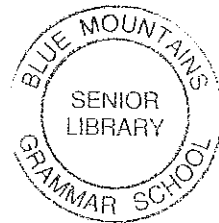


2010
Semester 1
**HIGHER SCHOOL CERTIFICATE
EXAMINATION**

Student Number

Mathematics Extension 1



General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 84

- Attempt questions 1-7
- All questions are of equal value

Question 1 (12 marks)

Start a new sheet of writing paper.

Marks

- (a) Find the perpendicular distance from the point (1, 4) to the line
 $4y = 3x - 2$ 2
- (b) Differentiate with respect to x
 $y = \sin 2x$ 1
- (c) Differentiate with respect to x
 $y = \ln \sqrt{\frac{2x-1}{3x+2}}$ 2
- (d) Prove
$$\frac{\tan 2\theta + \cot 2\theta}{\tan 2\theta - \tan \theta} = \cot^2 \theta$$
 3
- (e) Using the following function
 $y = \frac{\ln x}{x}$
- (i) Show that
$$\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$$
 1
- (ii) Evaluate
$$\int_e^{e^2} \frac{1 - \ln x}{x \ln x} \cdot dx = \ln(2 - 1)$$
 3

End of Question 1

Question 2 (12 marks)

Start a new sheet of writing paper.

Marks

- (a) Find

1

$$\lim_{x \rightarrow 0} \frac{4x}{\sin 5x}$$

- (b) Solve for x

2

$$\left| \frac{2x + 5}{3} \right| < 2$$

- (c) Consider the polynomial $P(x) = x^3 + Ax^2 - 2008$

2

If $(x - 2)$ is a factor of $P(x)$ find A .

- (d) The point P divides the line AB externally in the ratio $3:2$.
Find P if A is $(2, -5)$ and B is $(6, 1)$.

2

- (e) P is the point, other than the origin, where $y = ax^2$ meets the line $y = x$

- (i) Find the coordinates of P .

1

- (ii) Find, to the nearest minute, the size of the acute angle formed by the line $y = x$ and the tangent to $y = ax^2$ at P .

2

- (f) Two dice are rolled, find the probability that the sum of the two dice is $= 5$

1

- (g) Calculate the value of x to 3 dp in the following equation

1

$$5^x = 40$$

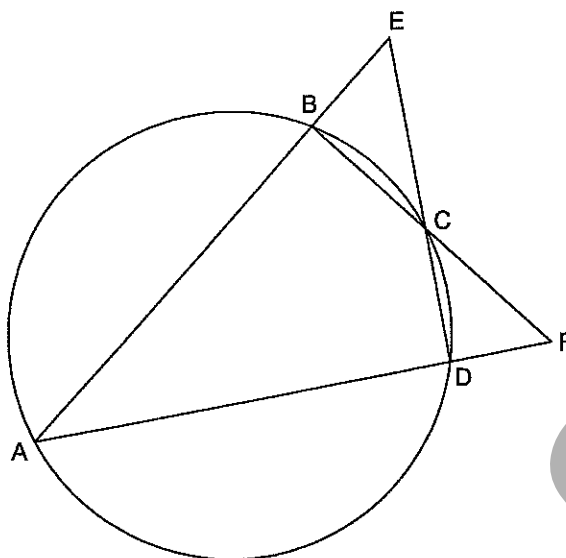
End of Question 2

Question 3 (12 marks)

Start a new sheet of writing paper.

Marks

(a)



In the diagram above
DBE, ADF, DCE and BCF are straight lines.
 $\angle AED = \angle BFA$.

- (i) Copy the diagram into your answer booklets and add in the relevant information. 3
Prove $\angle ABC = \angle ADC$
- (ii) Hence or otherwise prove AC is a diameter. 2
- (b) (i) Find 2
$$\frac{d}{dx}(\cos^3 x)$$
- (ii) Hence or otherwise evaluate 3
$$\int_0^{\frac{\pi}{3}} 3 \sin x \cos^2 x \cdot dx$$
- (c) If $t = \tan \frac{x}{2}$, 2
express $\frac{1 - \cos x}{1 + \cos x}$ in terms of t , in simplest form.

End of Question 3

Question 4 (12 marks)

Start a new sheet of writing paper.

