



CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NSW

Centre Number									
Student Number									

2011
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics Extension 1

Morning Session
Friday, 12 August 2011

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this page
- All necessary working should be shown in every question

Total marks – 84

- Attempt Questions 1–7
- All questions are of equal value

Disclaimer

Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

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STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Total marks – 84
Attempt Questions 1–7
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Solve $4^{x-3} = 27$. Give your answer correct to two decimal places. 2

(b) Evaluate $\int_0^1 \frac{1}{\sqrt{2-x^2}} dx$. 2

(c) How many arrangements of the letters of the word AUSTRALIA are possible? 2

(d) The graphs of the line $x+2y-1=0$ and the curve $y=1-x^3$ intersect at $(1, 0)$.
 Find the size of the acute angle between the line and the tangent to the curve at the point of intersection. 3

(e) Use the substitution $u=1+2x$, to find $\int \frac{x}{\sqrt{1+2x}} dx$. 3

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\int_0^{\frac{\pi}{6}} \sin^2 3x dx$. 3

(b) The polynomial $P(x)$ is given by $P(x) = 2x^3 + 7x^2 - 46x + 21$.

(i) Show that $P(x)$ is divisible by $(x-3)$. 1

(ii) Hence, fully factorise $P(x)$. 2

(c) A school swim squad consists of six girls and four boys. A team consisting of four of these students is to be chosen.

(i) In how many ways can the team be chosen? 1

(ii) Among the squad are a brother and sister.
 A team of two boys and two girls is chosen at random. What is the probability that it contains the brother and sister? 2

(d) The polynomial equation $2x^3 - 4x^2 + 5x - 1 = 0$ has 3 roots α, β and γ .

(i) Find $\alpha\beta + \beta\gamma + \alpha\gamma$. 1

(ii) Find $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma}$. 2

Question 3 (12 marks) Use a SEPARATE writing booklet.

- (a) State the domain and range of $y = 3 \cos^{-1} 2x$. 2

- (b) A pot of soup is to be refrigerated overnight. It needs to be cooled from its temperature of 100°C before it can be placed in the fridge.

The pot of soup is cooled in a sink full of cold water kept at a temperature of 5°C . The temperature of the pot of soup, $T^\circ\text{C}$, after t minutes in the sink satisfies the equation

$$\frac{dT}{dt} = -k(T-5).$$

- (i) Show that $T = 5 + 95e^{-kt}$ satisfies both this equation and the initial condition. 2

- (ii) After 10 minutes in the sink, the temperature of the pot of soup reduces from 100°C to 60°C . It must be cooled to a temperature of 20°C before it can be placed in the fridge. 3

How long (to the nearest minute) must the pot of soup remain in the sink full of cold water?

- (c) (i) Find the vertical and horizontal asymptotes of the hyperbola $y = \frac{x-1}{2x-3}$ and hence sketch the graph of $y = \frac{x-1}{2x-3}$. 3

- (ii) Hence, or otherwise, find the values of x for which $\frac{x-1}{2x-3} \geq -1$. 2

Question 4 (12 marks) Use a SEPARATE writing booklet.

- (a) Use mathematical induction to prove that $\sum_{r=1}^n (r^2+1)r! = n(n+1)!$ 3

- (b) Let $g(x) = 2x^3 + x + 4$.

- (i) Show that $g(x) = 0$ has a root between the integers -1 and -2 . 1

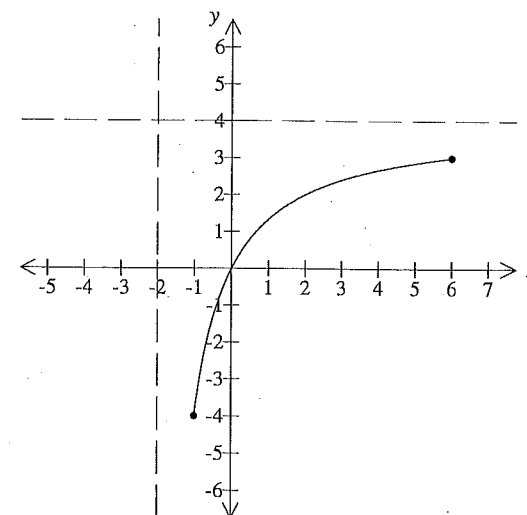
- (ii) Taking $x = -1.5$ as the first approximation to this root, use one application of Newton's method to obtain a better approximation for this root. 2

Give this approximation correct to 2 significant figures.

- (iii) Explain why the function $y = g(x)$ has only one x -intercept. 1

- (c) The diagram below shows a sketch of the graph of $y = f(x)$, where

$$f(x) = \frac{4x}{x+2} \text{ for } -1 \leq x \leq 6.$$



- (i) Copy or trace this diagram into your writing booklet. 2

On the same set of axes, sketch the graph of the inverse function, $y = f^{-1}(x)$.

