

① a) By Factor Theorem $P(-2) = 0$

$$\therefore (-2)^3 - 2(-2)^2 + a(-2) + 4 = 0$$

$$-12 - 2a = 0$$

$$a = -6 \quad [1]$$

b) i) $\frac{d}{dx} \ln(\cos 2x) = \frac{-2\sin 2x}{\cos 2x} \quad [1]$

$$= -2\tan 2x$$

ii) $\int_0^{\pi/6} \tan 2x = \left[-\frac{1}{2} \ln(\cos 2x) \right]_0^{\pi/6}$

$$= -\frac{1}{2} \ln(\cos \frac{\pi}{3}) + \frac{1}{2} \ln 1$$

$$= -\frac{1}{2} \ln(\frac{1}{2})$$

$$= \ln \sqrt{2} \text{ or } \frac{1}{2} \ln 2 \quad [2]$$

c) i) $\int \frac{e^{3x}}{2+e^{3x}} dx = \frac{1}{3} \ln(2+e^{3x}) + C$

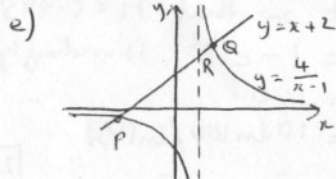
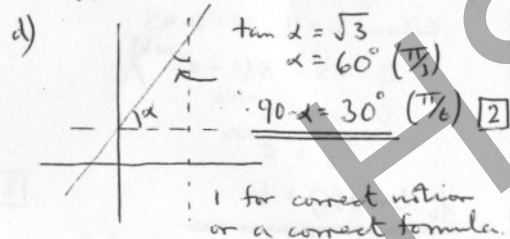
Where C is an undetermined constant. [1]

ii) $\int \frac{dx}{\sqrt{9-4x^2}} = \frac{1}{2} \int \frac{dx}{\sqrt{\frac{9}{4}-x^2}}$

$$= \frac{1}{2} \sin^{-1}\left(\frac{2x}{3}\right) + C \quad [2]$$

$$\left[\text{or } \frac{1}{2} \cos^{-1}\left(\frac{2x}{3}\right) + C \right]$$

Where C is an undetermined constant



From the diagram, $x+2 < \frac{4}{x-1}$

for values of x less than that at P and for values between R (non-inclusive) and Q .

P, Q found by solving $(x+2)(x-1) = 4$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$P \text{ is } (-3, -1) \quad Q \text{ is } (2, 4) \quad [3]$$

Soln. is $\{x: x < -3\} \cup \{x: 1 < x < 2\}$

② a) Let $u = x^{1/2}$

$$\frac{du}{dx} = \frac{1}{2x^{1/2}} = \frac{1}{2u}$$

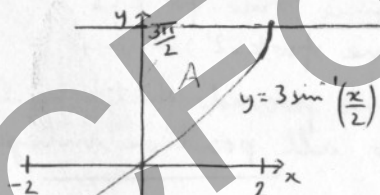
$$\therefore \int_0^{\pi/4} \frac{\sin \sqrt{x}}{\sqrt{x}} dx = \int_0^{\pi/4} 2 \sin u du$$

$$= [-2 \cos u]_0^{\pi/4}$$

$$= \left[-\frac{2}{\sqrt{2}} + 2 \right]$$

$$= 2 - \sqrt{2} \quad [3]$$

b) i)



ii) Area $A = \int_0^{\pi/2} x dy$

$$= \int_0^{\pi/2} 2 \sin \frac{y}{3} dy$$

$$= \left[-6 \cos \frac{y}{3} \right]_0^{\pi/2}$$

$$= 6 \text{ sq units.} \quad [2]$$

2 c) $\alpha + \beta + \gamma = \frac{2}{3}$

$$(\alpha\beta + \beta\gamma + \gamma\alpha = 1)$$

$$\alpha\beta\gamma = \frac{4}{3}$$

$$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = \frac{\alpha + \beta + \gamma}{\alpha\beta\gamma}$$

$$= \frac{2/3}{4/3} = \frac{1}{2} \quad [2]$$

d) i) $3! = 6$ [1]

ii) Initially treat vowels as 1 unit $\rightarrow 3!$

But vowels can be arranged in $3!$ ways.

$$\therefore \text{Total} = 3!3! = 36 \quad [1]$$

iii) Both answers will be divided by 2. [1]

③ a) $\cos 2x - \cos x = 0$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

$$\cos x = -\frac{1}{2} \text{ or } \cos x = 1$$

But $\cos \frac{\pi}{3} = \frac{1}{2}$ and $\cos 0 = 1$

$$\therefore x = 2n\pi \pm \frac{2\pi}{3} \text{ or } 2m\pi \quad [3]$$

When $n = \dots, -2, -1, 0, 1, 2, \dots$
 $m = \dots, -2, -1, 0, 1, 2, \dots$

b) (i) Tangent at P is $y = tx - at^2$
 $\therefore$ Tangent at Q is $y = ux - au^2$

Solve these equations

$$\text{Subtract } 0 = x(t-u) - a(t^2-u^2)$$

$$a(t+u) = x$$

Sub into first equation

$$y = at(t+u) - at^2$$

$$y = atu \quad [2]$$

ii) Referring to part 1,

$$u+t = 1$$

$$ut = -6$$

Solving, $u = 3, t = -2$

(or vice-versa)

