NAME: _____

TEACHER: _____

EXTENSION 1 MATHEMATICS

ASSESSMENT TASK 3

TIME ALLOWED : 45 MINUTES

INSTRUCTIONS

- Answer all questions in the space provided with all possible working.
- Board approved calculators may be used.
- Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \; n \neq -1; \; x \neq 0, \; if \; n < 0$$

$$\int \frac{1}{x} dx = \ln x, \; x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \; a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \; a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \; a \neq 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), \; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x, \; x > 0$

MARKS

1) Let $f(x) = x^3 + 2x^2 + 5x - 4$ .The equation $f(x) = 0$ has only one real root.

(i) Show that the root lies between $x = 0$ and $x = 1$. (2)

(ii) Use one application of the "halving the interval" method to find a smaller interval containing the root. (2)

(iii) Determine which end of the improved interval found in part (ii) is closer to the root. Explain your answer. (2)

2) Use one application of Newtons method to improve the initial estimate $x_0 = 2$ of a root of $f(x) = 2x^3 + 3x^2 - 30$ (3)

3) Consider the function $y = \frac{4}{x+2}$.

(i) Write down the domain and range of the function $y = \frac{4}{x+2}$ in the space provided. (2)

DOMAIN { }

RANGE { }

(ii) Find the inverse function of $y = \frac{4}{x+2}$. (1)

(iii) Find the domain and range of the inverse function. Write the domain and range in the space provided. (2)

DOMAIN { }

RANGE { }

4) Find the exact values of the following

(i) $\sin^{-1}(\frac{1}{2})$ (1)

(ii) $\cos^{-1}(-\frac{\sqrt{3}}{2})$ (1)

(iii) $\tan(\frac{-1}{\sqrt{3}})$ (1)

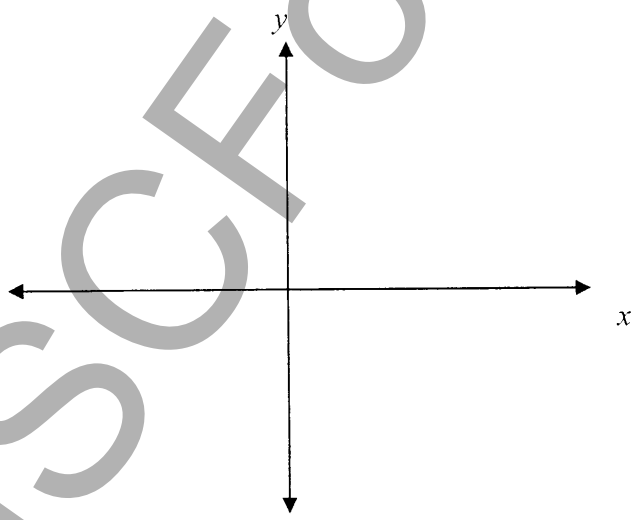
5)

Sketch the following inverse trigonometric functions
and state clearly the domain and range in the space provided

(i) $y = \tan^{-1} x$ (3)

DOMAIN{ }

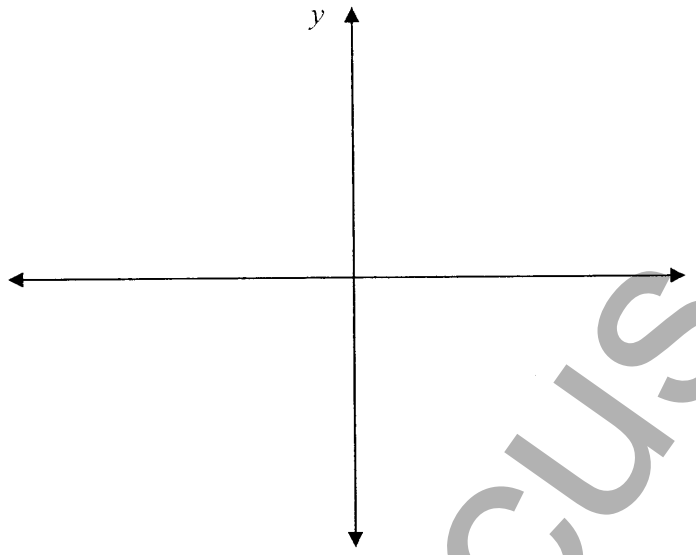
RANGE { }



(ii) $y = 3\cos^{-1} x$ (3)

DOMAIN{ }

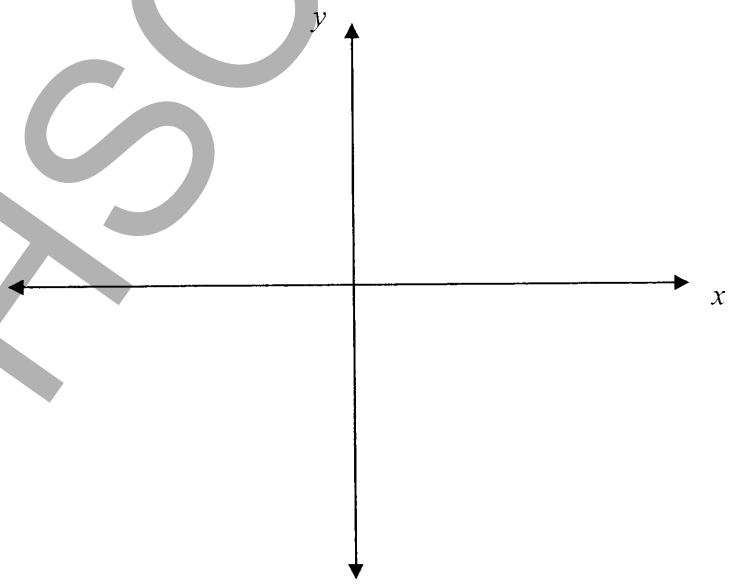
RANGE{ }



(iii) $y = 4\sin^{-1} 2x$ (3)

DOMAIN{ }

RANGE{ }



6) Find $\frac{dy}{dx}$ for each of the following.

(i) $y = \tan^{-1} 2x$ (1)

(ii) $y = 3\sin^{-1} x$ (1)

(iii) $y = \cos^{-1} 6x$ (1)

(iv) $y = e^{3x} \sin^{-1} 3x$ (2)

(v) $y = (4 + \tan^{-1} 4x)^4$ (2)

7) Find the equation of the tangent to the curve $y = 2\sin^{-1} x$ (3)
at the point where $x = \frac{\sqrt{3}}{2}$.

8) Find each of the following

(i) $\int \frac{1}{\sqrt{16 - x^2}}\, dx$ (1)

(ii) $\int \frac{1}{\sqrt{4 - 9x^2}}\, dx$ (2)

9) Evaluate the following (3)

$$\int_{\frac{-1}{2}}^{\frac{1}{2}} \frac{dt}{1+4t^2}$$

10) The area bounded by the curve $y = \frac{3}{\sqrt{x^2+4}}$,the x axis (3)
and the ordinates of $x = 0$ and $x = 2$, is rotated about the x axis.
Find the volume of the solid of revolution so formed.