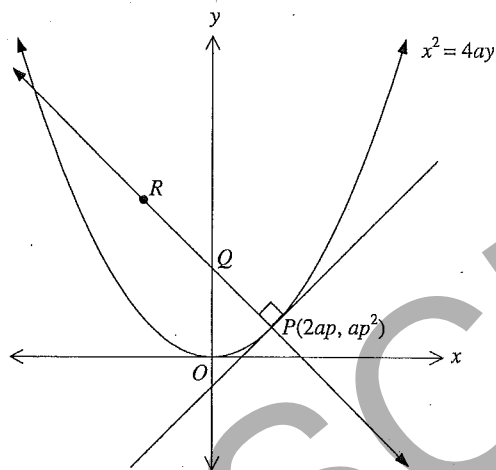
Clearly show any points of intersection with $y = f(x)$.

- (ii) Find an expression for $f^{-1}(x)$ in terms of x . 3

Question 5 (12 marks) Use a SEPARATE writing booklet.

- (a) Find the term independent of x in the expansion of $\left(x^2 - \frac{1}{2x}\right)^{12}$. 3
- (b) The velocity, $v \text{ ms}^{-1}$, of a particle moving in simple harmonic motion along the x -axis is given by $v^2 = 8 - 2x - x^2$, where x is in metres.
- Between which two points is the particle oscillating? 2
 - Find the centre of the motion. 1
 - Find the maximum speed. 1
 - Find an expression for the acceleration of the particle in terms of x . 1

(c)



The diagram shows a variable point $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$.

The normal to the parabola at P intersects the y -axis at Q . The point Q is the midpoint of PR .

The equation of the normal is $x + py - 2ap - ap^3 = 0$. (Do NOT prove this.)

- Find the coordinates of the point Q . 1
- The locus of the point R is a parabola. 3
Find the equation of this parabola in Cartesian form and state its vertex.

Question 6 (12 marks) Use a SEPARATE writing booklet.

- Express $\cos x - \sin x$ in the form $A \cos(x + \alpha)$, where $0 \leq \alpha \leq \frac{\pi}{2}$. 2
 - Hence, or otherwise, solve $\cos x - \sin x = 1$ for $0 \leq x \leq 2\pi$. 2
- In the binomial expansion of $\left(1 + \frac{x}{k}\right)^n$, the coefficient of x^3 is twice the coefficient of x^2 . 2
Prove that $n = 6k + 2$.
- On the way to work, Mathilda passes through a particular intersection. She notices that the traffic lights are red for 15 seconds, amber for 5 seconds and green for 30 seconds. 2
Mathilda passes through this intersection a total of five times in a week.
What is the probability that she is stopped by a red light on exactly one occasion?

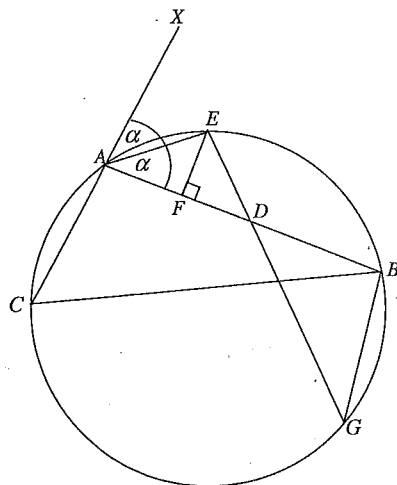
Question 6 continues on page 9

Question 6 (continued)

- (d) The diagram shows a circle which passes through the vertices A , B , and C of a triangle. The point E lies on the circle with AE bisecting the exterior angle, $\angle BAX$, of the triangle such that $\angle XAE = \angle BAE = \alpha$.

The point F lies on AB such that $EF \perp AB$.

The chord EG intersects AB at D such that $DF = AF$ and $\triangle EFD \cong \triangle EFA$.



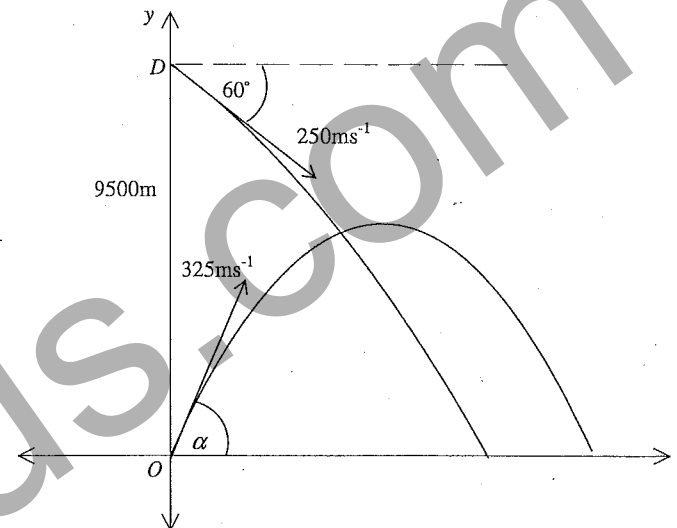
- (i) Explain why $\angle BGE = \angle BAE$.
- (ii) Hence, show that $\angle BDG = \angle BGD = \alpha$.
- (iii) Show that $\triangle CAB \cong \triangle GBA$.

