lengths of the tangents drawn from it to the two circles $x^2+y^2-4x-8y+11=0$, $x^2+y^2-10x-4y+13=0$ are in the ratio $1:\sqrt{2}$ respectively. Show the locus of the point is a circle, and prove it passes through the points of intersection of the given circles. $\{$*Hint*: *find the equation of the common chord of the circles in pairs.*$\}$

(iv) Prove that the circles $x^2 + y^2 - 6x - 2y = 30$, $x^2 + y^2 - 9x - 3y = 0$ touch each other, and find the coordinates of the point of contact. Show that there is only one common tangent to the two circles and find its equation. Find also the coordinates of a point on the outer circle, such that the length of a tangent from it to the inner circle is equal to the radius of the outer circle.

12. Prove that the locus of the point whose coordinates are $x = 3\cos\alpha + 4$, $y = 3\sin\alpha - 2$ is a circle. Draw the circle, indicating the points which are given by $\alpha = 30°$ and $\alpha = 120°$. Find the equations of the tangents at these points, and show that they intersect at $\{\frac{1}{2}(3\sqrt{2} + 5), \frac{1}{2}(3\sqrt{3} - 1)\}$.

13. Find the equations of the tangents to the two circles $x^2 + y^2 = 2ax$ and $x^2 + y^2 = 2by$ at their points of intersection, and verify that they cut at right angles at both points. (Such curves are called orthogonal)