

## Question 1

## a. Outcomes assessed : H5

## Marking Guidelines

Criteria	Marks
• rearranges in terms of known trigonometric limit	1
• evaluates limit	1

## Answer

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{2x} = \frac{3}{2} \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} = \frac{3}{2} \times 1 = \frac{3}{2}$$

## b. Outcomes assessed : H5

## Marking Guidelines

Criteria	Marks
• identifies $a$ and $r$ for the G.P	1
• applies formula for limiting sum	1

## Answer

$$\left(\frac{e}{e+1}\right) + \left(\frac{e}{e+1}\right)^2 + \left(\frac{e}{e+1}\right)^3 + \dots \quad \text{is G.P. with } a = \frac{e}{e+1}, \text{ and } r = \frac{e}{e+1} \Rightarrow 0 < r < 1$$

$$\therefore \text{Limiting sum is } \frac{a}{1-r} = \frac{e}{e+1} \div \frac{1}{e+1} = e$$

## c. Outcomes assessed : PE3

## Marking Guidelines

Criteria	Marks
• expresses sum of reciprocals of roots in terms of sums of products	1
• evaluates using relationships between roots and coefficients	1

## Answer

$$\alpha, \beta \text{ and } \gamma \text{ roots of } x^3 + 2x^2 + 3x + 6 = 0. \quad \therefore \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha\beta\gamma} = \frac{3}{-6} = -\frac{1}{2}$$

## d. Outcomes assessed : H5

## Marking Guidelines

Criteria	Marks
• substitutes values of gradients into formula for tangent of acute angle between the lines	1
• evaluates required angle	1

## Answer

$$\text{Acute angle } \theta \text{ between lines } y = 2x \text{ and } x + y - 3 = 0 \text{ is given by } \tan \theta = \left| \frac{2 - (-1)}{1 + 2 \cdot (-1)} \right| = 3$$

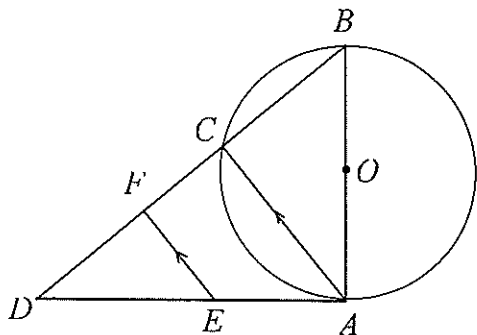
$$\therefore \theta \approx 72^\circ \text{ (to the nearest degree)}$$

e. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
i • quotes alternate segment theorem	1
ii • gives a sequence of deductions resulting in a test for a cyclic quadrilateral	1
• justifies these deductions by quoting geometric properties and tests	1
iii • explains why $BE$ subtends a right angle at $A$ or at $F$	1

Answer



Let  $O$  be the centre of the circle.

- The angle between the tangent at  $A$  and the chord  $AC$  is equal to the angle subtended by that chord in the alternate segment, hence  $\angle EAC = \angle ABC$ .
- $\angle EAC = \angle DEF$  (Corresp.  $\angle$ 's with parallel lines  $AC$ ,  $EF$  are equal)  
 $\therefore \angle DEF = \angle ABC$  (Both equal to  $\angle EAC$ )  
 $\therefore EABF$  is cyclic (Exterior  $\angle$  equal to interior opp.  $\angle$ )
- $\angle BAE = 90^\circ$  (Tangent to circle  $ABC$  at  $A$  is perpendicular to radius  $OA$  drawn to point of contact)  
 $\therefore BE$  is a diameter (subtends right  $\angle$  at circumference) of circle  $EABF$ .