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet.

- (a) During an army exercise, a surface to air missile is launched from the point O in order to intercept a dummy bomb that is released from a point D .

The point D is 9500 metres directly above O .



The dummy bomb is released at an angle of 60° below the horizontal, with a velocity of 250 ms^{-1} . It can be shown that the equations of motion of the dummy bomb are:

$$x_D = 125t \quad \text{and} \quad y_D = 9500 - 125\sqrt{3}t - 5t^2. \quad (\text{Do NOT prove this.})$$

- (i) Calculate how long it would take the dummy bomb to reach the ground (correct to the nearest second) and where it would strike the ground (correct to the nearest metre).

Question 7 continues on page 11

Question 7 (continued)

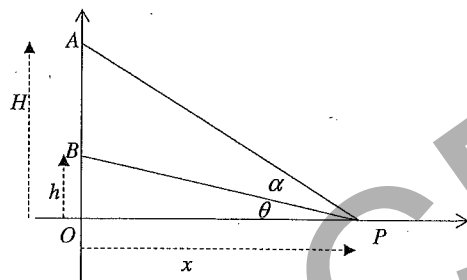
The missile is launched at the same time as the dummy bomb is released. It is launched with an initial velocity of 325 ms^{-1} and its angle of projection *above* the horizontal is α .

The equations of motion of the missile are:

$$x_M = 325t \cos \alpha \text{ and } y_M = 325t \sin \alpha - 5t^2. \text{ (Do NOT prove this.)}$$

- (ii) Show that in order for the missile to intercept the dummy bomb it must be launched with an angle of projection $\alpha = \cos^{-1}\left(\frac{5}{13}\right)$. 1
- (iii) How high above the ground, correct to the nearest metre, does the collision occur? 3
- (b) The diagram shows the point P on the horizontal axis, a variable distance x from the origin O .

The points A and B are fixed points on the vertical axis, with distances H and h respectively, from the origin O .



Let $\angle BPO = \theta$ and $\angle APB = \alpha$.

- (i) Show that $\alpha = \tan^{-1}\left(\frac{H}{x}\right) - \tan^{-1}\left(\frac{h}{x}\right)$. 1
- (ii) Show that α is a maximum when $x^2 = Hh$. 3
(There is no need to justify the maximum.)
- (iii) Using the expansion of $\tan(A - B)$, or otherwise, show that the maximum value of α occurs when $\tan \alpha = \frac{H - h}{2\sqrt{Hh}}$. 2

End of paper



CATHOLIC SECONDARY SCHOOLS ASSOCIATION OF NSW
2011 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS EXTENSION 1

Question 1 (12 marks)

(a) (2 marks)

Outcomes assessed: H3

Targeted Performance Bands: E2

Criteria	Marks
• Correct answer (no penalty for incorrect rounding)	2
• Substantial progress towards solution	1

Sample Answer:

$$4^{x-3} = 27$$

$$(x-3) \ln 4 = \ln 27$$

$$x-3 = \frac{\ln 27}{\ln 4} = 2.377..$$

$$x = 5.38(2dp)$$

(b) (2 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E2

Criteria	Marks
• Correct answer	2
• Chooses correct inverse trig function	1

Sample Answer:

$$\int_0^1 \frac{1}{\sqrt{2-x^2}} dx = \left[\sin^{-1} \frac{x}{\sqrt{2}} \right]_0^1$$

$$= \left[\sin^{-1} \frac{1}{\sqrt{2}} - \sin^{-1} 0 \right] = \frac{\pi}{4}$$

(c) (2 marks)

Outcomes assessed: PE3

Targeted Performance Bands: E2

Criteria	Marks
• Correct solution (no need to simplify)	2
• Finds correct number of permutations with no repetition	1

Sample Answer:

$$\text{Arrangements of the letters of the word AUSTRALIA} = \frac{9!}{3!} = 60480$$

(d) (3 marks)

Outcomes assessed: PE2, HE7

Targeted Performance Bands: E2-E3

Criteria	Marks
• Correct answer	3
• Progress towards finding the angle between the line and the curve	2
• Correctly determines the gradient of the curve at the given point	1

Sample Answer:

$$x+2y-1=0 \rightarrow m_1 = -\frac{1}{2}$$

$$y = 1-x^3 \rightarrow \frac{dy}{dx} = -3x^2$$

$$\therefore m_2 = -3(1)^2 = -3 \text{ at } (1,0)$$

$$\tan \theta = \left| \frac{-3 - \left(-\frac{1}{2}\right)}{1 + -3 \times \frac{-1}{2}} \right| = \left| \frac{-2\frac{1}{2}}{2\frac{1}{2}} \right| = 1$$

$$\theta = 45^\circ$$

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(e) (3 marks)

