

Student Number:

2006
HIGHER SCHOOL CERTIFICATE
Sample Examination Paper

MATHEMATICS

Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using blue or black pen
- Write your student number at the top of this page

Total marks – 84

- Attempt ALL questions.
- Show all necessary working, marks may be deducted for careless or untidy work.
- Board-approved calculators may be used.
- Additional Answer Booklets are available.

Directions to school or college

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QUESTION 1 (Start a new booklet)

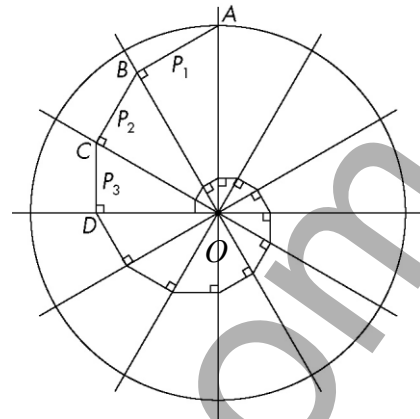
Marks

- (a) Find the exact value of $\int_0^1 \left(\frac{1}{1+x} + e^{-x} + \frac{1}{\sqrt{1-x^2}} \right) dx$ **4**
- (b) How many numbers greater than 5000 can be formed with the digits 4, 5, 6, 7 and 8 if no digit is used more than once in a number? **3**
- (c) Solve for x if $\log_{10} (10^x + 2) = 2x + 1$ **3**
- (d) Show that $\frac{1 - \cos 2x}{\sin 2x} = \tan x$ and hence express $\tan 15^\circ$ in simplest surd form. **2**

QUESTION 2 (Start a new booklet)

Marks

- (a) The figure shows a circle centre O and a radius 2 units. It is divided into 12 sectors and perpendiculars have been drawn as indicated. The points A, B, C , etc are shown as well as the distances $P_1, P_2, P_3, \dots$

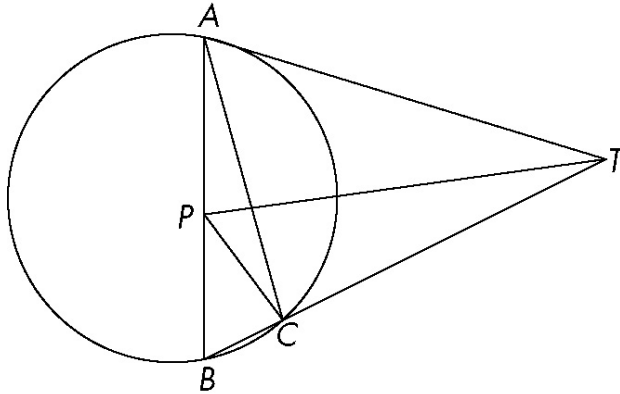


- (i) Find the value of P_1 by considering $\triangle OBA$. 1
- (ii) Show that the distances $P_1, P_2, P_3, \dots$ form a geometric series. 3
- (iii) Find the exact length of one spiral. ($P_1 + P_2 + P_3 + \dots + P_{12}$) 2
- (iv) If the spiral is continued indefinitely, show that its total length will not exceed 7.5 units. 1
- (b) Using the substitution $u = e^x$, find $\int \frac{e^x}{1 + e^{2x}} dx$. 2
- (c) Use Newton's method once to find a better root of the following equation $\cos x = \log_e x$ given that it has a root near $x = 1$. 3

QUESTION 3 (Start a new booklet)

Marks

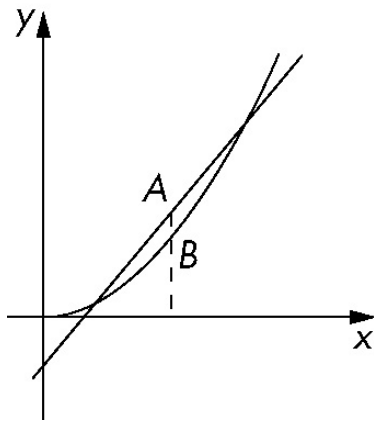
- (a) TA is a tangent to the circle at A , and AB is a chord.
 P is a point on AB such that $TA = PT$. The secant TB cuts the circle at C and AC and PC are joined.



