STUDENT NUMBER _____

TARA ANGLICAN SCHOOL FOR GIRLS

EXTENSION 1 MATHEMATICS
YEARLY EXAMINATION 2006

TIME ALLOWED: 2 Hours plus 5 minutes reading

GENERAL INSTRUCTIONS:
- Board approved calculators may be used.
- Write using a blue or black pen with diagrams in pencil
- Start each question in a new booklet

Question	
1	/14
2	/14
3	/14
4	/18
5	/13
6	/11
Total	/84

Outcomes to be assessed:

PE1 appreciates the role of Mathematics in the solution of practical problems

PE2 uses multi step deductive reasoning in a variety of contexts

PE3 solves problems involving inequalities and circle geometry

PE4 uses differentiation to identify geometric properties of parabolas

PE5 determines derivatives which require the application of more than one rule of differentiation

PE6 makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide variety of situations

Total marks – 84
Attempt Questions 1 – 6
Start each question in a new booklet

Question 1 (14 marks) **Marks**

(a) If $x + \sqrt{y} = (3\sqrt{2} - 2\sqrt{3})(2\sqrt{2} + 5\sqrt{3})$ find x and y. **3**

(b) Solve for a and b: **3**

$5^{a+b} = 125$

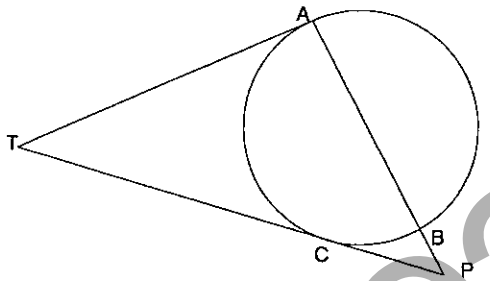
$7^{a-b} = 1$

(c) Solve for x:

$$\frac{4x-1}{x-1} \geq 3, \quad x \neq 1$$ **3**

(d) For n, find the least positive whole number for which $(1.017)^n > 2$. **1**

(e)



Not Drawn to Scale

AB is a diameter of a circle ABC. The tangents at A and C meet at T.
The lines TC and AB are produced to meet at P.

Copy the diagram into your examination booklet. Join AC and CB.

(i) Prove that $\angle CAT = 90^0 - \angle BCP$

(ii) Hence, or otherwise, prove that $\angle ATC = 2\angle BCP$ **4**

Turn page for Question 2

Question 2 (14 marks) Use a separate booklet **Marks**

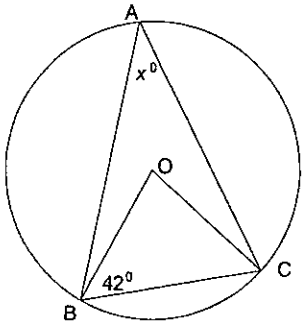
(a) Find the exact value of: $\dfrac{1+\cot^2 225^0}{\sec 300^0 \operatorname{cosec} 750^0}$ **3**

(b) Simplify $\left[\dfrac{x^a}{x^b}\right]^{a+b-c} \times \left[\dfrac{x^b}{x^c}\right]^{b+c-a} \times \left[\dfrac{x^c}{x^a}\right]^{c+a-b}$ **3**

(c) Solve for x: $\log_{x-2}(2x-1) = 2$ **2**

(d) Shade the region which satisfies $2x + 3y > 6$ and $|x-2| \leq 2$, simultaneously **4**

(e) Find the value of x giving reasons. **2**



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Turn page for question 3

Question 3 (14 marks) Use a separate booklet **Marks**

(a) State the natural (largest possible) domain of the function f for which $f(x) = \sqrt{9-x^2}$.

What is the range of this function? **2**

(b) If $f(u) = \frac{1}{u}$ express $f(u+1) - f(u)$ as a single fraction in its simplest form. **2**

(c) Evaluate (i) $\lim_{x \to 3} \frac{x^2-9}{x-3}$

(ii) $\lim_{x \to \infty} \frac{12x^2-9x+2}{7x^2+4x-9}$ **4**

(d) Point A is (3,1) and point B is (4,-2). If point P divides the interval AB externally in the ratio 1 : 3, find the co-ordinates of P. **2**

(e) Write down the expansion for tan (A-B) in terms of tan A and tan B.
The lines $y+x-4=0$ and $7y-x+20=0$ cut the y-axis at A and B respectively and intersect at P. Find the size of the angle APB. **4**

Question 4 (18 marks) Use a separate booklet. **Marks**

(a) Differentiate with respect to x:

(i) $y = x^2 - 2x - \frac{1}{x^2}$ (ii) $y = \frac{5}{\sqrt{x}}$

(iii) $y = \sqrt{2x+9}$ (iv) $y = (2x+3)(4x^2-7)^3$

(v) $y = \frac{-7x}{x^2+6}$ **15**

Turn Page for question 4

Question 4 cont'd

(b) Find the equation of the tangent to the curve $y = \frac{1}{2x}$ at the point $x = 1$. **3**

Question 5 (13 marks) Start a separate booklet. **Marks**

(a) (i) Write the expansion for $\cos(\alpha + \phi)$ **1**

(ii) Show that $\cos 2\phi = 2\cos^2\phi - 1$ **1**

(iii) Hence show that $\cos 15^0 = \frac{\sqrt{2+\sqrt{3}}}{2}$ **2**

(b) The elevation of the top of a hill at a place P due east of it is 40^0; at a place Q due south of P, the elevation is 25^0. If the distance from P to Q is 600 m, find the height of the hill, to the nearest metre. **4**

(c) (i) Express $3\cos\phi + 4\sin\phi$ in the form $R\cos(\phi - \alpha))$ where $R > 0$ **1**

(ii) Hence solve the equation $3\cos\phi + 4\sin\phi = 2$ for $0^0 \le \phi \le 360^0$ **3**

Turn page for question 6

Question 6 (11 marks) Use a separate booklet. **Marks**

(a) Explain briefly how to find the angle between the two different curves $y = f(x)$ and $y = g(x)$. **2**

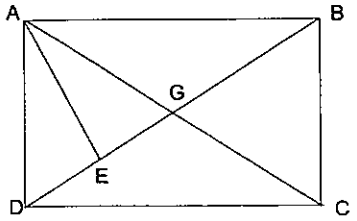
(b)

If $g(m) = \begin{cases} 1 - m^2 \text{ if } m > 1 \\ m^2 - 1 \text{ if } m \leq 1 \end{cases}$

find the value of $g(2) + g(-1) - g(0)$ and sketch the graph of $y = g(m)$. **4**

(c) If the straight line $y = mx + b$ is 1 unit from the origin, prove that $b^2 = m^2 + 1$ **2**

(d) ABCD is a rectangle with sides $AB = CD = p$ units and $BC = DA = q$ units where the diagonals intersect in G. AE is perpendicular to the diagonal BD.



(i) Show that $\cos\theta = \frac{q}{\sqrt{p^2+q^2}}$ where $\theta = ADE$

(ii) Show that $DE = \frac{q^2}{\sqrt{p^2+q^2}}$

(iii) Deduce that the area of ΔAEG is $\frac{pq(p^2-q^2)}{4(p^2+q^2)}$ units squared. **3**

End of Paper