Outcomes assessed: HE6

Targeted Performance Bands: E2

Criteria	Marks
• Correct solution (+c not essential)	3
• Finds the correct primitive	2
• Correctly uses substitution to rewrite the integral	1

Sample Answer:

$$u = 1 + 2x$$

$$\frac{du}{dx} = 2$$

$$dx = \frac{1}{2} du$$

$$\begin{aligned}\int \frac{x}{\sqrt{1+2x}} dx &= \frac{1}{2} \int \frac{u-1}{\sqrt{u}} \times \frac{1}{2} du \\ &= \frac{1}{4} \int u^{\frac{1}{2}} - u^{-\frac{1}{2}} du \\ &= \frac{1}{4} \left(\frac{2}{3} u^{\frac{3}{2}} - 2u^{\frac{1}{2}} \right) \\ &= \frac{1}{6} (1+2x)^{\frac{3}{2}} - \frac{1}{2} (1+2x)^{\frac{1}{2}} + c\end{aligned}$$

Question 2 (12 marks)

(a) (i) (3 marks)

Outcomes assessed: HE6

Targeted Performance Bands: E2–E3

Criteria	Marks
• Gives correct answer	3
• Correctly evaluates the integral apart from a minor error	2
• Uses the substitution $\sin^2 3x = \frac{1}{2}(1 - \cos 6x)$ OR • Correctly integrates a non-trivial expression used to replace $\sin^2 3x$	1

Sample Answer:

$$\begin{aligned}\int_0^{\frac{\pi}{6}} \sin^2 3x dx &= \frac{1}{2} \int_0^{\frac{\pi}{6}} (1 - \cos 6x) dx \\ &= \frac{1}{2} \left[x - \frac{1}{6} \sin 6x \right]_0^{\frac{\pi}{6}} \\ &= \frac{1}{2} \left[\left(\frac{\pi}{6} - \frac{1}{6} \sin \pi \right) - \left(0 - \frac{1}{6} \sin 0 \right) \right] = \frac{\pi}{12}\end{aligned}$$

(b) (i) (1 mark)

Outcomes assessed: PE3

Targeted Performance Bands: E2

Criteria	Mark
• Correct solution	1

Sample Answer:

$$P(x) = 2x^3 + 7x^2 - 46x + 21$$

$$P(3) = 2(3)^3 + 7(3)^2 - 46(3) + 21 = 0$$

$$\therefore x-3 \text{ is a factor}$$

(b) (ii) (2 marks)

Outcomes assessed: PE3

Targeted Performance Bands: E2

Criteria	Marks
• Correct factorisation	2
• Finds correct quadratic factor or equivalent progress	1

Sample Answer:

$$P(x) = 2x^3 + 7x^2 - 46x + 21$$

$$= (x-3)(2x^2 + 13x - 7)$$

$$= (x-3)(x+7)(2x-1)$$

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(c) (i) (1 mark)

Outcomes assessed: PE3

Targeted Performance Bands: E3

Criteria	Mark
• Correct answer	1

Sample Answer:

$${}^{10}C_4 = 210$$

(c) (ii) (2 marks)

Outcomes assessed: PE3, HE5

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	2
• Finds the number of teams including the brother and sister, or equivalent progress	1

Sample Answer:

$$\frac{{}^5C_1 {}^3C_1}{{}^6C_2 {}^4C_2} = \frac{1}{6}$$

(d) (i) (1 mark)

Outcomes assessed: PE3

Targeted Performance Bands: E2

Criteria	Mark
• Correct answer	1

Sample Answer:

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = \frac{5}{2}$$

(d) (ii) (2 marks)

Outcomes assessed: PE3

Targeted Performance Bands: E2

Criteria	Marks
• Correct answer	2
• Significant progress towards the correct answer	1

Sample Answer:

$$\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma} = \frac{2(\alpha\beta + \beta\gamma + \gamma\alpha)}{\alpha\beta\gamma} = \frac{2\left(\frac{5}{2}\right)}{\frac{1}{2}} = 10$$

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Question 3 (12 marks)

(a) (2 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3

Criteria	Marks
• Correct answers	2
• Either correct domain or correct range	1

Sample Answer:

$$y = 3 \cos^{-1} 2x$$

$$\text{Domain: } -1 \leq 2x \leq 1$$

$$\text{Range: } 0 \leq \cos^{-1} 2x \leq \pi$$

$$-\frac{1}{2} \leq x \leq \frac{1}{2}$$

$$0 \leq \cos^{-1} 2x \leq 3\pi$$

$$\therefore 0 \leq y \leq 3\pi$$

(b) (i) (2 marks)

Outcomes assessed: HE3, HE7

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	2
• Shows that the given equation satisfies the initial condition or equivalent merit	1

