Extension 1 Mathematics Trial HSC Solutions
2013 St. George Girls High School.

Section 1

Q.1. $\tan 45° = 1$ $\quad \left|\frac{3-m}{1+3m}\right| = 1$

$|1+3m| = |3-m|$

$1+3m = |3-m|$ since $1+3m > 0$ since gradient $m > 0$.

$1+3m = 3-m$ since $m < 3$, $\therefore 3-m > 0$

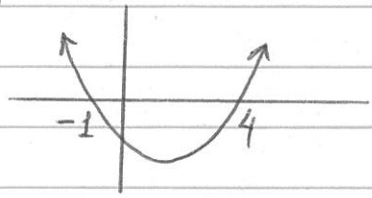
$4m = 2$

$m = \frac{1}{2}$ $\quad \therefore$ B is the correct choice

Q.2. $x^2 - 3x < 4$

$x^2 - 3x - 4 < 0$

$(x-4)(x+1) < 0$



From the graph: $-1 < x < 4$

$\therefore$ A is the correct choice.

Q.3. $\int \frac{5}{2+x^2}\,dx = 5\int \frac{dx}{(\sqrt{2})^2 + x^2} = \frac{5}{\sqrt{2}} \tan^{-1}\frac{x}{\sqrt{2}} + C$

$\therefore$ C is the correct choice.

Q.4 D is the correct choice

Q5. $-1 \le \frac{x}{2} \le 1 \implies -2 < x < 2$

$\therefore$ C is the correct choice

Q.6. $v^2 = -5 + 6x - x^2$

$= (5-x)(x-1)$

at extremes of the displacement $v = 0$

$\therefore v^2 = 0$

$x = 5$ and $x = 1$

$\therefore$ A is the correct choice.

Q.7. $\frac{dy}{dx} = -\sin(\ln x) \times \frac{1}{x} = \frac{-\sin(\ln x)}{x}$

$\frac{d^2y}{dx^2} = \frac{x \times (-\cos(\ln x)) \times \frac{1}{x} - (-\sin(\ln x)) \times 1}{x^2}$

$= \frac{\sin(\ln x) - \cos(\ln x)}{x^2}$

$\therefore$ D is the correct choice.

Q.8. $f'(x) = 1 - 2\cos x$

$f(1.7) = 1.7 - 2\sin 1.7$

$= -0.28332962\ldots$

$f'(1.7) = 1 - 2\cos(1.7)$

$= 1.25768989\ldots$

$x_1 = 1.7 - \frac{f(1.7)}{f'(1.7)} = 1.9252779\ldots$

$\doteqdot 1.93$

$\therefore$ B is the correct choice

Q.9. $T_r = (-1)^r \, {}^{11}C_r (2x)^{11-r} \times \left(\frac{1}{x}\right)^r$

$= (-1)^r \, {}^{11}C_r \, 2^{11-r} x^{11-2r}$

When $r = 4$

$(-1)^r = 1$

${}^{11}C_4 = 330$

$2^{11-4} = 2^7 = 128$

$11 - 2r = 3$

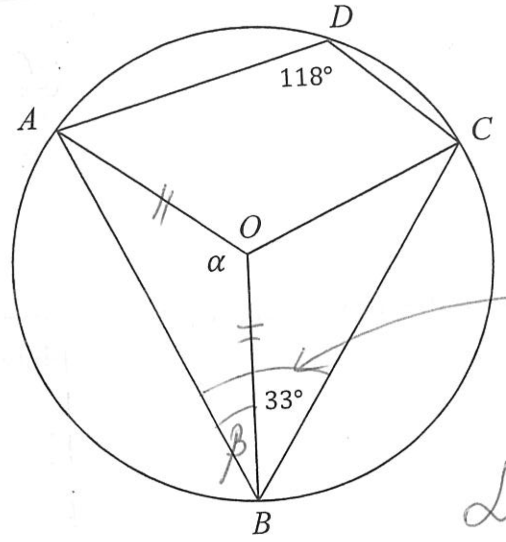
$2r = 8$

$r = 4$

Coeff. of $x^3 = 128 \times 330 = 42240$

$\therefore$ A is the correct choice

Q.10.



$180-118=62$

$\beta = 62-33$
$= 29°$

$\alpha = 180 - 2 \times 29$
$= \underline{\underline{122°}}$

Section II

Q.11.