- (i) Prove that $TAPC$ is a cyclic quadrilateral. 3
- (ii) Prove that TP is a tangent to circle PCB at P . 2
- (b) Differentiate with respect to x : $\cos^{-1}\left(\frac{1}{x}\right)$ 2
- (c) A bag contains twice as many white marbles as blue marbles. If a single marble is chosen at random, what is the probability that it is
- (i) white? 1
- (ii) blue? 1
- (iii) If two marbles are chosen with replacement, find the probability that they will be of different colours. 1
- (d) A particle is moving along the x -axis. Its velocity V at position x is given by $V = \sqrt{8x - x^2}$. Find the acceleration when $x = 3$. 2

QUESTION 4 (Start a new booklet)**Marks**

- (a) The diagram represent the graphs of $f(x) = x^2$ and $g(x) = 7x - 6$.



Not to scale

- (i) What is the maximum value of AB ? 3
- (ii) Show that the line AB at its maximum value divides the area between the curve and the straight line in half. 4
- (b) Given that $f(x) = x^2 + bx + c$ and $g(x) = c + x + bx^2$ and if both $f(x)$ and $g(x)$ are divided by $x - t$, the remainder is 2, prove that $t = \frac{2-c}{b+1}$ 2
- (c) (i) Express $\sin x + \sqrt{3} \cos x$ in the form $A \sin(x + \alpha)$ where α is in radians and $A > 0$. 2
- (ii) Hence or otherwise, sketch the graph of $y = \sin x + \sqrt{3} \cos x$ for $0 \leq x \leq \pi$. 1

QUESTION 5 (Start a new booklet)

Marks

- (a) (i) Draw graphs of $y = |\sin x|$ and $y = -\cos x$ for $0 \leq x \leq 360^\circ$. 1
- (ii) For what values of x will $|\sin x| + \cos x = 0$? 1
- (iii) For what values of x will $|\sin x| \cdot \cos x < 0$? 1
- (b) A bowl of hot soup at temperature $T^\circ\text{C}$, when placed in a cooler environment, loses heat according to the law

$$\frac{dT}{dt} = k(T - T_0)$$
 where t is the time elapsed in minutes and T_0 is the temperature of the environment in degrees Celsius.
- (i) A bowl of soup at 96°C is left to stand in a room at a temperature of 18°C . After 3 minutes the soup cools down to 75°C . Calculate the value of k to 4 decimal places. 2
- (ii) Susan wishes to enjoy her soup at a temperature of 60°C . How long should she wait? 2
- (c) On certain days of constant weather the variation of temperature each day follows a pattern of simple harmonic motion. If it is 13°C at 5 a.m. and 23°C at 5 p.m., at what daylight times would the temperature be
- (i) 18°C ? 3
- (ii) 15°C ? 1
- (iii) What would be the expected temperature at 1 p.m.? 1

QUESTION 6 (Start a new booklet)**Marks**

- (a) Differentiate $(x^2 + 2x + 2)e^{-x}$ and hence evaluate $\int_1^2 x^2 e^{-x} dx$ correct to 3 decimal places. 2
- (b) $P(2ap, ap^2)$ is a point on the parabola $x^2 = 4ay$.
- (i) Show that the equation of the normal at P is $x + py = 2ap + ap^3$. 2
- (ii) This normal cuts the y -axis at R . State the coordinates of R . 1
- (iii) From P a line PT is drawn perpendicular to the directrix meeting it in T . State the coordinates of T . 1
- (iv) If M is the midpoint of RT , find the coordinates of M . 1
- (v) Find the locus of M and show that it is a parabola with vertex at the focus of the original parabola. 3
- (c) A point P is moving on the curve $y = 2x^3$ in such a way that its x -coordinate is changing at a constant rate of 0.5 units/s. At what rate is the gradient changing when $x = 1$? 2

QUESTION 7 (Start a new booklet)

Marks

- (a) A particle is projected with a speed of 20 m/s and passes through a point P whose horizontal distance from the point of projection is 30 m and whose vertical height above the point of projection is $8\frac{3}{4}$ m. Taking $g = 10 \text{ m/s}^2$ and θ to be the angle of elevation, **8**
- (i) Prove that $x = 20t \cos \theta$ and $y = -5t^2 + 20t \sin \theta$.
- (ii) Find the angle of elevation of θ and the time taken for the particle to reach P .
- (b) In the expansion $(1 + x + kx^2)^9$ in ascending powers of x , the coefficient of x^2 is zero. Find the value of k . **4**