Sample Answer:

$$T = 5 + 95e^{-kt} \rightarrow 95e^{-kt} = T - 5$$

$$t = 0, T = 5 + 95e^0 = 5 + 95 = 100$$

$$\frac{dT}{dt} = -95ke^{-kt} = -k(T - 5)$$

$\therefore$ satisfies initial condition

(b) (ii) (3 marks)

Outcomes assessed: HE3, HE7

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	3
• Makes significant progress towards finding t	2
• Finds k or equivalent merit	1

Sample Answer:

$$t = 10, T = 60 \rightarrow$$

$$60 = 5 + 95e^{-10k}$$

$$T = 20, t = ?$$

$$20 = 5 + 95e^{-\left[\frac{1}{10} \ln\left(\frac{55}{95}\right)\right]t}$$

$$\frac{55}{95} = e^{-10k}$$

$$\ln\left(\frac{15}{95}\right) = \frac{1}{10} \ln\left(\frac{55}{95}\right)t$$

$$k = -\frac{1}{10} \ln\left(\frac{55}{95}\right)$$

$$t = \ln\left(\frac{15}{95}\right) \div \frac{1}{10} \ln\left(\frac{55}{95}\right)$$

$$\therefore t = 33.7727.. = 34 \text{ minutes (nearest minute)}$$

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(c) (i) (3 marks)

Outcomes assessed: PE5, HE7

Targeted Performance Bands: E2–E3

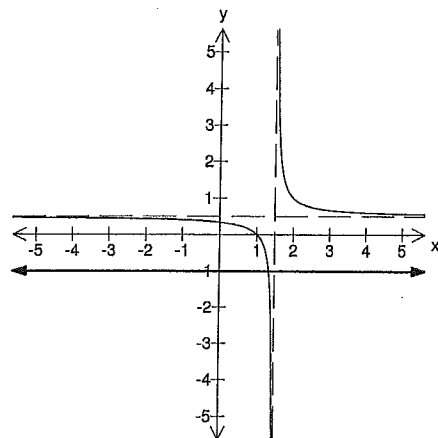
Criteria	Marks
• Correct solution	3
• Both asymptotes correct, or equivalent merit	2
• Correct horizontal or vertical asymptote	1

Sample Answer:

$$y = \frac{x-1}{2x-3} \quad \text{Vertical asymptote: } x = \frac{3}{2} \quad \text{Horizontal asymptote: } y = \frac{1}{2}$$

$$\lim_{x \rightarrow \infty} \frac{x-1}{2x-3} = \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{x}}{2 - \frac{3}{x}} = \frac{1}{2}$$

$$\therefore y = \frac{1}{2}$$



(c) (ii) (2 marks)

Outcomes assessed: PE6

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	2
• Finds the point of intersection of $y = -1$ with curve	1

Sample Answer:

$y = -1$ drawn on above graph

$$\frac{x-1}{2x-3} = -1 \rightarrow x-1 = -2x+3 \quad \frac{x-1}{2x-3} \geq -1$$

$$3x = 4 \quad \therefore x \leq \frac{4}{3}, x > \frac{3}{2}$$

$$x = \frac{4}{3}$$

This is where the curve is above the line, $y = -1$.

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Question 4 (12 marks)

(a) (3 marks)

Outcomes assessed: HE2

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	3
• Verifies the case for $n=1$ or equivalent merit and attempts to use the induction assumption with minor errors	2
• Verifies the case for $n=1$ or equivalent merit	1

Sample Answer:

$$\sum_{r=1}^n (r^2+1)r! = n(n+1)!$$

$$(1^2+1)1! + (2^2+1)2! + (3^2+1)3! + \dots + (n^2+1)n! = n(n+1)!$$

when $n=1$, $LHS = (1^2+1)1! = 2$

$$RHS = 1(1+1)! = 2 \quad \therefore \text{true for } n=1$$

Assume true for $n=k$: $(1^2+1)1! + (2^2+1)2! + (3^2+1)3! + \dots + (k^2+1)k! = k(k+1)!$

$$S_k = k(k+1)!$$

when $n=k+1$, $S_{k+1} = S_k + ((k+1)^2+1)(k+1)!$

$$= k(k+1)! + (k^2+2k+1+1)(k+1)!$$

$$= (k+1)! [k + k^2 + 2k + 1 + 1]$$

$$= (k+1)! [k^2 + 3k + 2]$$

$$= (k+1)! [(k+2)(k+1)] = (k+2)!(k+1)$$

$\therefore$ true for $n=k+1$,

and since true for $n=1$, then by induction true for all integral values of $n \geq 1$.