Question 2

a. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• finds primitive	1
• evaluates in surd form	1

Answer

$$\int_0^{\frac{\pi}{8}} \sec 2x \tan 2x \, dx = \frac{1}{2} \left[ \sec 2x \right]_0^{\frac{\pi}{8}} = \frac{1}{2} (\sqrt{2} - 1)$$

b. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
• counts arrangements for one possible pattern of B's and G's	1
• adds number of arrangements for the second possible pattern of B's and G's	1

Answer

B G B G G B or B G G B G B  $\therefore 2 \times 3! \times 3! = 72$  ways

c. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• finds $x$ coordinate of $P$	1
• finds $y$ coordinate of $P$	1

Answer

$$\begin{array}{ccc}
 A(-2, 3) & & B(6, -1) \\
 & \times & \\
 3 & : & 2 \\
 \hline
 P\left(\frac{3 \times 6 + 2 \times (-2)}{3+2}, \frac{3 \times (-1) + 2 \times 3}{3+2}\right)
 \end{array}$$

$\therefore P$  has coordinates  $P\left(\frac{14}{5}, \frac{3}{5}\right)$

d. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• simplifies $1 - \cos x$ in terms of $t$	1
• completes simplification of given expression in terms of $t$ to establish required result	1

Answer

$$t = \tan \frac{x}{2}$$

$$\begin{aligned}
 1 - \cos x &= 1 - \frac{1-t^2}{1+t^2} \\
 &= \frac{2t^2}{1+t^2} \\
 \sin x &= \frac{2t}{1+t^2}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{\sin x}{1 - \cos x} &= \frac{2t}{1+t^2} \times \frac{1+t^2}{2t^2} \\
 &= \frac{1}{t} \\
 &= \cot \frac{x}{2}
 \end{aligned}$$

e. Outcomes assessed : PE3, PE4

Marking Guidelines

Criteria	Marks
i • finds $\frac{dy}{dx}$ as a function of $t$	1
• finds equation of normal in required form	1
ii • finds coordinates of $M$	1
• finds equation of locus of $M$	1

Answer

i.

$$\begin{aligned}
 y &= at^2 \Rightarrow \frac{dy}{dt} = 2at \\
 x &= 2at \Rightarrow \frac{dx}{dt} = 2a
 \end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = t$$

$\therefore$  Normal at  $P$  has gradient  $-\frac{1}{t}$  and equation

$$y - at^2 = -\frac{1}{t}(x - 2at)$$

$$ty - at^3 = -x + 2at$$

$$x + ty = 2at + at^3$$

$$\begin{aligned}
 \text{ii. } N(0, 2a + at^2) \quad \therefore M(at, a + at^2) \\
 P(2at, at^2)
 \end{aligned}$$

$$\begin{aligned}
 \text{Locus of } M \text{ has equation } y &= a + a\left(\frac{x}{a}\right)^2 \\
 x^2 &= a(y - a)
 \end{aligned}$$

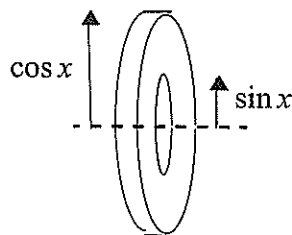
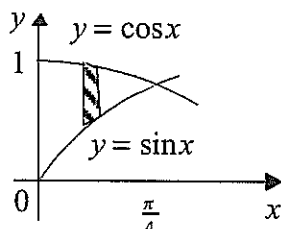
### Question 3

a. Outcomes assessed : H5

#### Marking Guidelines

Criteria	Marks
• writes definite integral for the volume in terms of $\cos x$ and $\sin x$	1
• evaluates the integral.	1

Answer



$$\begin{aligned}
 V &= \pi \int_0^{\pi/4} (\cos^2 x - \sin^2 x) dx \\
 &= \pi \int_0^{\pi/4} \cos 2x dx \\
 &= \frac{1}{2} \pi [\sin 2x]_0^{\pi/4} \\
 &= \frac{1}{2} \pi (1 - 0)
 \end{aligned}$$

Volume is  $\frac{\pi}{2}$  cubic units.