End of paper

STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1} x^{n+1}, n \neq -1; x \neq 0, \text{ if } n < 0$
$\int \frac{1}{x} dx$	$= \ln x, x > 0$
$\int e^{ax} dx$	$= \frac{1}{a} e^{ax}, a \neq 0$
$\int \cos ax dx$	$= \frac{1}{a} \sin ax, a \neq 0$
$\int \sin ax dx$	$= -\frac{1}{a} \cos ax, a \neq 0$
$\int \sec^2 ax dx$	$= \frac{1}{a} \tan ax, a \neq 0$
$\int \sec ax \tan ax dx$	$= \frac{1}{a} \sec ax, a \neq 0$
$\int \frac{1}{a^2 + x^2} dx$	$= \frac{1}{a} \tan^{-1} \frac{x}{a}, a \neq 0$
$\int \frac{1}{\sqrt{a^2 - x^2}} dx$	$= \sin^{-1} \frac{x}{a}, a > 0, -a < x < a$
$\int \frac{1}{\sqrt{x^2 - a^2}} dx$	$= \ln(x + \sqrt{x^2 + a^2}), x > a > 0$
$\int \frac{1}{\sqrt{x^2 + a^2}} dx$	$= \ln(x + \sqrt{x^2 + a^2})$
NOTE: $\ln x = \log_e x, x > 0$	

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2006 MATHEMATICS EXTENSION 1 HSC TRIAL
Examination Mapping Grid

Question	Marks	Content	Syllabus Outcomes	Targeted Performance Bands
1(a)	4	Integration of various functions	HE4	E2 – E3
1(b)	3	Permutations and combinations	PE3	E2 – E3
1(c)	3	Logarithmic functions	H3	E2 – E3
1(d)	2	Trigonometric functions	PE2 H5	E2 – E4
2(a)	7	Sequences and series	H5	E2 – E3
2(b)	2	Integration by substitution	HE6	E2 – E4
2(c)	3	Polynomials	PE3	E2 – E4
3(a)	5	Circle Geometry	PE3	E2 – E4
3(b)	2	Differentiation of inverse trigonometric functions	HE4/5	E2 – E3
3(c)	3	Probability	H5	E2 – E3
3(d)	2	Applications of calculus to the physical world	HE5	E2 – E3
4(a)	7	Calculus	HE1 HE4 PE6	E2 – E3
4(b)	2	Real functions of a real variable	PE3	E2 – E4
4(c)	3	Trigonometry	HE1 H9	E2 – E3
5(a)	3	Trigonometry	PE2 H5	E2 – E4
5(b)	4	Applications of calculus to the physical world	HE3	E2 – E3
5(c)	5	Applications of calculus to the physical world	HE3	E2 – E3
6(a)	2	Calculus	HE1	E2 – E4
6(b)	8	Parametric representation	PE3	E2 – E4
6(c)	2	Calculus	HE5	E2 – E4
7(a)	8	Applications of calculus to the physical world	HE3	E2 – E4
7(b)	4	Binomial theorem	HE3	E2 – E4

SOLUTIONS

QUESTION 1

$$(a) \int_0^1 \left(\frac{1}{1+x} + e^{-x} + \frac{1}{\sqrt{1-x^2}} \right) dx$$

$$= \left[\log(1+x) - e^{-x} + \sin^{-1} x \right]_0^1$$

$$= \log 2 - e^{-1} + \sin^{-1} 1 - (\log 1 - e^0 + \sin^{-1} 0)$$

$$= \log 2 - \frac{1}{e} + \frac{\pi}{2} + 1$$

(b) If all 5 digits are used: number of permutations = 5!

$$= 120$$

If only 4 digits are used

4	4	3	2
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$$\text{Number of permutations} = 4 \times 4 \times 3 \times 2 = 96$$

$$\therefore \text{Total} = 216$$

$$(c) \log_{10}(10^x + 2) = 2x + 1$$

$$10^x + 2 = 10^{2x+1}$$

$$10^x + 2 = 10(10^{2x})$$

$$\text{Let } 10^x = a$$

$$\therefore 10a^2 - a - 2 = 0$$

$$(5a + 2)(2a - 1) = 0$$

$$a = -\frac{2}{5}, a = \frac{1}{2}$$

$$10^x = -\frac{2}{5}; 10^x = \frac{1}{2}$$

$$\text{No solution; } x = \log_{10} \frac{1}{2}$$

$$\begin{aligned} \text{(d)} \quad \frac{1 - \cos 2x}{\sin 2x} &= \frac{1 - (1 - 2\sin^2 x)}{2\sin x \cos x} \\ &= \frac{2\sin^2 x}{2\sin x \cos x} \\ &= \frac{\sin x}{\cos x} \\ &= \tan x \\ \tan 15^\circ &= \frac{1 - \cos 30^\circ}{\sin 30^\circ} \\ &= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} \\ &= 2 - \sqrt{3} \end{aligned}$$

