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# NEW SOUTH WALES

## Higher School Certificate

# Mathematics Extension 2

## Exercise 5/67

by James Coroneos*

**Operations of ordered pairs.**

- $(a,b)+(c,d)=(a+c,b+d)$
- $(a,b)-(c,d)=(a-c,b-d)$
- $(a,b)\times(c,d)=(ac-bd,bc+ad)$
- $\frac{(a,b)}{(c,d)}=\left(\frac{ac+bd}{c^2+d^2},\frac{bc-ad}{c^2+d^2}\right)$
- $k(a,b)=(ka,kb)$

The following exercises are to be done using only the rules given above for ordered pairs.

**1.** **(a)** $(7,4)+(1,5)$ **(b)** $(7,4)-(1,5)$ **(c)** $(7,4)(1,5)$ **(d)** $(7,4)\div(1,5)$

**2.** **(a)** $(-2,3)+(-5,-1)$ **(b)** $(-2,3)-(-5,-1)$ **(c)** $(-2,3)(-5,-1)$
**(d)** $(-2,3)\div(-5,-1)$

**3.** **(a)** $(3,-4)^2$ **(b)** $(-1,-2)^2$ **(c)** $(2,-3)^3$ **(d)** $(1,0)^4$ **(e)** $(0,1)^4$

**4.** **(a)** $(4,-6)+(-2,5)+(4,-6)(-2,5)$
**(b)** $(8,-11)+2(-3,-4)-(5,-8)$

**5.** **(a)** $(3,2)^2-(2,3)^2$ **(b)** $\{(5,-3)-(3,-2)\}^2$ **(c)** $(-\frac{1}{2},\frac{\sqrt{3}}{2})^3$

**6.** **(a)** $\frac{(-2,-3)(3,4)}{(2,5)}$ **(b)** $\frac{(2,5)}{(-2,-3)(3,4)}$ **(c)** $\frac{(3,-2)(2,1)}{(1,-2)(-4,1)}$

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*Other resources by James Coroneos are available. Write to P.O. Box 25, Rose Bay, NSW, 2029, Australia, for a catalogue. Typeset by $\mathcal{AMS}$-TEX.

**7.** Express answers in both $(x, y)$ and $x + iy$ forms:
**(a)** $\frac{(-1,-5)-(3,-4)}{(-1,-5)(3,-4)}$ **(b)** $\frac{(2,-4)^2}{(-2,-4)^2}$ **(c)** $\frac{(5,-2)(0,1)^2-(-2,4)(1,0)}{(3,-2)}$

**8.** If $z_1 = (1, 0), z_2 = (0, 1), z_3 = (-1, 0), z_4 = (0, -1)$ find the value of
**(i)** $z_1^2$ **(ii)** $z_2^2$ **(iii)** $z_3^2$ **(iv)** $z_4^2$ **(v)** $z_1z_2 + z_3z_4$ **(vi)** $2z_1 - 3z_2 - 5z_3$

**9.** Evaluate the following, expressing answers in both forms $(x, y)$ and $x + iy$.
**(i)** $(a, b)^2$ **(ii)** $(a, -b)^2$ **(iii)** $(a, b)(a, -b)$ **(iv)** $(a, b) - (a, -b)$ **(v)** $(a, b) \div (a, b)$
**(vi)** $(a, b) \div (a, -b)$ **(vii)** $(a, b)^3$ **(viii)** $(a, -b)^2 \div (-a, b)$ **(ix)** $(a, b)^3 + (a, -b)^3$
**(x)** $\{(a, b) + (-a, b)\}^2$

**10.** Find real numbers $x$ and $y$ such that
**(i)** $x(3, 5) - y(4, 2) + (-11, -9) = (0, 0)$ **(ii)** $(x, y)^2 = (-21, 20)$
**(iii)** $(1, 1)(x, y) = (2, 1)$

**11.** Show the number pairs $(2, 5)$ and $(2, -5)$ both satisfy the equation
$x^2 - 4x + (29, 0) = (0, 0)$

**12.** If $z = (3, -2)$ and $w = (-1, -4)$ are number pairs, calculate
**(i)** $z^2$ **(ii)** $w^2$ **(iii)** $\frac{z}{w}$ **(iv)** $\frac{w}{z}$ **(v)** $zw$ **(vi)** $(2z - 3w)^2$

**13.** If $[a, b][c, d] = [0, 0]$, prove that $ac - bd = 0$ and $bc + ad = 0$, and hence that $(a^2 + b^2)(c^2 + d^2) = 0$. Deduce that either $[a, b] = [0, 0]$ or $[c, d] = [0, 0]$.

**14.** Show that if $z = (a, b), w = (c, d)$ then
**(i)** $z^2 - w^2 = (z - w)(z + w)$ **(ii)** $(z + w)^2 = z^2 + 2zw + w^2$
**(iii)** $z^3 - w^3 = (z - w)(z^2 + zw + w^2)$