(b) (i) (1 mark)

Outcomes assessed: HE7

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	1

Sample Answer:

$$g(x) = 2x^3 + x + 4$$

$$g(-1) = 2(-1)^3 + (-1) + 4 = 1 > 0$$

$$g(-2) = 2(-2)^3 + (-2) + 4 = -14 < 0$$

Since there is a sign change and the curve is continuous, then there is a root between $x = -1$ and $x = -2$

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(b) (ii) (2marks)

Outcomes assessed: HE7

Targeted Performance Bands: E3

Criteria	Marks
• Correct solution	2
• Significant progress towards correct approximation	1

Sample Answer:

$$g(x) = 2x^3 + x + 4 \quad \therefore g'(x) = 6x^2 + 1$$

$$\text{For } x_1 = -1.5 \quad g(x_1) = 2(-1.5)^3 + (-1.5) + 4 = -4.25.$$

$$g'(x_1) = 6(-1.5)^2 + 1 = 14.5.$$

$$\begin{aligned} \therefore x_2 &= x_1 - \frac{g(x_1)}{g'(x_1)} \\ &= -1.5 - \frac{-4.25}{14.5} \\ &= -1.206.. \\ &= -1.2 \quad (2 \text{ s.f.}) \end{aligned}$$

(b) (iii) (1 mark)

Outcomes assessed: HE7

Targeted Performance Bands: E3

Criteria	Mark
• Correct solution	1

Sample Answer:

Since $g'(x) = 6x^2 + 1 > 0$ for all x then $g(x)$ is always increasing and therefore can only intersect the x -axis once.

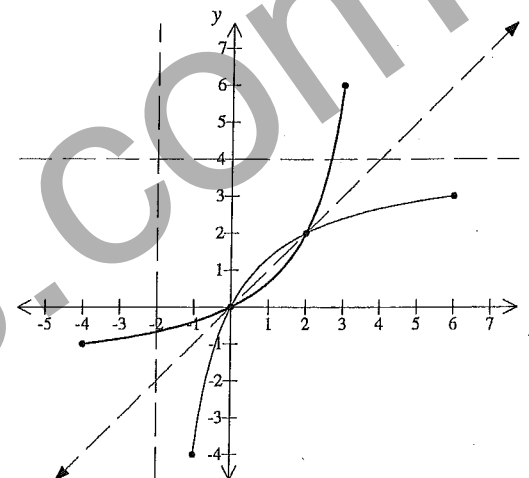
(c) (i) (2 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct graph with clearly identifiable points of intersection (no penalty for incorrect domain)	2
• Attempts to reflect the graph in the line $y = x$	1

Sample Answer:



(c) (ii) (3 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3-E4

Criteria	Marks
• Correct expression for $f^{-1}(x)$ with correct domain	3
• Correct expression for $f^{-1}(x)$	2
• Obtains $x = \frac{4y}{y+2}$, or equivalent merit	1

Sample Answer:

$$x = \frac{4y}{y+2}$$

$$xy + 2x = 4y$$

$$y(4-x) = 2x$$

$$y = \frac{2x}{4-x}, \quad -4 \leq x \leq 3$$

Question 5 (12 marks)

(a) (3 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Marks
• Finds the correct term	3
• Establishes the value of n which gives the correct term	2
• Writes a correct expression for T_{n+1}	1

Sample Answer:

$$T_{n+1} = \binom{12}{n} (x^2)^{12-n} \left(\frac{-1}{2x}\right)^n = \binom{12}{n} x^{24-2n} \left(\frac{-1}{2}\right)^n x^{-n}$$

$$= \binom{12}{n} \left(\frac{-1}{2}\right)^n x^{24-3n}$$

For the term independent of x : $24 - 3n = 0 \therefore n = 8$,

$$\therefore \text{The term independent of } x \text{ is } \binom{12}{8} \left(\frac{-1}{2}\right)^8 = \frac{495}{256}$$

(b) (i) (2 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	2
• Attempts to find when $v^2 = 0$ or equivalent merit	1

Sample Answer:

$$8 - 2x - x^2 = 0$$

$$(4+x)(2-x) = 0 \quad \text{The particle is oscillating between } x = -4 \text{ and } x = 2$$

$$x = -4 \text{ or } x = 2$$

(b) (ii) (1 mark)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Mark
• Correct answer	1

Sample Answer:

The centre of motion is $x = -1$.

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(b) (iii) (1 mark)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Mark
• Correct answer	1

Sample Answer:

The maximum speed occurs at the centre of motion.

$$v^2 = 8 - 2(-1) - (-1)^2 = 9 \quad \therefore \text{Max speed } v = 3$$

(b) (iv) (1 mark)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Mark
• Correct solution	1

Sample Answer:

$$\ddot{x} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right) = \frac{d}{dx} \left(4 - x - \frac{x^2}{2} \right)$$

$$= -1 - x$$

(c) (i) (1 mark)

Outcomes assessed: P4

Targeted Performance Bands: E2

Criteria	Mark
• Correct answer	1

Sample Answer:

$$x + py - 2ap - ap^3 = 0 \rightarrow Q(0, ?) \quad \therefore py - 2ap - ap^3 = 0$$

$$py = 2ap + ap^3$$

$$y = 2a + ap^2 \rightarrow Q(0, 2a + ap^2)$$

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(c) (ii) (3marks)

Outcomes assessed: PE3

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	3
• Finds the Cartesian equation of the locus of R or equivalent merit	2
• Finds coordinates of R or equivalent merit	1