QUESTION 2

(a) (i) $OA = 2$ $\angle AOB = 30^\circ$ $\therefore P_1 = 1$ and $OB = \sqrt{3}$

(ii) $OB = \sqrt{3}$ $\angle BOC = 30^\circ$ $\therefore P_2 = \sqrt{3} \sin 30^\circ = \frac{\sqrt{3}}{2}$

and $OC = OB \cos 30^\circ = \sqrt{3} \cdot \frac{\sqrt{3}}{2} = \frac{3}{2}$

$OC = \frac{3}{2}$, $\angle COD = 30^\circ$, $\therefore P_3 = \frac{3}{2} \sin 30^\circ = \frac{3}{4}$

and $OD = OC \cos 30^\circ = \frac{3}{2} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$

$$\frac{P_3}{P_2} = \frac{\frac{3}{4}}{\frac{\sqrt{3}}{2}} = \frac{3}{4} \times \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\frac{P_2}{P_1} = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$$

$\therefore P_1, P_2, P_3, \dots$ form a G.P.

(iii) Length of 1 spiral $S_{12} = \frac{a(r^{12} - 1)}{r - 1} = \frac{a(1 - r^{12})}{1 - r}$

$$= \frac{1 \left[1 - \left(\frac{\sqrt{3}}{2} \right)^{12} \right]}{1 - \frac{\sqrt{3}}{2}}$$

$$= \frac{\left[1 - \frac{729}{4096} \right]}{\frac{2 - \sqrt{3}}{2}}$$

$$= \frac{(3367) \times 2}{4096(2 - \sqrt{3})}$$

$$= \frac{3367}{2048(2 - \sqrt{3})}$$

$$\begin{aligned}
 \text{(iv) } S_{\infty} &= \frac{a}{1-r} \\
 &= \frac{1}{1-\frac{\sqrt{3}}{2}} \\
 &= \frac{2}{2-\sqrt{3}} \\
 &\approx 7.464 < 7.5
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } u &= e^x \\
 du &= e^x dx \\
 \therefore \int \frac{du}{1+u^2} &= \tan^{-1} u + c \\
 &= \tan^{-1}(e^x) + c
 \end{aligned}$$

$$\text{(c) Let } f(x) = \cos x - \log x$$

$$f'(x) = -\sin x - \frac{1}{x}$$

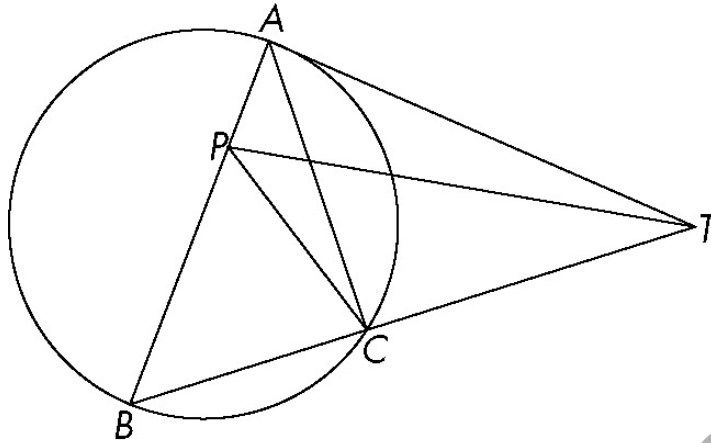
$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1 - \frac{\cos 1 - \log 1}{-\sin 1 - 1}$$

$$\approx 1.29$$

QUESTION 3

(a) (i)



$$\angle PAT = \angle TPA \quad (TA = TP)$$

$$\therefore \angle PAC + \angle TAC = \angle TPA$$

$$\angle TPA = \angle ABT + \angle PTB \quad (\text{exterior angle of } \triangle PBT)$$

$$\therefore \angle ABT + \angle PTB = \angle PAC + \angle TAC$$

But $\angle TAC = \angle TBA$ (angle between tangent and chord)

$$\therefore \angle PTB = \angle PAC$$

$\therefore TACP$ is a cyclic quadrilateral (angles subtended by arc PC are equal).

