Gosford High School

2010
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time - 5 minutes.
- Working Time - 2 hours.
- Write using a blue or black pen.
- Board Approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question in a new booklet

Total marks (**84**)

- Attempt Questions 1-7.
- All questions are of equal value.

Question 1. **(Begin a new booklet)**

a) Find the value of $\lim_{x \to 0} \frac{\sin 3x}{2x}$ (1)

b) Find the acute angle between the lines $y = 3x - 2$ and $y = 2 - x$, giving your answer to the nearest degree. (2)

c) Solve $\frac{(x+1)}{x} > 0$. (2)

d) Find the number of ways in which 3 boys and 3 girls can be arranged in a straight line so that the tallest boy and tallest girl occupy the two middle positions. (2)

e) Find the value of k such that $(x-2)$ is a factor of $P(x) = x^3 + 2x + k$ (2)

f) Evaluate $\int_0^{\pi} \sin^2 x \, dx$. (3)

Question 2. (Begin a new booklet)

a) The interval AB is divided externally in the ratio 1:4. If A and B are the points (1,3) and (6,-2) respectively, find the coordinates of the point of division. (2)

b) Given that a root of $y = x + \ln x - 2$ lies close to $x = 1.5$, use Newton's method once to find an improved value of that root. (3)

c) Using the substitution $u = x^2 - 2$ or otherwise, find $\int \frac{x}{\sqrt{x^2 - 2}}\,dx$. (2)

d) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$. The equation of the tangents at P and Q respectively are $y = px - ap^2$ and $y = qx - aq^2$.

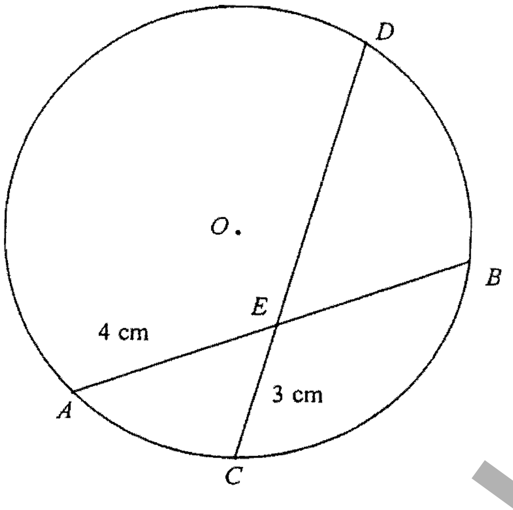
(i) The tangents at P and Q meet at the point R. Show that the coordinates of R are $(a(p+q), apq)$. (2)

(ii) The equation of the chord PQ is $y = \frac{p+q}{2}x - apq$
(Do NOT show this.) If the chord PQ passes through $(0, a)$, show that $pq = -1$. (1)

(iii) Find the equation of the locus of R if the chord PQ passes through $(0, a)$ (2)

Question 3. (Begin a new booklet)

a) In the circle centred at O, the chords AB and CD intersect at E.
The length of AB is x cm and of CD is y cm.
$AE = 4$ cm and $CE = 3$ cm.



Show that $4x = 3y + 7$ (2)

b) Consider the function $f(x) = \dfrac{3x}{x^2 - 1}$

i. Show that the function is odd. (1)

ii. Show that the function is decreasing for all values of x. (1)

iii. Sketch the graph of the function showing clearly the equations of any asymptotes (2)

c) If $\tan A$ and $\tan B$ are the roots of the quadratic equation $3x^2 - 5x - 1 = 0$, find the value of $\tan(A + B)$. (2)

d) Solve $\cos 2A = \cos A$ where $0 \le A \le 2\pi$ (2)

e) Write down the general solutions of $\tan(x - \dfrac{\pi}{3}) = -1$ (2)

Question 4. (Begin a new booklet)

a) At time t hours after an oil spill occurs, a circular oil slick has a radius r km, where $r = \sqrt{t+1} - 1$. Find the rate at which the area of the slick is increasing when its radius is 1 km, giving your answer correct to 2 decimal places. (3)

b) Use Mathematical Induction to show that $3^n - 2n - 1$ is divisible by 4 for all positive integers $n \geq 2$ (4)

c) At time t years after observation begins, the number N of birds in a colony is given by $N = 100 + 400e^{-0.1t}$

i. Sketch the graph of N as a function of t showing clearly the initial population size and the limiting population size. (2)

ii. Find the time taken for the population size to fall to half its initial value, giving the answer correct to the nearest year (2)

d) Given $f(x) = \log_e\left(\sqrt{9-x^2}\right)$, state the domain of $f(x)$. (1)

Question 5. (Begin a new booklet)

a) Use the substitution $t = \tan\frac{x}{2}$ to show that $\operatorname{cosec} x + \cot x = \cot\frac{x}{2}$ (2)