Marks

- (a) Use Mathematical Induction to show that if x is a positive integer then $(1+x)^n - 1$ is divisible by x for all integers n such that $n \geq 1$.
(Hint in step 2 let $(1+x)^n - 1 = x \times P(x)$)

4

- (b) (i) Prove that

2

$$\int_0^{\frac{\pi}{4}} \sin^2 x \cdot dx = \frac{\pi}{8} - \frac{1}{4}$$

- (ii) Prove that

2

$$\frac{d}{dx}(x \sin^2 x) - \sin^2 x = x \sin 2x$$

- (iii) Hence, or otherwise, prove

2

$$\int_0^{\frac{\pi}{4}} x \sin 2x \cdot dx = \frac{1}{4}$$

- (c) The probability of a cure with drug A is 0.6 and the probability of a cure with drug B is 0.8. If drug A is administered to one patient and drug B to another patient, what is the probability that neither patient will be cured?

2

End of Question 4

Question 5 (12 marks)

Start a new sheet of writing paper.

Marks

- (a) Four cards marked with the numbers 1, 2, 3 and 4 are placed in a box. Two cards are selected at random, one after the other without replacement, to form a two-digit number.
- (i) Draw a tree diagram to show the possible outcomes. 1
 - (ii) How many different two-digit numbers can be formed? 1
 - (iii) What is the probability that the number formed is less than 34? 1
 - (iv) What is the probability that the number formed is divisible by 3? 1
- (b) Graph $y = \sin 3\theta$ showing all relevant points for $-\pi \leq \theta \leq \pi$. 3
- (c) Find $\frac{d}{dx} \frac{\sin 2x}{\cos 4x}$ 2
- (d) Solve $\sqrt{3} \cos 2\theta - \sin 2\theta = 1$ where $0 \leq \theta \leq 2\pi$ 3

End of Question 5

Question 6 (12 marks)

Start a new sheet of writing paper.

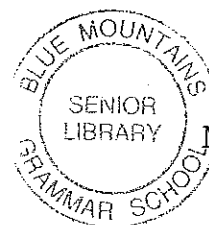
Marks

- (a) (i) State the domain of the function
 $\ln(2x + 3) + \ln(x - 2) = 2\ln(x + 4)$ 1
- (ii) Find all of the real numbers such that
 $\ln(2x + 3) + \ln(x - 2) = 2\ln(x + 4)$ 2
- (b) For the function
$$f(x) = \ln\left(\frac{x^2}{x - 1}\right)$$
- (i) State the domain of $f(x)$ 1
- (ii) Find the coordinates and nature of the stationary point on the curve
 $y = f(x)$ 3
- (iii) Find the coordinates of the point of inflexion. 3
- (iv) Sketch the graph of showing the coordinates of the stationary point and the equations of any asymptotes. 2

End of Question 6

Question 7 (12 marks)

Start a new sheet of writing paper.



Marks

- (a) Show that

$$\frac{\tan 2\theta - \tan \theta}{\tan 2\theta + \cot \theta} = \tan^2 \theta$$

3

- (b) Use log laws to find

$$\frac{d}{dx} (\ln \sqrt{e^x})$$

(hint $\sqrt{e^x} = e^{\frac{1}{2}x}$)

1

- (c)

$$\int \frac{e^{3x} + e^x - 5}{e^{2x}} \cdot dx$$

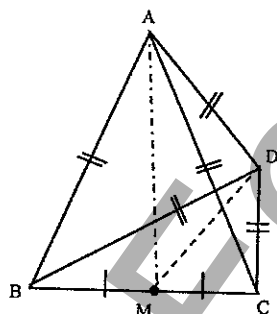
2

- (d) If α, β, γ are the roots of the equation $2x^3 - 14x - 1 = 0$
Find the value of

2

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

- (e) The figure below ABCD is a regular tetrahedron, with



$AB = AC = BC = BD = DC = AD = 20\text{cm}.$
(2M)

- (i) Draw the triangle BCD in your writing booklet and mark 'M' the midpoint of BC. Prove that $DM = 10\sqrt{3}$

2

- (ii) Determine the size of $\angle AMD$

2

End of Question 7

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$