b. Outcomes assessed : HE2

#### Marking Guidelines

Criteria	Marks
• defines an appropriate sequence of statements $S(n)$ and shows the first member is true	1
• writes the LHS of $S(k+1)$ in terms of RHS of $S(k)$ , conditional on truth of $S(k)$	1
• rearranges conditional expression for LHS of $S(k+1)$ to obtain RHS	1
• completes proof by Mathematical Induction	1

Answer

Let  $S(n)$ ,  $n = 2, 3, 4, \dots$ , be the sequence of statements defined by

$$S(n): 2 \times 1 + 3 \times 2 + 4 \times 3 + \dots + n(n-1) = \frac{n(n^2-1)}{3}$$

Consider  $S(2)$ :  $LHS = 2 \times 1 = 2$ ;  $RHS = \frac{2(2^2-1)}{3} = 2$ . Hence  $S(2)$  is true.

If  $S(k)$  is true:  $2 \times 1 + 3 \times 2 + 4 \times 3 + \dots + k(k-1) = \frac{k(k^2-1)}{3}$  \*

$$\begin{aligned}
 \text{Consider } S(k+1): LHS &= \{2 \times 1 + 3 \times 2 + 4 \times 3 + \dots + k(k-1)\} + (k+1)k \\
 &= \frac{k(k^2-1)}{3} + (k+1)k \quad \text{if } S(k) \text{ is true, using } * \\
 &= \frac{k(k+1)\{(k-1)+3\}}{3} \\
 &= \frac{(k+1)\{k^2+2k\}}{3} \\
 &= \frac{(k+1)\{(k+1)^2-1\}}{3} \\
 &= RHS
 \end{aligned}$$

Hence if  $S(k)$  is true then  $S(k+1)$  is true. But  $S(2)$  is true, and hence  $S(3)$  is true and so on. Hence by Mathematical Induction,  $S(n)$  is true for all positive integers  $n \geq 2$ .

c. Outcomes assessed : HE4

Marking Guidelines

Criteria	Marks
i • rearranges and interchanges $x$ and $y$ to obtain equation of inverse function	1
ii • sketches graph of $y = f(x)$ showing endpoints and intercepts	1
• sketches inverse function by reflection in $y = x$	1
• shows endpoints and intercepts for inverse function	1
iii • writes equation for $x$	1
• solves for $x$ in simplest exact form	1

Answer

i.  $f(x) = (x+2)^2 - 9$ ,  $-2 \leq x \leq 2$ .

$$(x+2)^2 = y+9 \quad \text{and} \quad 0 \leq x+2 \leq 4$$

$$x+2 = +\sqrt{y+9}$$

$$\therefore x = -2 + \sqrt{y+9}, \quad -9 \leq y \leq 7$$

$$\therefore x \leftrightarrow y \Rightarrow f^{-1}(x) = -2 + \sqrt{x+9}, \quad -9 \leq x \leq 7$$

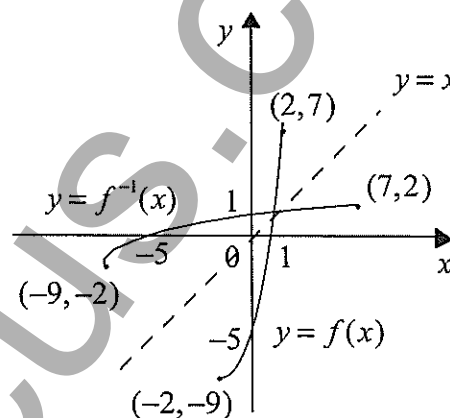
iii. Graphs intersect on the line  $y = x$ .