(ii) $TAC = \angle TPC$ ($TAPC$ is a cyclic quadrilateral)

But $\angle TAC = \angle ABC$ (angle between tangent and chord)

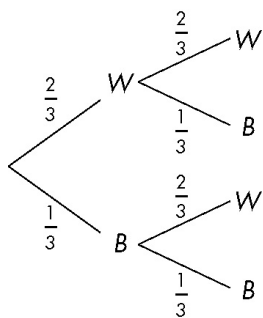
$$\therefore \angle TPC = \angle ABC$$

$\therefore PT$ is a tangent to circle PCB at P .

(b) Let $u = \frac{1}{x}$

$$\begin{aligned} y &= \cos^{-1} u \\ \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} = -\frac{1}{\sqrt{1-u^2}} \left(-\frac{1}{x^2} \right) \\ &= -\frac{1}{\sqrt{1-\frac{1}{x^2}}} \left(-\frac{1}{x^2} \right) \\ &= \frac{-x}{\sqrt{x^2-1}} \left(-\frac{1}{x^2} \right) \\ &= \frac{1}{x\sqrt{x^2-1}} \end{aligned}$$

(c)



$$(i) \quad P(E) = \frac{2}{3}$$

$$(ii) \quad P(E) = \frac{1}{3}$$

$$(iii) \quad P(E) = \frac{2}{3}\left(\frac{1}{3}\right) + \frac{1}{3}\left(\frac{2}{3}\right) \\ = \frac{4}{9}$$

$$(d) \quad v = \sqrt{8x - x^2}$$

$$\therefore v^2 = 8x - x^2$$

$$\frac{1}{2}v^2 = 4x - \frac{x^2}{2}$$

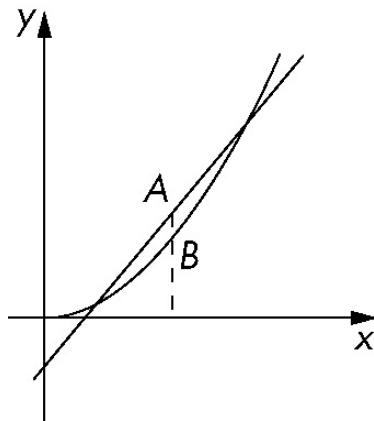
$$a = \frac{d}{dx}\left(4x - \frac{x^2}{2}\right)$$

$$a = 4 - x$$

$$\text{When } x = 3, a = 1$$

QUESTION 4

(a) (i)



Let distance $AB = s$

$$s = 7x - 6 - x^2$$

$$\frac{ds}{dx} = 7 - 2x = 0 \text{ for a maximum or minimum}$$

$$x = \frac{7}{2}$$

$$\frac{d^2s}{dx^2} = -2 \therefore \text{maximum}$$

$$\begin{aligned} s \text{ maximum} &= 7 \cdot \frac{7}{2} - 6 - \left(\frac{7}{2}\right)^2 \\ &= \frac{49}{2} - 6 - \frac{49}{4} \\ &= \frac{25}{4} \end{aligned}$$

(ii) The parabola and the line meet when $x^2 = 7x - 6$

$$x^2 - 7x + 6 = 0$$

$$(x - 6)(x - 1) = 0$$

$$x = 6, x = 1$$

Area between the parabola and the line is

$$\begin{aligned} &\int_1^6 (7x - 6 - x^2) dx \\ &= \left[\frac{7x^2}{2} - 6x - \frac{x^3}{3} \right]_1^6 \\ &= 126 - 36 - 72 - \left(\frac{7}{2} - 6 - \frac{1}{3} \right) \\ &= \frac{125}{6} \end{aligned}$$

Area between parabola and line and line $x = \frac{7}{2}$ is

$$\begin{aligned}\int_1^{\frac{7}{2}} (7x - 6 - x^2) dx &= \left[\frac{7x^2}{2} - 6x - \frac{x^3}{3} \right]_1^{\frac{7}{2}} \\&= \frac{343}{8} - 21 - \frac{343}{24} - \left(\frac{7}{2} - 6 - \frac{1}{3} \right) \\&= \frac{125}{12}\end{aligned}$$

$\therefore$ Line AB divides the area in half.