(a) $\dfrac{4}{5-x} \leq 1$ $\quad \times (5-x)^2$

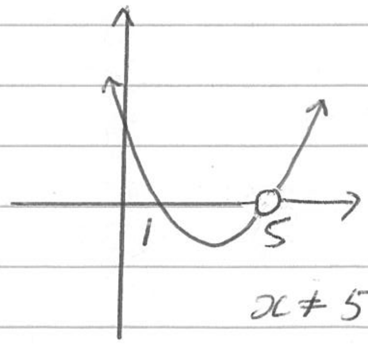
$4(5-x) \leq (5-x)^2 \,,\quad x \neq 5$

$-(5-x)^2 + 4(5-x) \leq 0$

$(5-x)^2 - 4(5-x) \geq 0$

$(5-x)(5-x-4) \geq 0$

$(5-x)(1-x) \geq 0$



From the graph:

$x \leq 1$ or $x > 5$

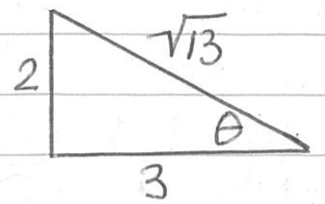
(b) $\dfrac{\sec\theta + \operatorname{cosec}\theta}{\sin\theta + \cos\theta} = \dfrac{\frac{1}{\cos\theta} + \frac{1}{\sin\theta}}{\sin\theta + \cos\theta}$

$= \dfrac{\frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}}{\sin\theta + \cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$

$= \dfrac{1}{\frac{2}{\sqrt{13}} \times \frac{3}{\sqrt{13}}}$

$= \dfrac{13}{6}$ //



(c) $$\int_0^{3/2} \frac{dx}{\sqrt{3^2-x^2}} = \left[\sin^{-1}\frac{x}{3}\right]_0^{3/2}$$

$$= \sin^{-1}\frac{1}{2} - \sin^{-1}0$$

$$= \sin^{-1}\frac{1}{2}$$

$$= \frac{\pi}{6}$$

(d) $\alpha+\beta+\gamma = 1$

$\alpha\beta+\alpha\gamma+\beta\gamma = 4$

$\alpha\beta\gamma = 1$

$$(\alpha+1)(\beta+1)(\gamma+1) = \alpha\beta\gamma + (\alpha\beta+\alpha\gamma+\beta\gamma) + (\alpha+\beta+\gamma) + 1$$

$$= 1+4+1+1$$

$$= 7$$

(e) $k : l = -5 : 2$

$$x = \frac{l \times x_1 + k \times x_2}{k+l} = \frac{2\times(-3) + (-5)\times 2}{-3} = \frac{16}{3}$$

$$y = \frac{l \times y_1 + k \times y_2}{k+l} = \frac{2\times(-1) + (-5)\times 4}{-3} = \frac{22}{3}$$

$\therefore$ P has coordinates $\left(\frac{16}{3}, \frac{22}{3}\right)$

or $\left(5\frac{1}{3}, 7\frac{1}{3}\right)$

(f)

(i) $f'(x) = \dfrac{-2}{\sqrt{2-x^2}} - \dfrac{-2x}{\sqrt{1-(1-x^2)^2}}$

$= \dfrac{2x}{\sqrt{(1-(1-x^2))(1+(1-x^2))}} - \dfrac{2}{\sqrt{2-x^2}}$

$= \dfrac{2x}{\sqrt{x^2(2-x^2)}} - \dfrac{2}{\sqrt{2-x^2}}$

$= \dfrac{2\cancel{x}}{\cancel{x}\sqrt{2-x^2}} - \dfrac{2}{\sqrt{2-x^2}}$

$= \underline{\underline{0}}$

(ii) Since $f'(x)=0$ $f(x)$ must be a constant.

Domain of $f(x)$ is $-\sqrt{2} \le x \le \sqrt{2}$ and there are no discontinuities at which $f(x)$ could change sign.

$\therefore$ substituting any value of x within the domain will give us the value of $f(x)$ for any x in the domain.

Let $x = \sqrt{2}$

$f(x) = f(\sqrt{2}) = 2\cos^{-1}\dfrac{\sqrt{2}}{\sqrt{2}} - \sin^{-1}(1-2)$

$= 2\times 0 - \sin^{-1}(-1)$

$= 0 - \left(-\dfrac{\pi}{2}\right)$

$= \dfrac{\pi}{2}$, as required. ☺

(g) $\frac{dx}{dt} = 5(x-3)$

(i) Method 1

If $x = 3 + Ae^{5t}$

$LHS = \frac{d(3+Ae^{5t})}{dt}$

$= 5Ae^{5t}$

$RHS = 5(3+Ae^{5t}-3)$

$= 15 + 5Ae^{5t} - 15$

$= 5Ae^{5t}$

$= LHS$, as required.