$\therefore P$ is $(-4a, 4a)$

and Q is $(6a, 9a)$

R is $(a, -6a)$

$$PQ = \sqrt{25a^2 + 100a^2} = a5\sqrt{5}$$

$$PR = \sqrt{25a^2 + 100a^2} = a5\sqrt{5}$$

$\therefore \Delta PQR$ is isosceles [3]

c) i) $(5+2x)^{12} = 5^{12} + {}^{12}C_1 5^{11} (2x)^1 + \dots$

$$= \sum_{k=0}^{12} a_k x^k \quad [2]$$

where $a_k = {}^{12}C_k 2^k 5^{12-k}$

ii) $\frac{a_{k+1}}{a_k} = \frac{{}^{12}C_{k+1} 2^{k+1} 5^{12-k-1}}{{}^{12}C_k 2^k 5^{12-k}}$

$$= \frac{12!}{(k+1)!(12-(k+1))!} \times 2$$

$$= \frac{12!}{k!(12-k)!} \times 5$$

$$= \frac{k!(12-k)! \times 2}{(k+1)!(12-(k+1))! \times 5}$$

$$= \frac{2(12-k)}{5(k+1)}$$

$$= \frac{24-2k}{5k+5} \quad [2]$$

$$t = 10 \ln 100 / \ln(5/3)$$

Ans) 1) B will follow path

$$y = -\frac{g}{2V^2} \sec^2 \beta + x \tan \beta$$

$$\text{Total Prob} = 0.036 + 0.115 + 0.074 = 0.225$$

Collision will occur when y values same and x values same.

$$-\frac{g}{2V^2} \sec^2 \beta + x \tan \beta = -\frac{g}{2V^2} \sec^2 \alpha + x \tan \alpha$$

$$\frac{g}{2V^2} (\sec^2 \alpha - \sec^2 \beta) = x (\tan \alpha - \tan \beta)$$

Neglect the zero at $x=0$ (start point)

$$x = \frac{2V^2 (\tan \alpha - \tan \beta)}{g (\sec^2 \alpha - \sec^2 \beta)}$$

$$= \frac{2V^2 (\tan \alpha - \tan \beta)}{g (\tan^2 \alpha - \tan^2 \beta)} (\sec^2 \alpha - \sec^2 \beta)$$

$$= \frac{2V^2}{g (\tan \alpha + \tan \beta)}$$

$$= \frac{2V^2 \cos \alpha \cos \beta}{g (2 \sin \alpha \cos \beta)} \approx \frac{2V^2 \cos \alpha \cos \beta}{g \sin (\alpha + \beta)}$$

ii) For particle B, $x_B = V(t-T) \cos \beta$ (t measured from when A fired)

Collision occurs when both x values are equal, at some time t, to the form from part (ii)

$$V \cos \alpha = \frac{2V^2 \cos \alpha \cos \beta}{g \sin (\alpha + \beta)} \Rightarrow t = \frac{2V \cos \beta}{g \sin (\alpha + \beta)}$$

$$\text{So } V(t-T) \cos \beta = \frac{2V^2 \cos \alpha \cos \beta}{g \sin (\alpha + \beta)} \Rightarrow t - T = \frac{2V \cos \alpha}{g \sin (\alpha + \beta)}$$

$$\therefore T = \left\{ t - (t-T) \right\} = \frac{2V (\cos \beta - \cos \alpha)}{g \sin (\alpha + \beta)}$$

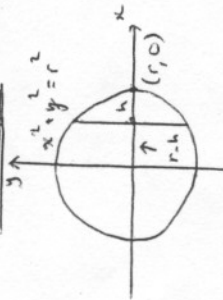
(7) a) This can be achieved in three ways, the individual probabilities of which must be added.

$$\text{Prob} (2 \text{ from } A, 0 \text{ from } B) = {}^1C_2 (0.05)^2 (0.95)^0$$

$$\text{Prob} (1 \text{ from } A, 1 \text{ from } B) = {}^1C_1 (0.05) (0.95) {}^1C_1 (0.05) (0.95)$$

$$\text{Prob} (0 \text{ from } A, 2 \text{ from } B) = {}^1C_0 (0.05)^0 (0.95)^2$$

b) i)



$$V dC = \pi \int y^2 dx$$

$$= \pi \int_{r-h}^r x^2 dx = \pi \left[r^2 x - \frac{x^3}{3} \right]_{r-h}^r = \pi \left(\frac{2r^3}{3} - \left\{ r^2(r-h) - \frac{(r-h)^3}{3} \right\} \right) = \frac{\pi h^2}{3} (3r-h)$$

(7) b) ii)

$$CT = \sqrt{64-28} = 2\sqrt{10} = 6.32$$

Total height = 8+6+16 = 30 cm



$$V = \frac{8\pi h^2}{3} - \frac{\pi h^3}{3}$$

$$\frac{dV}{dt} = 16\pi h - \pi h^2 \frac{dh}{dt}$$

$$\text{But } \frac{dV}{dt} = 100$$

$$\therefore \frac{dh}{dt} = \frac{100}{\pi h (16-h)} \text{ cm/sec}$$

$$\text{When } h=14, \frac{dh}{dt} = \frac{100}{2.8\pi} = 1.14 \text{ cm/sec}$$

At this rate it will take $\frac{16}{1.14} \text{ sec} \approx 14 \text{ seconds}$