(b) $f(x) = x^2 + bx + c$ $f(t) = t^2 + tb + c = 2 \dots (1)$

$g(x) = c + x + bx^2$ $g(t) = c + t + bt^2 = 2 \dots (2)$

From (1) $t^2 = 2 - tb - c$

Sub in (2) $c + t + b(2 - tb - c) = 2$

$$c + t + 2b - tb^2 - bc = 2$$

$$t(1 - b^2) = 2 - c - 2b + bc$$

$$= 2 - c - b(2 - c)$$

$$t(1 - b)(1 + b) = (2 - c)(1 - b)$$

$$t(1 + b) = 2 - c$$

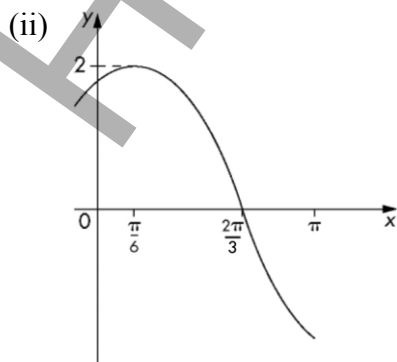
$$t = \frac{2 - c}{1 + b}$$

(c) (i) $A = \sqrt{1+3} = 2$

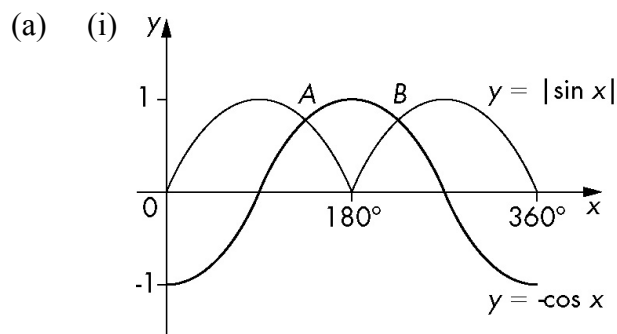
$$\tan \alpha = \frac{\sqrt{3}}{1}$$

$$\therefore \alpha = \frac{\pi}{3}$$

$$\therefore y = 2 \sin\left(x + \frac{\pi}{3}\right)$$



QUESTION 5



(ii) The solutions are found where the graphs intersect (see A and B)

$$|\sin x| = -\cos x$$

when $x = 135^\circ$ or 225°

(iii) $\cos x |\sin x| < 0$

when $-\cos x |\sin x| > 0$

i.e. when $90^\circ < x < 180^\circ$ and

$180^\circ < x < 270^\circ$

(b) (i) The solution of the differential equation is

$$T = T_0 + Ae^{-kt} \text{ where } A \text{ is a constant}$$

When $t = 0$, $T = 96$, $T_0 = 18$ so that $A = 78$

$$T = 18 + 78e^{-kt}$$

When $t = 3$, $T = 75^\circ$

$$75 = 18 + 78e^{-3k}$$

$$57 = 78e^{-3k}$$

$$e^{-3k} = \frac{57}{78}$$

$$-3k = \frac{\ln 57}{78}$$

$$k = \frac{\ln \frac{57}{78}}{-3}$$

$$\approx 0.1046$$

$$(ii) \quad 60 = 18 + 78e^{-0.1046t}$$

$$42 = 78e^{-0.1046t}$$

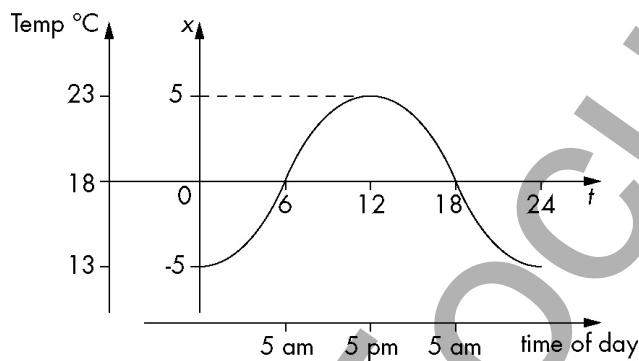
$$\frac{42}{78} = e^{-0.1046t}$$

$$-0.1046t = \ln \frac{42}{78}$$

$$t = \frac{\ln \frac{42}{78}}{-0.1046}$$

$$\approx 5.92 \text{ minutes}$$

- (c) Let x be the number of degrees by which the temperature differs from the mean temperature at time t after 5 a.m. (This places the origin of the temperature at mean temperature and the origin of time at 5 a.m.)



