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NEW SOUTH WALES

Higher School Certificate

Mathematics Extension 2

Exercise 50/67

by James Coroneos*

1. Determine which of the following equations represent real circles, and find the centre and radius of those that do.
(a) $x^2 + y^2 = 12$ **(b)** $x^2 + y^2 + 1 = 0$ **(c)** $x^2 + (y - 3)^2 = 16$
(d) $x^2 + y^2 + 4x - 2y - 11 = 0$ **(e)** $x^2 + y^2 + 6x + 4y + 13 = 0$
(f) $x^2 + y^2 - 5x = 0$ **(g)** $8x^2 + 8y^2 - 12x + 20y - 1 = 0$

2. Find the equation of the circle with centre
(a) the origin O, radius 10 units **(b)** $(-7, 4)$ radius $3\frac{1}{2}$ units
(c) O to pass through $(-3, -4)$ **(d)** $(-3, -4)$ to pass through $(2, -6)$
(e) $(3, 5)$ touching the y-axis **(f)** $(6, -1)$ touching the line $3x - 4y = 12$

3. Find the equation of the circle
(a) with the points $(-1, -7)$ and $(3, -2)$ as the ends of a diameter;
(b) through the three points $(2, 1), (-3, -4), (-6, 5)$;
(c) through $(2, 0)$ and $(1, 7)$, with centre on the line $3x + y = 19$;
(d) through the points $(-3, -1), (4, -2)$ with radius 5 units;
(e) with centre on $x + y = 1$ and touching the x-axis at $(3, 0)$;
(f) touching the coordinate axes and passing through $(-6, 3)$.

4. **(i)** Show that the point $A(-1, 3)$ lies on the circle $x^2 + y^2 - 2x - 4y = 0$, and find the coordinates of the other end of the diameter through A.
(ii) Find the points of intersection of the line $x + y = 4$ and the circle $x^2 + y^2 + 4x - 2y - 20 = 0$.

*Other resources by James Coroneos are available. Write to P.O. Box 25, Rose Bay, NSW, 2029, Australia, for a catalogue. Typeset by $\mathcal{AMS}$-TeX.

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5. The circles $x^2 + y^2 + 2x - 7 = 0, x^2 + y^2 + 4x - 2y - 5 = 0$ intersect at the points A, B. Find the

- **(a)** equation of the circle through A, B and also the point $(4, 1)$;
- **(b)** equation of the chord AB and the coordinates of A, B.

6. The line $3x - 4y = 10$ cuts the circle $x^2 + y^2 = 16$ at P, Q. Find the

- **(a)** length of the chord PQ **(b)** coordinates of the midpoint of PQ
- **(c)** equation of the circle on PQ as diameter.

7.

- **(i)** Find the length of the chord intercepted on the line $x + y = 1$ by the circle $x^2 + y^2 - 4x + 6y + 2 = 0$
- **(ii)** Show that the circles $x^2 + y^2 + 4x - 8y = 29, x^2 + y^2 - 2x = 3$ touch one another.

8. Prove that the circles $x^2 + y^2 - 6x + 8y + 9 = 0, x^2 + y^2 = 1$ touch externally, and find the coordinates of P, the point of contact. The circle $x^2 + y^2 = 1$ and a circle of radius $3\frac{1}{2}$ units touch internally at P; find the equation of this circle.

9. O, A, B are the points $(0, 0), (5, 2), (7, 4)$ respectively. The circles with diameters OA, OB intersect at O, P. Find the coordinates of P and show that the points A, B, P are collinear.

10. The line $x \cos\alpha + y \sin\alpha = p$ $(p > 0)$ intersects the circle $x^2 + y^2 = a^2$ at A, B. Find the length of AB, the coordinates of the midpoint of AB and the equation of the circle on AB as diameter.

11. The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts the

- **(i)** x-axis at P, Q and the y-axis at R, S. Prove analytically that $OP.OQ = OR.OS$, where O is the origin;
- **(ii)** parabola $x = at^2, y = 2at$ at the points A, B, C, D with parameters t_1, t_2, t_3, t_4. Show that $t_1 + t_2 + t_3 + t_4 = 0$. Hence deduce that the centroid (mean centre) of these points lies on the axis of the parabola and that the chords AB, CD are equally inclined to the axis.

12.

- **(i)** If the line $y = mx + c$ cuts off a chord of length $2b$ on the circle $x^2 + y^2 = a^2$, show that $c^2 = (1 + m^2)(a^2 - b^2)$
- **(ii)** Write down the equation of the common chord of the circles $x^2 + y^2 - ax = 0$, $x^2 + y^2 - by = 0$, and show that the circle with this chord as diameter has equation $(a^2 + b^2)(x^2 + y^2) - ab(bx + ay) = 0$

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13. Find the equation of the chord joining the points P, Q with parameters t_1, t_2 on the parabola $x^2 = 4ay$, and determine the condition for PQ to pass through the focus. Write down the equation of the circle on diameter PQ. If this circle cuts the x-axis in A, B show that $OA.OB$ is independent of t_1, t_2 (O is the origin).

14. **(i)** Find the centre and radius of the circles whose parametric equations are **(a)** $x = 7\cos\theta$, $y = 7\sin\theta$ **(b)** $x = 5 + 3\cos\theta$, $y = -1 + 3\sin\theta$

(ii) A is the point $(a, 0)$ and P ia a variable point on the circle $x^2 + y^2 = a^2$. Find the locus of the midpoint of AP.

15. Show that any point on the circumference of the circle $x^2 + y^2 = a^2$ may be represented by the coordinates $(a\cos\alpha, a\sin\alpha)$. If A and B are two points on the circle for which α has the values p, q respectively, show that the equation of the chord PQ is $x(\sin p - \sin q) - y(\cos p - \cos q) = a\sin(p - q)$. If this line passes through a point D whose coordinates are $(0, b)$ prove that $\cos\frac{1}{2}(p - q) : \sin\frac{1}{2}(p + q) = b : a$.

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