Hence evaluate $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\operatorname{cosec} x + \cot x)\,dx$ (3)

b) Find the term independent of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^9$ (3)

c) Show, using sketches on separate sets of axes :

(i) the area enclosed between $y = \sin^{-1}x$, the x axis, and the line $x = 1$ (1)

(ii) the area enclosed between $y = \sin x$, the x axis, and the line $x = \frac{\pi}{2}$. (1)

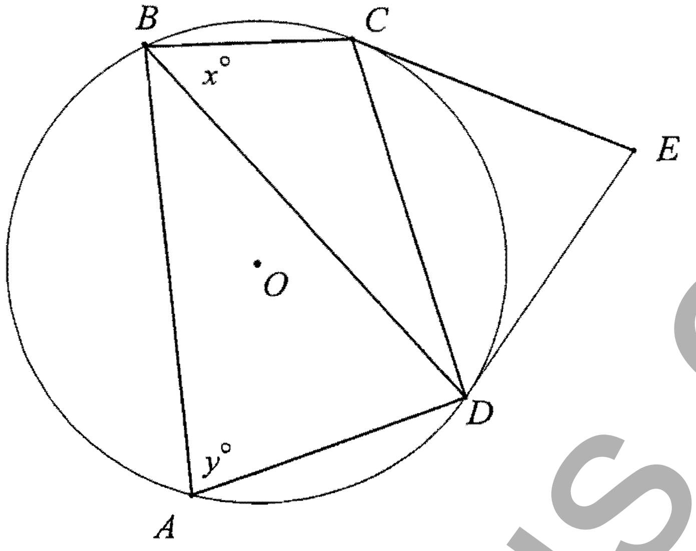
(ii) Using the graphs, explain why

$$\int_0^1 \sin^{-1} x\,dx = \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \sin x\,dx.$$

(2)

Question 6. (Begin a new booklet)

a) The circle $ABCD$ has centre O. Tangents are drawn from an external point E to contact the circle at C and D. $\angle CBD = x^\circ$ and $\angle BAD = y^\circ$.



i. Copy the diagram into your examination booklet

ii. Show that $\angle CED = (180 - 2x)^\circ$. (2)

iii. Show that $\angle BDC = (y - x)^\circ$. (2)

b) A particle moving in a straight line is performing Simple Harmonic Motion. At the time t seconds it has displacement x metres from a fixed point O on the line, where $x = 4\cos^2 t - 1$

i. Show that its acceleration is given by $\ddot{x} = -4(x-1)$ (2)

ii. Sketch the graph of x as a function of t for $0 \le t \le \pi$, clearly showing the times when the particle passes through O. (2)

iii. For $0 \le t \le \pi$, find the time when the velocity of the particle is increasing most rapidly, and find this rate of increase in the velocity. (2)

c) Differentiate $2x^2 \cos^{-1} 2x$. (2)

Question 7. **(Begin a new booklet)**

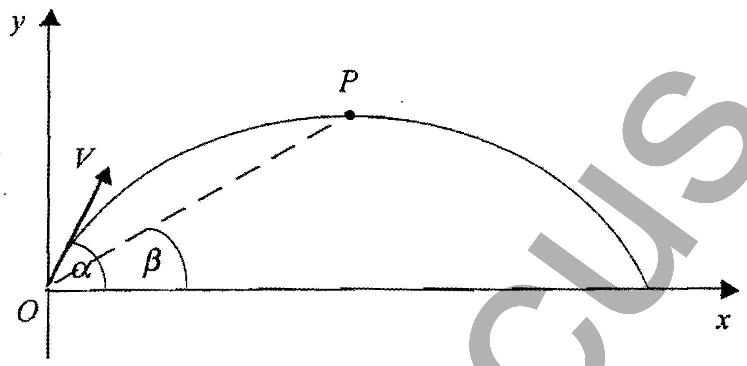
a) The polynomial $P(x) = 2x^3 - 5x^2 - 3x + 1$ has zeros α, β and γ.
Find the values of

(i) $3\alpha + 3\beta + 3\gamma - 4\alpha\beta\gamma$ (2)

(ii) $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$ (1)

(iii) $\alpha^2 + \beta^2 + \gamma^2$ (1)

b)



A particle is projected from a point O with speed V ms^{-1} at an angle α above the horizontal, where $0 < \alpha < \frac{\pi}{2}$. It moves in a vertical plane subject to gravity where the acceleration due to gravity is 10 ms^{-2}. At time t seconds it has horizontal and vertical displacements x metres and y metres respectively from O. At point P where it attains its greatest height the angle of elevation of the particle from O is β radians.

(i) Use integration to show that $x = Vt\cos\alpha$ and $y = Vt\sin\alpha - 5t^2$. (2)

(ii) Show that $\tan\beta = \frac{1}{2}\tan\alpha$. (3)

(iii) If the particle has greatest height 80 m above O at a horizontal distance 120 m from O, find the exact values of α and V. (3)