Hence  $(x+2)^2 - 9 = x$

$$x^2 + 3x - 5 = 0$$

$$\therefore x > 0 \Rightarrow x = \frac{-3 + \sqrt{29}}{2}$$

ii. Graphs of inverse functions are reflections of each other in  $y = x$



Question 4

a. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
• writes expression for probability in terms of binomial coefficients	1
• evaluates required probability	1

Answer

$$P(\text{none in common}) = \frac{{}^{34}C_6}{{}^{40}C_6} \approx 0.35 \quad (\text{to 2 decimal places})$$

b. Outcomes assessed : HE6

Marking Guidelines

Criteria	Marks
• writes $du$ in terms of $dx$ and converts limits for $x$ into limits for $u$	1
• finds equivalent definite integral in terms of $u$	1
• finds primitive and substitutes limits	1
• simplifies exact answer	1

# Answer

$$u = \sin^2 x$$

$$du = 2 \sin x \cos x \, dx$$

$$du = \sin 2x \, dx$$

$$x = \frac{\pi}{4} \Rightarrow u = \frac{1}{2}$$

$$x = \frac{\pi}{3} \Rightarrow u = \frac{3}{4}$$

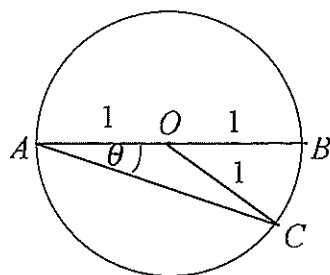
$$\begin{aligned} \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin 2x}{1 + \sin^2 x} \, dx &= \int_{\frac{1}{2}}^{\frac{3}{4}} \frac{1}{1+u} \, du \\ &= \left[ \ln(1+u) \right]_{\frac{1}{2}}^{\frac{3}{4}} \\ &= \ln \frac{7}{4} - \ln \frac{3}{2} \\ &= \ln \frac{7}{6} \end{aligned}$$

## c. Outcomes assessed : H5, PE3

### Marking Guidelines

Criteria	Marks
i • finds area of $\triangle AOC$ in terms of $\sin 2\theta$	1
• uses area information to complete equation for $\theta$	1
ii • shows that $f(0.4)$ , $f(0.5)$ have opposite signs	1
• notes that $f$ is continuous, and deduces equation has one root $\theta$ , $0.4 < \theta < 0.5$	1
iii • applies Newton's rule to write numerical expression for next approximation	1
• evaluates this approximation	1

# Answer



$$\begin{aligned} \text{i. } \angle OCA &= \theta \quad (\angle\text{'s opp. equal sides are equal in } \triangle AOC) \\ \angle AOC &= \pi - 2\theta \quad (\angle \text{sum of } \triangle \text{ is } \pi) \\ \angle BOC &= 2\theta \quad (\text{adj. supp. } \angle\text{'s add to } \pi) \\ \text{Area sector } BOC + \text{Area } \triangle AOC &= \frac{1}{4} \text{ Area circle} \\ \therefore \frac{1}{2} \times 1^2 \times 2\theta + \frac{1}{2} \times 1^2 \times \sin(\pi - 2\theta) &= \frac{1}{4} \times \pi \times 1^2 \\ \theta + \frac{1}{2} \sin 2\theta - \frac{\pi}{4} &= 0 \end{aligned}$$

$$\begin{aligned} \text{ii. Let } f(\theta) &= \theta + \frac{1}{2} \sin 2\theta - \frac{\pi}{4} \\ f(0.4) &\approx -0.03 < 0 \\ f(0.5) &\approx 0.14 > 0 \quad \text{and } f \text{ is continuous} \\ \text{Also } f'(\theta) &= 1 + \cos 2\theta > 0 \Rightarrow f \text{ monotonic increasing} \\ \therefore f(\theta) &= 0 \text{ for exactly one value of } \theta, 0.4 < \theta < 0.5 \end{aligned}$$

$$\begin{aligned} \text{iii. Since } f'(\theta) &= 1 + \cos 2\theta, \\ \theta &\approx 0.4 - \frac{f(0.4)}{f'(0.4)} \\ &\approx 0.4 - \frac{-0.0267}{1.6967} \\ &\approx 0.42 \quad (\text{to 2 dec. pl.}) \end{aligned}$$

