



**SYDNEY BOYS HIGH
SCHOOL**
MOORE PARK, SURRY HILLS

2005

YEAR 12

**TRIAL HIGHER SCHOOL
CERTIFICATE EXAMINATION**

Mathematics Extension 1

General Instructions

- Working time – 2 Hours.
- Reading Time – 5 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work
- Hand in your answer booklets in 4 sections. Section A (Questions 1 and 2), Section B (Questions 3 and 4), Section C (Questions 5 and 6) and Section D (Question 7)

Total Marks - 84

- Attempt questions 1 – 7
- All QUESTIONS are of equal value.

Examiner: *A. Fuller*

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate

Total marks - 84

Attempt Questions 1 - 7

All questions are of equal value

Answer each SECTION in a SEPARATE writing booklet.

Section A

Marks

Question 1 (12 marks)

- (a) Simplify $\frac{3^n}{3^{n+1} - 3^n}$ 1
- (b) Evaluate $\lim_{x \rightarrow 0} \frac{\sin 5x}{4x}$ 1
- (c) The remainder when $x^3 - 3x^2 + px - 14$ is divided by $x - 3$ is 1. Find the value of p . 2
- (d) Given that $\log_a 2 = x$, find $\log_a(2a)$ in terms of x . 2
- (e) Find the coordinates of the point P that divides the interval from $A(-1, 5)$ to $B(6, -4)$ externally in the ratio $3 : 2$. 2
- (f) Find, to the nearest minute, the acute angle between the lines $3x + 2y - 5 = 0$ and $x - 5y + 7 = 0$. 2
- (g) Solve the inequality $\frac{2}{x} \leq 1$ 2

Question 2 (12 marks)

(a) Differentiate with respect to x

(i) $y = \tan^3(5x + 4)$ 2

(ii) $y = \ln\left(\frac{2x+3}{3x+4}\right)$ 2

(iii) $y = \cos(e^{1-5x})$ 2

(b) 30 girls, including Miss Australia, enter a Miss World Competition. The first six places are announced.

(i) How many different announcements are possible? 1

(ii) How many different announcements are possible if Miss Australia is assured a place in the first six? 2

(c) If $f(x) = \tan^{-1}(2x)$ evaluate:

(i) $f\left(\frac{1}{2}\right)$ 1

(ii) $f'\left(\frac{1}{2}\right)$ 2

End of Section

Section B (Use a SEPARATE writing booklet)

Marks

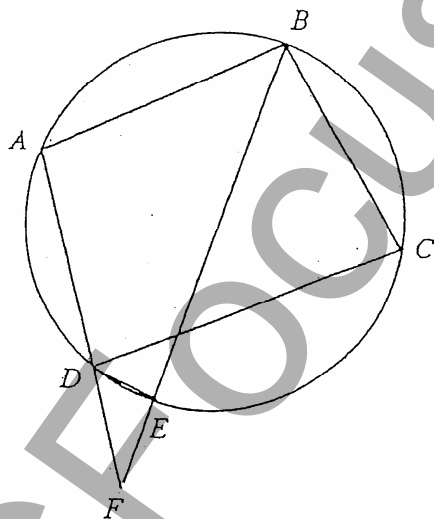
Question 3 (12 marks)

- (a) (i) State the natural domain and the corresponding range of $y = 3 \cos^{-1}(x - 2)$ 2
- (ii) Hence, or otherwise sketch $y = 3 \cos^{-1}(x - 2)$ 1
- (b) Find $\int x\sqrt{16 + x^2} dx$ using the substitution $u = 16 + x^2$ 2
- (c) Find the general solution of $\sin 2\theta = \sqrt{3} \cos 2\theta$ 2
- (d) The roots of the equation $4x^3 + 6x^2 + c = 0$, where c is a non-zero constant, are α , β , and $\alpha\beta$. 5
- (i) Show that $\alpha\beta \neq 0$.
- (ii) Show that $\alpha\beta + \alpha^2\beta + \alpha\beta^2 = 0$ and deduce the value of $\alpha + \beta$.
- (iii) Show that $\alpha\beta = -\frac{1}{2}$.