Method 2

$\frac{dx}{dt} = 5(x-3)$

$\frac{dt}{dx} = \frac{1}{5}\frac{1}{x-3}$

$t = \frac{1}{5}\int \frac{1}{x-3}\,dx$

$t = \frac{1}{5}(\ln|x-3| + c)$

$5t - c = \ln|x-3|$

$e^{5t-c} = x-3, \quad x>3$

$x = 3 + e^{5t}$

$x = 3 + Ae^{5t}$

(where $A = e^{-c}$)

as required ☺

(ii)

$20 = 3 + Ae^{0}$

$A = 20 - 3$

$= \underline{\underline{17}}$

(h) $(5+x)x = 6^2$

$x^2 + 5x - 36 = 0$

$(x+9)(x-4) = 0$

$x = -9$ or $x = 4$

Since $x \neq -9$ since x is a distance

$x = 4$ only.

Q.12.

(a) $3\tan^3\theta + 3\tan^2\theta - \tan\theta - 1 = 0 \qquad 0 \le \theta \le \pi$

$3\tan^2\theta(\tan\theta+1) - (\tan\theta+1) = 0$

$(3\tan^2\theta - 1)(\tan\theta + 1) = 0$

$3\tan^2\theta - 1 = 0$ or $\tan\theta + 1 = 0$

$(\sqrt{3}\tan\theta - 1)(\sqrt{3}\tan\theta + 1) = 0 \qquad \tan\theta = -1$

$\theta = \frac{3\pi}{4}$

$\tan\theta = \frac{1}{\sqrt{3}}$ or $\tan\theta = -\frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$ or $\theta = \frac{5\pi}{6}$

$\therefore$ Solutions : $\frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{6}$

(b) $\lim_{x\to 0} \frac{1-\cos 2x}{x^2} = \lim_{x\to 0} \frac{1-(1-2\sin^2 x)}{x^2}$

$= \lim_{x\to 0} \frac{2\sin^2 x}{x^2}$

$= 2\lim_{x\to 0} \frac{\sin x \sin x}{x \times x}$

$= 2\lim_{x\to 0} \frac{\sin x}{x} \times \lim_{x\to 0} \frac{\sin x}{x}$

$= 2 \times 1 \times 1$

$= 2$

(c)(i) gradient of $PQ = \frac{aq^2 - ap^2}{2aq - 2ap}$

$= \frac{a(q-p)(q+p)}{2a(q-p)}$

$= \frac{q+p}{2}$

$\therefore$ Using point - gradient form, equation of PQ:

$$y - ap^2 = \frac{p+q}{2}(x - 2ap)$$

$$y - ap^2 = \frac{p+q}{2}x - (p+q)ap$$

$$y - \cancel{ap^2} = \frac{p+q}{2}x - \cancel{ap^2} - apq$$

$$y = \frac{p+q}{2}x - apq$$, as required

(ii) $(4a, 0)$ satisfies the equation of PQ:

$$0 = \frac{p+q}{2} \times 4a - apq$$

$$\cancel{a}pq = 2(p+q)\cancel{a}$$

$$pq = 2(p+q)$$, as required

(d) at $t=0$ $\dot{x} = 30\cos 60°$ and $\dot{y} = 30\sin 60°$

$\ddot{x} = 0$ (1)

$\dot{x} = 30\cos 60°$

$\dot{x} = 15$ m/s (3)

$x = 15t + C_2$

Since at $t=0$

$x = 0$

$x = 15t$ (5)

$\ddot{y} = -10$ (2)

$\dot{y} = -10t + C_1$

Since at $t=0$

$\dot{y} = 30\sin 60°$

$= \frac{30\sqrt{3}}{2}$

$= 15\sqrt{3}$

$\dot{y} = -10t + 15\sqrt{3}$ (4)

$y = -5t^2 + 15\sqrt{3}t + C_3$

Since at $t=0$ $y = 50$

$y = -5t^2 + 15\sqrt{3}t + 50$ (6)