## Question 5

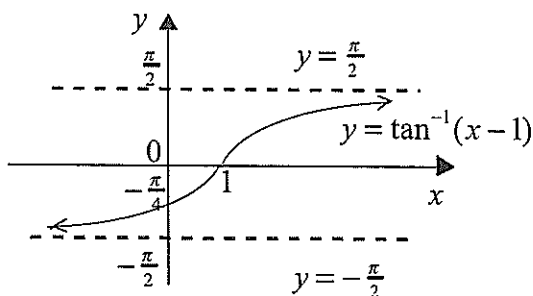
### a. Outcomes assessed : HE4

### Marking Guidelines

Criteria	Marks
i • shows correct shape and asymptotes	1
• shows intercepts on coordinate axes	1
ii • finds $\frac{dy}{dx}$ and evaluates for $x = 1$	1
• finds equation of tangent	1

# Answer

i.



$$\text{ii. } y = \tan^{-1}(x-1)$$

$$\frac{dy}{dx} = \frac{1}{1+(x-1)^2}$$

$$\therefore \frac{dy}{dx} = 1 \text{ when } x = 1$$

$\therefore$  Tangent at  $(1, 0)$  has gradient 1 and equation  $y = x - 1$

## b. Outcomes assessed : HE5

### Marking Guidelines

Criteria	Marks
i • shows by differentiation that $a$ is constant	1
ii • integrates to find a primitive function for $t$ in terms of $x$	1
• evaluates constant of integration using initial conditions then writes $x$ as a function of $t$	1
iii • evaluates $x$ at $t = 2$ and $t = 3$ to find distance travelled in third second.	1

# Answer

$$\text{i. } v = \sqrt{x} \Rightarrow \frac{1}{2}v^2 = \frac{1}{2}x$$

$$\therefore a = \frac{d}{dx}(\frac{1}{2}v^2) = \frac{1}{2} \text{ for all } x$$

Hence  $a$  is independent of  $x$ .

$$\text{ii. } \frac{dx}{dt} = x^{\frac{1}{2}}$$

$$\frac{dt}{dx} = x^{-\frac{1}{2}}$$

$$t = 2x^{\frac{1}{2}} + c$$

$$\left. \begin{array}{l} t = 0 \\ x = 1 \end{array} \right\} \Rightarrow c = -2$$

$$\therefore t = 2\sqrt{x} - 2$$

$$x = \frac{1}{4}(t+2)^2$$

iii. Between  $t = 2$  and  $t = 3$ , particle moves right from  $x = 4$  to  $x = \frac{25}{4}$

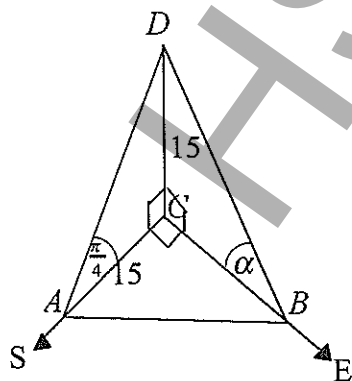
Distance travelled in third second is  $2.25 \text{ m}$ .

## c. Outcomes assessed : H5, HE5, HE7

### Marking Guidelines

Criteria	Marks
i • finds $AC$ and finds $BC$ in terms of $\cot \alpha$	1
• uses Pythagoras' theorem and an appropriate trig. identity to find $AB$ in terms of $\csc \alpha$	1
ii • differentiates $AB$ with respect to $t$ using chain rule or implicit differentiation	1
• substitutes given values and interprets result	1

# Answer



i. In  $\triangle ACD$ ,

$$\angle DAC = \angle ADC = \frac{\pi}{4}$$

$$\therefore AC = 15.$$

In  $\triangle BCD$ ,  $BC = 15 \cot \alpha$ .