The motion is simple harmonic with amplitude 5°C and period 24 hours

$$\therefore T = \frac{2\pi}{n} \text{ and } n = \frac{\pi}{12}$$

$$\therefore x = -5 \cos \frac{\pi}{12}t$$

- (i) When temperature = 18°C

$$x = 0$$

$$\therefore \cos \frac{\pi t}{12} = 0$$

$$\frac{\pi t}{12} = \frac{\pi}{2}$$

$$t = 6 \text{ h}$$

$\therefore$ Time would be 11 a.m.

(ii) $-5 \cos \frac{\pi t}{12} = -3$

$$\cos \frac{\pi t}{12} = 0.6$$

$$\frac{\pi t}{12} = 0.927$$

$$t = 3 \text{ h } 32 \text{ mins}$$

$\therefore$ Time would be 8.32 a.m.

(iii) At 1 p.m. $t = 8$

$$x = -5 \cos \frac{\pi t}{12}$$

$$x = -5 \cos \frac{8\pi}{12}$$

$$= -5 \cos \frac{2\pi}{3}$$

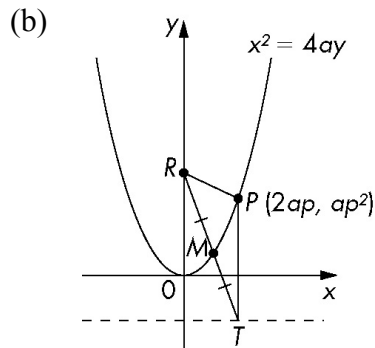
$$= -5 \left(-\frac{1}{2} \right)$$

$$= \frac{5}{2}$$

$\therefore$ Temperature is $18 + 2.5$
 $= 20.5^\circ\text{C}$

QUESTION 6

$$\begin{aligned}
 \text{(a)} \quad \frac{d}{dx} [x^2 + 2x + 2] e^{-x} &= (2x + 2)e^{-x} - e^{-x}(x^2 + 2x + 2) \\
 &= e^{-x}(2x + 2 - x^2 - 2x - 2) \\
 &= e^{-x}(-x^2) \\
 \therefore \int_1^2 x^2 e^{-x} dx &= \left[\frac{x^2 + 2x + 2}{e^x} \right]_1^2 \\
 &= \frac{5}{e} - \frac{10}{e^2} \\
 &\approx 0.486
 \end{aligned}$$



(i) $x^2 = 4ay$

$$y = \frac{x^2}{4a}$$

$$\frac{dy}{dx} = \frac{x}{2a}$$

At $x = 2ap$, $\frac{dy}{dx} = \frac{2ap}{2a} = p$

$\therefore$ Gradient of normal at P is $-\frac{1}{p}$

$\therefore$ Equation of normal at P is

$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$

$$py - ap^3 = -x + 2ap$$

i.e. $x + py = ap^3 + 2ap$

(ii) Normal cuts y -axis when $x = 0$

$$\text{i.e. } py = ap^3 + 2ap$$

$$\text{i.e. } y = ap^2 + 2a$$

$\therefore$ Coordinates of $R(0, ap^2 + 2a)$

(iii) Coordinates of $T(2ap, -a)$

(iv) Coordinates of $M\left(ap, \frac{ap^2 + a}{2}\right)$

(v) $x = ap$

$$y = \frac{ap^2 + a}{2}$$

$$\therefore p = \frac{x}{a}$$

$$y = \frac{a\left(\frac{x}{a}\right)^2 + a}{2}$$

$$2y = \frac{x^2}{a} + a$$

$$2ay = x^2 + a^2$$

$$x^2 = 2ay - a^2$$

$x^2 = 2a(y - a)$ which is a parabola with vertex at $(0, a)$, the focus of the original parabola

(c) Gradient $= m = \frac{dy}{dx}$
 $= 6x^2$
 $\frac{dm}{dt} = \frac{dm}{dx} \cdot \frac{dx}{dt}$
 $= 12x(0.5)$
 $= 12(1)(0.5)$
 $= 6 \text{ u / s}$