$\therefore x(t) = 15t$ and $y(t) = -5t^2 + 15\sqrt{3}t + 50$

(ii) $y = 0$

$$-5t^2 + 15\sqrt{3}t + 50 = 0$$
$$-5(t^2 - 3\sqrt{3}t - 10) = 0$$
$$t^2 - 3\sqrt{3}t - 10 = 0$$
$$\Delta = (-3\sqrt{3})^2 - 4 \times 1 \times (-10)$$
$$= 27 + 40$$
$$= 67$$
$$t = \frac{3\sqrt{3} \pm \sqrt{67}}{2}$$

Since $t > 0$ the time of flight is $t = \frac{3\sqrt{3} + \sqrt{67}}{2}$

Range is horizontal displacement at the time when the ball hits the grassed area.

$$\therefore t = \frac{3\sqrt{3} + \sqrt{67}}{2}$$
$$x = 15 \times \left(\frac{3\sqrt{3} + \sqrt{67}}{2}\right)$$
$$= \frac{45\sqrt{3} + 15\sqrt{67}}{2}$$

(e) $$f\left(\frac{3\pi}{4}\right) - f\left(\frac{\pi}{4}\right) = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2 x \, dx$$
$$= \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1 - \cos 2x)\, dx$$
$$= \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$
$$= \frac{1}{2}\left(\left(\frac{3\pi}{4} - \frac{\sin\frac{3\pi}{2}}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin\frac{\pi}{2}}{2}\right)\right)$$
$$= \frac{1}{2}\left(\frac{\pi}{2} + \frac{1}{2} + \frac{1}{2}\right)$$
$$= \frac{\pi}{4} + \frac{1}{2}$$

Q.13.

(a) (i) $\left(x - \frac{1}{3}\right)^3 = x^3 - 3x^2 \times \frac{1}{x} + 3x \times \frac{1}{x^2} - \frac{1}{x^3}$

$$= x^3 - 3x + \frac{3}{x} - \frac{1}{x^3}$$

$$= x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

(ii) Since $\left(x - \frac{1}{x}\right) = 1$ $\quad \left(x - \frac{1}{x}\right)^3 = 1$

$$\therefore \quad x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right) = 1$$

$$x^3 - \frac{1}{x^3} - 3 \times 1 = 1$$

$$x^3 - \frac{1}{x^3} = 1 + 3$$

$$\therefore \quad x^3 - \frac{1}{x^3} = 4$$

(b) $\displaystyle\int_0^3 \frac{3x}{\sqrt{1+x}}\,dx$

Let $u = 1 + x$

$\frac{du}{dx} = 1$

$du = dx$

$x = u - 1$

When $x = 0$ $u = 1$

$x = 3$ $u = 4$

$$= 3\int_1^4 \frac{u-1}{\sqrt{u}}\,du$$

$$= 3\int_1^4 u^{\frac{1}{2}} - u^{-\frac{1}{2}}\,du$$

$$= 3\left[\frac{2u^{\frac{3}{2}}}{3} - \frac{2u^{\frac{1}{2}}}{1}\right]_1^4$$

$$= 3\left(\left(\frac{2 \times 4^{\frac{3}{2}}}{3} - \frac{2 \times 4^{\frac{1}{2}}}{1}\right) - \left(\frac{2 \times 1^{\frac{3}{2}}}{3} - \frac{2 \times 1^{\frac{1}{2}}}{1}\right)\right)$$

$$= 3\left(\frac{16}{3} - 4 - \frac{2}{3} + 2\right)$$

$$= 3\left(\frac{14}{3} - 2\right) = \underline{\underline{8}}$$

(c) $y^2 = \frac{1}{3+x^2}$

$$V = \pi \int_1^3 \frac{1}{3+x^2}\,dx$$

$$= \pi \left[\frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} \right]_1^3$$

$$= \pi \left[\left(\frac{1}{\sqrt{3}} \tan^{-1} \sqrt{3} - \frac{1}{\sqrt{3}} \tan^{-1} \frac{1}{\sqrt{3}} \right) \right.$$