$\therefore$  In  $\triangle ABC$ ,

$$AB^2 = 15^2 + 15^2 \cot^2 \alpha$$

$$= 15^2(1 + \cot^2 \alpha)$$

$$= 15^2 \csc^2 \alpha$$

$$\therefore AB = 15 \csc \alpha$$

ii. When  $\alpha = \frac{\pi}{3}$ ,

$$\frac{dAB}{dt} = -15 \csc \alpha \cot \alpha \frac{d\alpha}{dt}$$

$$= -15 \times \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}} \times 0.01$$

$$= -0.1$$

$\therefore AB$  is decreasing at a rate of  $0.1 \text{ ms}^{-1}$

### Question 6

a. Outcomes assessed : HE3

#### Marking Guidelines

Criteria	Marks
i • integrates $v$ with respect to $t$ to find expression for $x$	1
• uses initial conditions to evaluate the constant of integration, giving $x$ as a function of $t$	1
• differentiates $v$ with respect to $t$ to get $\ddot{x}$ then expresses $\ddot{x}$ in terms of $x$	1
ii • states period	1
• states extremities	1
iii • solves trig. equation to find time to first return	1

#### Answer

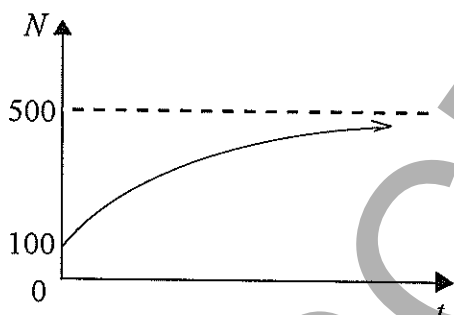
- i.  $v = -12\sin(2t + \frac{\pi}{3})$   
 $x = 6\cos(2t + \frac{\pi}{3}) + c$   
 $t = 0, x = 5 \Rightarrow c = 2$   
 $\therefore x = 2 + 6\cos(2t + \frac{\pi}{3})$   
 $\ddot{x} = -24\cos(2t + \frac{\pi}{3})$   
 $\therefore \ddot{x} = -4(x - 2)$
- ii. Period is  $\pi$  seconds.  $-4 \leq x \leq 8$
- iii.  $x = 5 \Rightarrow \cos(2t + \frac{\pi}{3}) = \frac{1}{2}$   
 $2t + \frac{\pi}{3} = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, \dots$   
 $t = 0, \frac{2\pi}{3}, \dots$   
 First return after  $\frac{2\pi}{3}$  seconds.

b. Outcomes assessed : HE3

#### Marking Guidelines

Criteria	Marks
i • sketches graph of correct shape with correct vertical intercept	1
• shows asymptote for limiting population size	1
ii • differentiates with respect to $t$	1
iii • writes and solves equation for $N$	1

#### Answer

- i. 
- ii.  $N = 500 - 400e^{-0.1t}$   
 $\frac{dN}{dt} = 0.1 \times 400e^{-0.1t}$   
 $= 0.1(500 - N)$
- iii. Initial rate of growth is  
 $0.1(500 - 100) = 0.1 \times 400$   
 $\therefore$  want  $N$  such that  
 $0.1(500 - N) = 0.1 \times 200$   
 $500 - N = 200$   
 $N = 300$

c. Outcomes assessed : H5, HE4

#### Marking Guidelines

Criteria	Marks
• uses inverse trig. identity to simplify equation	1
• uses trig. expansion to evaluate $x$ in terms of $k$	1

#### Answer

$$\cos^{-1} x - \sin^{-1} x = k, \quad -\frac{\pi}{2} \leq k \leq \frac{3\pi}{2}$$

$$\cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$$

$$\therefore 2\cos^{-1} x = k + \frac{\pi}{2}$$

$$\cos^{-1} x = \frac{k}{2} + \frac{\pi}{4}$$

$$x = \cos(\frac{k}{2} + \frac{\pi}{4})$$

$$\therefore x = \cos \frac{k}{2} \cos \frac{\pi}{4} - \sin \frac{k}{2} \sin \frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}} (\cos \frac{k}{2} - \sin \frac{k}{2})$$