QUESTION 7

(a) (i) $\ddot{x} = 0, \dot{x} = 20 \cos \theta, x = 20t \cos \theta$

$$\dot{y} = -10, y = -10t + 20 \sin \theta, y = -5t^2 + 20t \sin \theta$$

(ii) At P $30 = 20t \cos \theta$

$$3 = 2t \cos \theta \quad \therefore t = \frac{3}{2 \cos \theta}$$

$$\frac{35}{4} = -5t^2 + 20t \sin \theta$$

$$35 = -20t^2 + 80t \sin \theta$$

$$7 = -4t^2 + 16t \sin \theta$$

$$\therefore 7 = -4 \left(\frac{3}{2 \cos \theta} \right)^2 + 16 \left(\frac{3}{2 \cos \theta} \right) \sin \theta$$

$$= -9 \sec^2 \theta + 24 \tan \theta$$

$$= -9(\tan^2 \theta + 1) + 24 \tan \theta$$

$$\therefore 9 \tan^2 \theta - 24 \tan \theta + 16 = (3 \tan \theta - 4)^2 = 0$$

$$\theta = \tan^{-1} \frac{4}{3} \quad \text{and} \quad \cos \theta = \frac{3}{5}$$

$$= 53^\circ 08'$$

$$t = \frac{3}{2 \left(\frac{3}{5} \right)}$$

$$= 2.5 \text{ s}$$

(b) $(1 + x + kx^2)^9 = 1 + \binom{9}{1}(x + kx^2) + \binom{9}{2}(x + kx^2)^2 + \dots$

$$= 1 + 9x + 9kx^2 + 36(x + 2kx^3 + \dots) + \dots$$

$$= 1 + 9x + (9k + 36)x^2 + \dots$$

$$\therefore 9k + 36 = 0$$

$$k = -4$$

2006 Mathematics Extension 1 HSC Trial
Marking Guidelines

Questions			Marks	Criteria
1	(a)		3	Integration
			1	Evaluation
1	(b)		1	All digits
			1	Four digits
			1	Total
1	(c)		1	Index form
			1	Quadratic equation
			1	Solution
1	(d)		1	Identity
			1	Substitution
2	(a)	(i)	1	Values of P_1
		(ii)	3	Ratio constant
		(iii)	2	Length of one spiral
		(iv)	1	Evaluation
2	(b)		1	Substitution
			1	Integration
2	(c)		1	First derivative
			1	Correct substitution into formulae
			1	Evaluation
3	(a)	(i)	3	Statements with reasons
		(ii)	2	Statements with reasons
3	(b)		1	Chain rule
			1	Algebra
3	(c)	(i)	1	Correct answer
		(ii)	1	Correct answer
		(iii)	1	Correct answer
3	(d)		1	Acceleration
			1	Evaluation
4	(a)	(i)	1	Expression for AB
			1	Differentiation
			1	Maximum value
		(ii)	1	Coordinates of intersection
			1	First area
			1	Second area
			1	Conclusion
4	(b)		1	$f(t)$ and $g(t)$
			1	Various methods of obtaining result
4	(c)	(i)	1	Value of A
			1	Value of α
		(ii)	1	Graph
5	(a)	(i)	1	Graphs
			1	Answer
			1	Answer
5	(b)	(i)	1	Solution of differential equation
			1	Value of k
		(ii)	1	Correct substitution

Questions			Marks	Criteria
			1	Answer
5	(c)	(i)	1	Value of n
			1	Trigonometric equation
			1	Time at 18°C
		(ii)	1	Time at 16°C
		(iii)	1	Temperature at 1 o'clock
6	(a)		1	Differentiation
			1	Integration
6	(b)	(i)	1	Gradient of normal
			1	Equation of normal
		(ii)	1	Coordinates of R
		(iii)	1	Coordinates of T
		(iv)	1	Coordinates of M
		(v)	3	Locus of M and vertex
	(c)		1	Gradient
			1	Rate
7	(a)	(i)	1	Horizontal motion
			1	Vertical motion
		(ii)	1	T in terms of θ
			1	Quadratic equation of t
			1	Quadratic equation of $\tan \theta$
			1	Factorisation and solution
			1	θ
			1	Time
7	(b)		1	Expansion
			1	Terms containing x^2
			1	Equation
			1	Solution