$$= \pi \left(\frac{\pi}{3\sqrt{3}} - \frac{\pi}{6\sqrt{3}} \right)$$

$$= \pi \left(\frac{2\pi}{6\sqrt{3}} - \frac{\pi}{6\sqrt{3}} \right)$$

$$= \frac{\pi^2}{6\sqrt{3}}$$

$$= \frac{\sqrt{3}\pi^2}{18} \quad //$$

(d) (i) General term of the expansion $(3+2x)^n$ is

$${}^nC_r \times 3^{n-r} (2x)^r = {}^nC_r \times 3^{n-r} 2^r x^r$$

When $r = 5$ coefficient of x^5 is

$${}^nC_5 \times 3^{n-5} \times 2^5$$

When $r = 6$ coefficient of x^6 is

$${}^nC_6 \times 3^{n-6} \times 2^6$$

If the above coefficients equal, then

$$\frac{{}^nC_5 \times 3^{n-5} \times 2^5}{{}^nC_6 \times 3^{n-6} \times 2^6} = 1$$

$$\frac{{}^nC_5 \times 3}{{}^nC_6 \times 2} = 1$$

$$\frac{{}^nC_5}{{}^nC_6} = \frac{2}{3}$$

$$\frac{n!}{5!(n-5)!} \times \frac{6!(n-6)!}{n!} = \frac{2}{3}$$

$$\frac{6}{n-5} = \frac{2}{3}$$

$$\frac{n-5}{6} = \frac{3}{2}$$

$$n-5 = 9$$

$$n = 14$$

(ii) In the expansion $(3+2x)^{14}$ r^{th} term is ${}^{14}C_r\, 3^{14-r} \times 2^r x^r$

and $(r+1)^{th}$ term is

$$ {}^{14}C_{r+1} \times 3^{14-(r+1)}\, 2^{r+1} x^{r+1}$$

$$= {}^{14}C_{r+1} \times 3^{13-r} \times 2^{r+1} x^{r+1}$$

Coefficients of the expansion are increasing while

$$\frac{{}^{14}C_{r+1} \times 3^{13-r} \times 2^{r+1}}{{}^{14}C_r \times 3^{14-r} \times 2^r} > 1$$

$$\frac{{}^{14}C_{r+1}}{{}^{14}C_r} \times \frac{2}{3} > 1$$

$$\frac{{}^{14}C_{r+1}}{{}^{14}C_r} > \frac{3}{2}$$

$$\frac{14!}{(r+1)\,r!\,(14-r-1)!} \times \frac{r!\,(14-r)!}{14!} > \frac{3}{2}$$

$$\frac{14-r}{r+1} > \frac{3}{2}$$

$$2(14-r) > 3(r+1)$$

$$28-2r > 3r+3$$

$$5r < 25$$

$$r < 5$$

Which mean that the coefficients are increasing up to the term containing x^5, we have already found that the coefficients of x^5 and x^6 are equal and after this term the coefficients will start decreasing.

$\therefore$ Coefficients of x^5 and x^6 are the greatest coefficients in the expansion.

(e) $\cos 2x = \sin x$

$$1 - 2\sin^2 x = \sin x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$2\sin^2 x + 2\sin x - \sin x - 1 = 0$$

$$2\sin x(\sin x + 1) - (\sin x + 1) = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$2\sin x = 1$ or $\sin x = -1$

$\sin x = \frac{1}{2}$

$\therefore x = \frac{\pi}{6} + 2n\pi\ ;\ \frac{5\pi}{6} + 2n\pi;\ \frac{3\pi}{2} + 2n\pi,$

where n is an integer.

(f) (i) $\frac{d}{dx} \log_e(\sec x + \tan x)$

$$= \frac{1}{\sec x + \tan x} \times \sec x \tan x + \sec^2 x$$

$$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$$

$$= \sec x$$

(ii) $\int_0^{\frac{\pi}{4}} \sec x \, dx = \left[\ln(\sec x + \tan x)\right]_0^{\frac{\pi}{4}}$

$$= \ln\left(\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right) - \ln(1+0)$$

$$= \ln(\sqrt{2}+1)$$

Q.14.

(a) $\tan 20^\circ = \frac{h}{PR} \quad \therefore PR = \frac{h}{\tan 20^\circ}$

$\tan 15^\circ = \frac{h}{QR} \quad \therefore QR = \frac{h}{\tan 15^\circ}$

$PR \perp PQ$ (given)

$\therefore QR^2 = 60^2 + PR^2$

$$\frac{h^2}{\tan^2 15^\circ} = 60^2 + \frac{h^2}{\tan^2 20^\circ}$$

$$h^2\left(\frac{1}{\tan^2 15^\circ} - \frac{1}{\tan^2 20^\circ}\right) = 60^2$$

$$h^2 = \frac{60^2 \tan^2 15^\circ \tan^2 20^\circ}{\tan^2 20^\circ - \tan^2 15^\circ}$$

$h = 23.7550\ldots$

$h \approx 24\text{ m}$ (to nearest metre).