### Question 7

a. Outcomes assessed : HE3

#### Marking Guidelines

Criteria	Marks
i • uses integration to find expressions for $\dot{x}$ and $x$	1
• uses integration to find expressions for $\dot{y}$ and $y$	1
ii • writes simultaneous equations for $V$ and $\theta$	1
• finds the value of $V$	1
• finds the value of $\theta$	1
iii • finds the values of $\dot{x}$ and $\dot{y}$ just before impact	1
• uses Pythagoras' theorem to find the magnitude of $v$	1
• uses trigonometry to find the direction of $v$ as an angle relative to the horizontal	1

#### Answer

i.

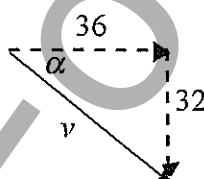
$$\begin{aligned}
 \ddot{x} &= 0 & x &= Vt \cos \theta + c_2 & \ddot{y} &= -10 & y &= -5t^2 + Vt \sin \theta + c_4 \\
 \dot{x} &= c_1 & t=0 & \left. \begin{array}{l} x=0 \end{array} \right\} \Rightarrow c_2 = 0 & \dot{y} &= -10t + c_3 & t=0 & \left. \begin{array}{l} y=0 \end{array} \right\} \Rightarrow c_4 = 0 \\
 \therefore \dot{x} &= V \cos \theta & \therefore x &= Vt \cos \theta & t=0 & \left. \begin{array}{l} \dot{y} = V \sin \theta \end{array} \right\} \Rightarrow c_3 = V \sin \theta & \therefore y &= Vt \sin \theta - 5t^2 \\
 & & & & \therefore \dot{y} &= -10t + V \sin \theta & &
 \end{aligned}$$

ii. When  $t = 8$

$$\begin{aligned}
 \left. \begin{array}{l} x = 288 \\ y = 64 \end{array} \right\} &\Rightarrow \begin{array}{l} 8V \cos \theta = 288 \\ 8V \sin \theta = 384 \end{array} & \therefore V^2 (\cos^2 \theta + \sin^2 \theta) &= 36^2 + 48^2 & \tan \theta &= \frac{48}{36} = \frac{4}{3} \\
 & & V &= 60 & \theta &= \tan^{-1} \frac{4}{3}
 \end{aligned}$$

iii. When  $t = 8$

$$\begin{aligned}
 \dot{x} &= 60 \times \frac{3}{5} = 36 \\
 \dot{y} &= -80 + 60 \times \frac{4}{5} = -32
 \end{aligned}$$



$$\begin{aligned}
 v^2 &= 36^2 + 32^2 & \tan \alpha &= \frac{8}{9} \\
 v &= 4\sqrt{145} & \alpha &\approx 41.6^\circ
 \end{aligned}$$

Velocity of rocket just before impact is approximately  $48 \text{ ms}^{-1}$  inclined at  $42^\circ$  below the horizontal.

b. Outcomes assessed : HE3

#### Marking Guidelines

Criteria	Marks
i • writes a typical term in $x^r$ in the expansion of the RHS of the identity	1
• collects like terms to find coefficient of $x^r$ , then equates to coefficient of $x^r$ on LHS	1
ii • writes single binomial coefficient for sum on LHS	1
• writes single binomial coefficient for sum on RHS then deduces result	1

## Answer

i.  $(1+x)^{m+n} = (1+x)^m(1+x)^n$

For  $0 \leq r \leq m$  and  $0 \leq r \leq n$ ,

terms in  $x^r$  in expansion of the RHS have the form  ${}^m C_k x^k \times {}^n C_{r-k} x^{r-k}$ ,  $k = 0, 1, 2, \dots, r$ .