Question 4 (12 marks)

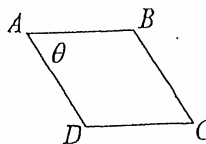
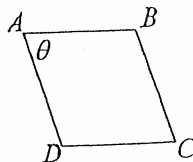
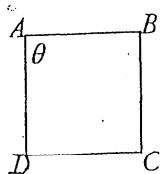
- (a) If $\tan \theta = 2$ and $0 < \theta < \frac{\pi}{2}$ evaluate $\sin\left(\theta + \frac{\pi}{4}\right)$. 3

- (b) In the diagram ABCD is a cyclic quadrilateral. The bisector of $\angle ABC$ cuts the circle at E, and meets AD produced at F.



- (i) Copy the diagram showing the above information
- (ii) Give a reason why $\angle CDE = \angle CBE$ 1
- (iii) Show that DE bisects $\angle CDF$ 3

(c)



A square ABCD of side 1 unit is gradually 'pushed over' to become a rhombus. The angle at A (θ) decreases at a constant rate of 0.1 radians per second.

- (i) At what rate is the area of the rhombus ABCD decreasing when $\theta = \frac{\pi}{6}$?

2

- (ii) At what rate is the shorter diagonal of the rhombus ABCD decreasing when $\theta = \frac{\pi}{3}$?

3

End of Section

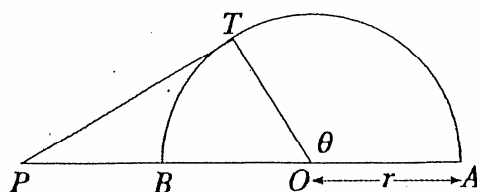
Section C (Use a SEPARATE writing booklet)

Marks

Question 5 (12 marks)

- (a) Two boys decide to settle an argument by taking turns to toss a die. The first person to throw a six wins.
- (i) What is the probability that the first person wins on his second throw? 1
- (ii) What is the probability that the first person will win the argument? 2
- (b) $P(2at, at^2)$, $t > 0$ is a point on the parabola $x^2 = 4ay$.
The normal to the parabola at P cuts the x axis at X and the y axis at Y.
- (i) Show that the normal at P has equation $x + ty - 2at - at^3 = 0$ 2
- (ii) Find the co-ordinates of X and Y 1
- (iii) Find the value of t such that P is the midpoint of XY 2

(c)



The point T lies on the circumference of a semicircle, radius r and diameter AB , as shown. The point P lies on AB produced and PT is the tangent at T .

4

The arc AT subtends an angle of θ at the centre, O , and the area of $\triangle OPT$ is equal to that of the sector AOT .

(i) Show that $\theta + \tan \theta = 0$.

(ii) Taking 2 as an approximation to θ , use Newton's method once to find a better approximation to two decimal places.

Question 6 (12 marks)

- (a) A particle is oscillating in simple harmonic motion such that its displacement x metres from a given origin O satisfies the equation $\frac{d^2x}{dt^2} = -4x$ where t is the time in seconds
- (i) Show that $x = \alpha \cos(2t + \beta)$ is a possible equation of motion for this particle, where α and β are constants 2
- (ii) The particle is observed initially to have a velocity of 2 metres per second and a displacement from the origin of 4 metres. Find the amplitude of the oscillation. 2
- (iii) Determine the maximum velocity of the particle 2
- (b) Prove by Mathematical Induction that 3
- $$\sum_{r=1}^n r^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4}n^2(n+1)^2$$
- (c) Consider the function $f(x) = \frac{x}{\sqrt{1-x^2}}$
- (i) Find the domain of $f(x)$ 1
- (ii) Find $f^{-1}(x)$, the inverse function of $f(x)$ 2

End of Section

Section D (Use a SEPARATE writing booklet)

Marks

Question 7 (12 marks)

- (a) A projectile fired with velocity V and at an angle of 45° to the horizontal, just clears the tops of two vertical posts of height $8a^2$, and the posts are $12a^2$ apart. There is no air resistance, and the acceleration due to gravity is g .
- (i) If the projectile is at a point $P(x, y)$ at time t ,
Derive expressions for x and y in terms of t . 2
- (ii) Hence, show that the equation of the path of the projectile
is $y = x - \frac{gx^2}{V^2}$ 2
- (iii) Using the information in (ii) show that the range of the
projectile is $\frac{V^2}{g}$ 2
- (iv) If the first post is b units from the origin, show that 2
- (α) $\frac{V^2}{g} = 2b + 12a^2$
- (β) $8a^2 = b - \frac{gb^2}{V^2}$
- (v) Hence or otherwise prove that $V = 6a\sqrt{g}$ 4

End of paper