(b) (i) $\sqrt{2}\left(\frac{1-t^2}{1+t^2}\right) + \frac{2t}{1+t^2} \quad \left(t = \tan\frac{\theta}{2}\right)$

(ii) $\sqrt{2}\left(\frac{1-t^2}{1+t^2}\right) + \frac{2t}{1+t^2} = 1$

$$\sqrt{2}(1-t^2) + 2t = 1 + t^2$$
$$\sqrt{2} - \sqrt{2}t^2 + 2t - 1 - t^2 = 0$$
$$t^2 + \sqrt{2}t^2 - 2t + 1 - \sqrt{2} = 0$$
$$(1+\sqrt{2})t^2 - 2t + (1-\sqrt{2}) = 0$$

$$\Delta = 4 - 4(1+\sqrt{2})(1-\sqrt{2})$$
$$= 4 + 4$$
$$= 8$$
$$\sqrt{\Delta} = 2\sqrt{2}$$

$$t = \frac{2 \pm 2\sqrt{2}}{2(1+\sqrt{2})}$$
$$= \frac{1 \pm \sqrt{2}}{1+\sqrt{2}}$$

$t = 1$ or $t = \frac{(1-\sqrt{2})^2}{1-2}$

$$= \frac{(1-\sqrt{2})^2}{-1}$$
$$= \frac{1-2\sqrt{2}+2}{-1}$$
$$= 2\sqrt{2} - 3$$

$\therefore \tan\frac{\theta}{2} = 1$ or $\tan\frac{\theta}{2} = 2\sqrt{2} - 3$

since $0 \le \theta \le 360°$

$0 \le \frac{\theta}{2} \le 180°$ or $0 \le \frac{\theta}{2} \le \pi$

$\frac{\theta}{2} = 45°$ or $\frac{\theta}{2} = \pi - \tan^{-1}(3-2\sqrt{2})$

$\theta = 90°$

$$\theta = 2\pi - 2\tan^{-1}(3-2\sqrt{2})$$
$$\doteqdot 5.943348 \text{ radians}$$
$$= 340°32'$$

$\therefore$ Solutions are : $\theta = 90°$ or $\theta \doteqdot 340°32'$

(c)

(i) $A_1 = 20000 \times 1.1 - 2400$

$A_2 = A_1 \times 1.1 - 2400$

$= (20000 \times 1.1 - 2400) \times 1.1 - 2400$

$= 20000 \times 1.1^2 - 2400 \times 1.1 - 2400$

$A_3 = A_2 \times 1.1 - 2400$

$= 20000 \times 1.1^3 - 2400 \times 1.1^2 - 2400 \times 1.1 - 2400$

$= 20000 \times 1.1^3 - 2400(1.1^2 + 1.1 + 1)$

$\vdots$

$A_n = 20000 \times 1.1^n - 2400(\underbrace{1.1 + 1.1^{n-1} + \dots + 1.1 + 1}$)

GP with $a = 1$, $r = 1.1$ and n terms, where $S_n = \dfrac{1.1^n - 1}{1.1 - 1}$

$\therefore$

$A_n = 20000 \times 1.1^n - 2400 \times \dfrac{1.1^n - 1}{1.1 - 1}$

$= 20000 \times 1.1^n - 2400 \times \dfrac{1.1^n - 1}{0.1}$

$= 20000 \times 1.1^n - 24000(1.1^n - 1)$

$= 20000 \times 1.1^n - 24000 \times 1.1^n + 24000$

$= 1.1^n(20000 - 24000) + 24000$

$= 24000 - 4000 \times 1.1^n$, as required.

(ii) $A_n = 0$

$\therefore \quad 24000 - 4000 \times 1.1^n = 0$

$1.1^n = 6$

$n \ln 1.1 = \ln 6$

$n = \dfrac{\ln 6}{\ln 1.1}$

$n = 18.799\ldots$

$n = 18$

$\therefore$ The number of years is $\underline{\underline{18}}$.

(d) $A = \pi r^2$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$= 2\pi r \times \frac{dr}{dt}$$

$$= 2\pi \times 40 \times 0.02$$

$$= 8 \times \pi \times 0.2$$

$$= 1.6\pi \quad cm^2/s$$

$$\doteqdot 5.03 \quad cm^2/s$$