Collecting such like terms gives the coefficient of  $x^r$  as  $\sum_{k=0}^r {}^m C_k {}^n C_{r-k}$ .

The coefficient of  $x^r$  in the expansion of the LHS is  ${}^{m+n} C_r$ .

Hence equating coefficients of  $x^r$  on both sides of the identity gives  ${}^{m+n} C_r = \sum_{k=0}^r {}^m C_k {}^n C_{r-k}$ .

ii. Using i., for  $m \geq 2$  and  $n \geq 2$ ,

$${}^{m+1} C_0 {}^n C_2 + {}^{m+1} C_1 {}^n C_1 + {}^{m+1} C_2 {}^n C_0 = {}^{(m+1)+n} C_2 \quad \text{and} \quad {}^m C_0 {}^{n+1} C_2 + {}^m C_1 {}^{n+1} C_1 + {}^m C_2 {}^{n+1} C_0 = {}^{m+(n+1)} C_2$$

$$\therefore {}^{m+1} C_0 {}^n C_2 + {}^{m+1} C_1 {}^n C_1 + {}^{m+1} C_2 {}^n C_0 = {}^m C_0 {}^{n+1} C_2 + {}^m C_1 {}^{n+1} C_1 + {}^m C_2 {}^{n+1} C_0 = {}^{m+n+1} C_2$$

Question	Marks	Content	Syllabus Outcomes	Targeted Performance Bands
1 a	2	Trigonometric functions	H5	E2-E3
b	2	Series and applications	H5	E2-E3
c	2	Polynomials	PE3	E2-E3
d	2	Angle between two lines	H5	E2-E3
e i	1	Circle geometry	PE3	E2-E3
ii	2	Circle geometry	PE2, PE3	E2-E3
iii	1	Circle geometry	PE3	E2-E3
2 a	2	Trigonometric functions	H5	E2-E3
b	2	Permutations and combinations	PE3	E2-E3
c	2	Division of an interval	H5	E2-E3
d	2	Trigonometric functions	H5	E2-E3
e i	2	Parametric representation	PE3, PE4	E2-E3
ii	2	Parametric representation	PE3, PE4	E2-E3
3 a	2	Trigonometric functions	H5	E2-E3
b	4	Induction	HE2	E3-E4
c i	1	Inverse functions	HE4	E2-E3
ii	3	Inverse functions	HE4	E2-E3
iii	2	Inverse functions	HE4	E2-E3
4 a	2	Further probability	HE3	E2-E3
b	4	Methods of integration	HE6	E2-E3
c i	2	Trigonometric functions	H5	E2-E3
ii	2	Polynomials	PE3	E2-E3
iii	2	Polynomials	PE3	E2-E3
5 a i	2	Inverse functions	HE4	E2-E3
ii	2	Inverse functions	HE4	E2-E3
b i	1	Velocity and acceleration as functions of displacement	HE5	E2-E3
ii	2	Velocity and acceleration as functions of displacement	HE5	E2-E3
iii	1	Velocity and acceleration as functions of displacement	HE5	E2-E3
c i	2	Trigonometric functions	H5	E3-E4
ii	2	Rates of change	HE5, HE7	E3-E4
6 a i	3	Simple harmonic motion	HE3	E3-E4
ii	2	Simple harmonic motion	HE3	E3-E4
iii	1	Simple harmonic motion	HE3	E3-E4
b i	2	Exponential growth and decay	HE3	E2-E3
ii	1	Exponential growth and decay	HE3	E2-E3
iii	1	Exponential growth and decay	HE3	E2-E3
c	2	Trigonometric functions, inverse functions	H5, HE4	E2-E3
7 a i	2	Projectile motion	HE3	E3-E4
ii	3	Projectile motion	HE3	E3-E4
iii	3	Projectile motion	HE3	E3-E4
b i	2	Binomial theorem	HE3	E3-E4
ii	2	Binomial theorem	HE3	E3-E4

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