Sample Answer:

$$\begin{aligned} R = (x, y) \rightarrow 0 &= \frac{x+2ap}{2} & 2a+ap^2 &= \frac{y+ap^2}{2} \\ 0 &= x+2ap & 4a+2ap^2 &= y+ap^2 \\ x &= -2ap & y &= 4a+ap^2 \\ \therefore p &= \frac{-x}{2a} & y &= 4a + a\left(\frac{-x}{2a}\right)^2 \\ y &= 4a + \frac{x^2}{4a} & \rightarrow x^2 &= 4a(y-4a) \\ \text{Vertex} &(0, 4a) \end{aligned}$$

Question 6 (12 marks)

(a) (i) (2 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct answer	2
• Correctly obtains A or α , or equivalent progress	1

Sample Answer:

$$\begin{aligned} A \cos(x+\alpha) &= A \cos x \cos \alpha - A \sin x \sin \alpha \\ \therefore \begin{cases} A \cos \alpha = 1 \\ A \sin \alpha = 1 \end{cases} &\left\{ \tan \alpha = 1 \rightarrow \alpha = \frac{\pi}{4} \right. & A &= \sqrt{1^2+1^2} = \sqrt{2} \\ \therefore \cos x - \sin x &= \sqrt{2} \cos\left(x + \frac{\pi}{4}\right) \end{aligned}$$

(a) (ii) (2 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	2
• Obtains one correct result for $x + \alpha$	1

Sample Answer:

$$\begin{aligned} \cos x - \sin x &= 1 \\ \sqrt{2} \cos\left(x + \frac{\pi}{4}\right) &= 1 \\ \cos\left(x + \frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}} & \therefore x + \frac{\pi}{4} &= \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4} \\ & & x &= 0, \frac{3\pi}{2}, 2\pi \end{aligned}$$

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(b) (2 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	2
• Sets up a correct identity, or equivalent merit	1

Sample Answer:

$$\left(1 + \frac{x}{k}\right)^n \rightarrow \text{coefficient of } x^3 = 2 \times \text{coefficient of } x^2$$
$$\binom{n}{3} \left(\frac{1}{k}\right)^3 = 2 \times \binom{n}{2} \left(\frac{1}{k}\right)^2$$
$$\frac{n!}{3!(n-3)!k^3} = \frac{2(n!)}{2!(n-2)!k^2} \rightarrow \frac{n(n-1)(n-2)}{6k^3} = \frac{n(n-1)}{k^2}$$
$$n-2=6k \quad \therefore n=6k+2$$

(c) (2 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct answer	2
• Correctly sets up binomial probability, or equivalent merit	1

Sample Answer:

$$P(\text{red}) = \frac{15}{50} = \frac{3}{10} \quad P(\text{not red}) = \frac{7}{10}$$

$$P(\text{exactly one red in 5 times}) = {}^5C_1 \left(\frac{3}{10}\right) \left(\frac{7}{10}\right)^4 = \frac{7203}{20000} = 0.36015$$

(d) (i) (1 mark)

Outcomes assessed: PE3

Targeted Performance Bands: E2

Criteria	Mark
• Correct reason	1

Sample Answer:

Angles at the circumference subtended by the same arc EB are equal.

(d) (ii) (1 mark)

Outcomes assessed: PE3

Targeted Performance Bands: E2

Criteria	Mark
• Correct solution	1

Sample Answer:

$\angle EAF = \angle EDF = \alpha$ (corresponding angles of congruent triangles $\triangle EFD \cong \triangle EFA$)

$\angle EDF = \angle BDG = \alpha$ (vertically opposite angles)

$\therefore \angle BDG = \alpha = \angle BGD$ (proven in (i))

(d) (iii) (2 marks)

Outcomes assessed: HE5

Targeted Performance Bands: E2–E3

Criteria	Marks
• Correct solution	2
• Significant progress towards correct solution	1

Sample Answer:

$\angle CAB = 180^\circ - 2\alpha$ (adj angles on a straight angle)

$\angle GBA = 180^\circ - 2\alpha$ (angle sum of $\triangle DBG$)

$\angle CAB = \angle GBA$

$\angle ACB = \angle BGA$ (angles subtended by the same arc AB)

$AB = BA$ (common side)

$\therefore \triangle CAB \cong \triangle GBA$ (AAS)

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Question 7 (12 marks)

(a) (i) (2 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	2
• Finds when the dummy bomb hits the ground, or uses incorrect time to find where it strikes the ground	1

Sample Answer:

$$\begin{aligned}
 0 &= 9500 - 125\sqrt{3}t - 5t^2 \\
 t^2 + 25\sqrt{3}t - 1900 &= 0 \\
 \text{Hits the ground } y_D &= 0 \quad t = \frac{-25\sqrt{3} \pm \sqrt{(25\sqrt{3})^2 - 4 \times -1900}}{2} \\
 &= \frac{-25\sqrt{3} \pm \sqrt{9475}}{2} \\
 t &= 27.01917..., -70.3204.. \\
 &\text{Reaches the ground after 27 seconds (nearest second)}
 \end{aligned}$$

$$\text{When } t = 27.01917..., x_D = 125(27.01917...) = 3377.396..$$

Strikes the ground at 3377 m (or 3375 m if $t = 27$ is used)

(a) (ii) (1 mark)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Mark
• Correct solution	1

Sample Answer:

$$\begin{aligned}
 x_D = x_M &\rightarrow 125t = 325t \cos \alpha \\
 \cos \alpha &= \frac{125t}{325t} = \frac{5}{13} \\
 \therefore \alpha &= \cos^{-1}\left(\frac{5}{13}\right)
 \end{aligned}$$

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(a) (iii) (3 marks)

Outcomes assessed: HE3

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	3
• Correctly finds the value of t	2
• Equates $y_D = y_M$ and attempts to find t or correctly finds the value of $\sin \alpha$	1

Sample Answer:

$$\begin{aligned}
 &\text{Collision occurs when} \\
 y_D = y_M &\rightarrow 9500 - 125\sqrt{3}t - 5t^2 = 325t \sin \alpha - 5t^2 \\
 9500 - 125\sqrt{3}t &= 325t \sin \alpha \quad \sin \alpha = \frac{12}{13} \text{ (from (ii))} \\
 9500 - 125\sqrt{3}t &= 325t \left(\frac{12}{13}\right) \\
 9500 &= 300t + 125\sqrt{3}t \\
 t &= \frac{9500}{300 + 125\sqrt{3}} = 18.3928... \\
 y_M &= 325(18.3928...) \frac{12}{13} - 5(18.3928...)^2 \\
 &= 3826.3649.. \\
 &= 3826 \text{ m}
 \end{aligned}$$

(b) (i) (1 mark)

Outcomes assessed: HE4

Targeted Performance Bands: E3–E4

Criteria	Mark
• Correct solution	1

Sample Answer:

$$\begin{aligned}
 \text{In } \triangle APO, \tan(\theta + \alpha) &= \frac{H}{x} \rightarrow \tan^{-1}\left(\frac{H}{x}\right) = \theta + \alpha \\
 \text{In } \triangle BPO, \tan \theta &= \frac{h}{x} \rightarrow \tan^{-1}\left(\frac{h}{x}\right) = \theta \\
 \therefore \alpha &= (\theta + \alpha) - \theta = \tan^{-1}\left(\frac{H}{x}\right) - \tan^{-1}\left(\frac{h}{x}\right)
 \end{aligned}$$

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(b) (ii) (3 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	3
• Finds an expression for $\frac{d\alpha}{dx}$	2
• Correctly differentiates either $\tan^{-1}\left(\frac{H}{x}\right)$ or $\tan^{-1}\left(\frac{h}{x}\right)$	1

Sample Answer:

$$\text{For max } \alpha, \frac{d\alpha}{dx} = 0$$

$$\frac{d\alpha}{dx} = \frac{\frac{-H}{x^2}}{1 + \left(\frac{H}{x}\right)^2} - \frac{\frac{-h}{x^2}}{1 + \left(\frac{h}{x}\right)^2}$$

$$= \frac{-H}{x^2 + H^2} + \frac{h}{x^2 + h^2}$$

$$\therefore \frac{d\alpha}{dx} = 0 \quad \text{when} \quad \frac{H}{x^2 + H^2} = \frac{h}{x^2 + h^2}$$

$$Hx^2 + Hh^2 = hx^2 + H^2h$$

$$x^2(H - h) = Hh(H - h)$$

$$x^2 = Hh \quad (\text{since } H \neq h \text{ for maximum } \alpha)$$

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(b) (iii) (2 marks)

Outcomes assessed: HE4

Targeted Performance Bands: E3–E4

Criteria	Marks
• Correct solution	2
• Applies the expansion of $\tan(A - B)$, or equivalent merit	1

Sample Answer:

$$\alpha \text{ is a max when } x = \sqrt{Hh} \quad (\text{taking } x > 0)$$

$$\alpha = \tan^{-1}\left(\frac{H}{\sqrt{Hh}}\right) - \tan^{-1}\left(\frac{h}{\sqrt{Hh}}\right) \quad (\text{using (i)})$$

$$\therefore \tan \alpha = \tan \left[\tan^{-1}\left(\frac{H}{\sqrt{Hh}}\right) - \tan^{-1}\left(\frac{h}{\sqrt{Hh}}\right) \right]$$

$$= \frac{\frac{H}{\sqrt{Hh}} - \frac{h}{\sqrt{Hh}}}{1 + \frac{H}{\sqrt{Hh}} \times \frac{h}{\sqrt{Hh}}}$$

$$= \frac{\frac{H-h}{\sqrt{Hh}}}{2}$$

$$= \frac{H-h}{2\sqrt{Hh}}$$

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