

--	--	--	--	--

Centre Number

--	--	--	--	--	--	--	--	--	--

Student Number



**CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES**

**2003
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION**

Mathematics

Extension 1

Afternoon Session
Tuesday 12 August 2003

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- All necessary working should be shown in every question

Total marks (84)

- Attempt Questions 1 – 7
- All questions are of equal value

Disclaimer

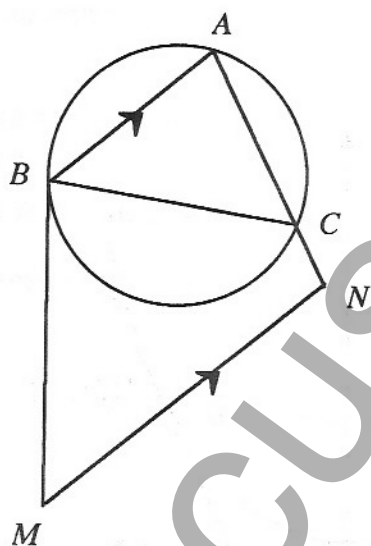
Every effort has been made to prepare these 'Trial' Higher School Certificate Examinations in accordance with the Board of Studies documents, *Principles for Setting HSC Examinations in a Standards-Referenced Framework* (BOS Bulletin, Vol 8, No 9, Nov/Dec 1999), and *Principles for Developing Marking Guidelines Examinations in a Standards Referenced Framework* (BOS Bulletin, Vol 9, No 3, May 2000). No guarantee or warranty is made or implied that the 'Trial' Examination papers mirror in every respect the actual HSC Examination question paper in any or all courses to be examined. These papers do not constitute 'advice' nor can they be construed as authoritative interpretations of Board of Studies intentions. The CSSA accepts no liability for any reliance use or purpose related to these 'Trial' question papers. Advice on HSC examination issues is only to be obtained from the NSW Board of Studies.

Question 1

Begin a new page

Marks

- (a) Find the value of $\lim_{n \rightarrow \infty} \frac{5(10^n) + 3}{2(10^n) + 1}$. 2
- (b) $A(-2, 5)$ and $B(7, -1)$ are two points. Find the coordinates of the point $M(x, y)$ which divides the line AB internally in the ratio $2 : 1$. 2
- (c) Solve the inequality $\frac{2}{x} > x - 1$. 4
- (d)



ABC is a triangle inscribed in a circle. M is a point on the tangent to the circle at B and N is a point on AC produced so that MN is parallel to BA .

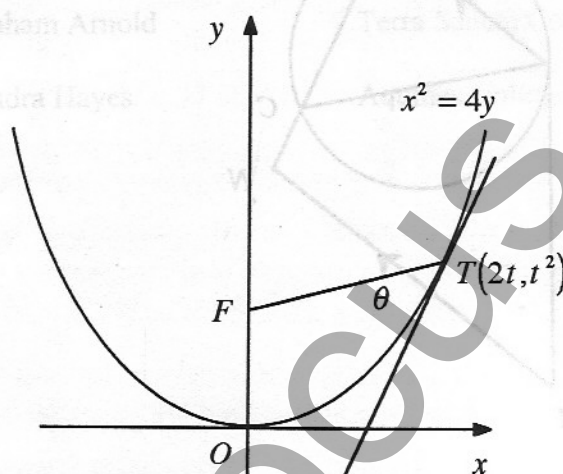
- (i) Copy the diagram. 1
- (ii) Give a reason why $\angle MBC = \angle BAC$. 3
- (iii) Show that $MNCB$ is a cyclic quadrilateral.

Question 2

Begin a new page

Marks

- (a) If $y = (x^2 + 1)^5$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. 2
- (b) Find the value of $\sum_{n=2}^5 {}^nC_2$. 2
- (c)
- (i) Show that $(\sin A - \cos A)^2 = 1 - \sin 2A$. 2
- (ii) Hence find the exact value of $\sin 15^\circ - \cos 15^\circ$. 2
- (d)



$T(2t, t^2)$ is a point on the parabola $x^2 = 4y$ with focus F . The tangent to the parabola at T makes an acute angle θ with the line FT .

- (i) Show that the tangent to the parabola at T has gradient t . 1
- (ii) Find $\tan \theta$ in simplest form in terms of t . 3

Question 3

Begin a new page.

- (a) Consider the function $f(x) = x - e^{-x}$.
- Show that the function is increasing and its graph is concave down for all values of x in its domain. 2
 - Use one application of Newton's Method with an initial approximation of $x = 0.5$ to find the value of the x intercept on the graph of $y = f(x)$, giving the answer correct to one decimal place. 2
 - Sketch the graph of $y = f(x)$ showing clearly the intercepts on the axes and the equations of any asymptotes. 3
- On the same diagram sketch the graph of the inverse function $y = f^{-1}(x)$.
- (b) A particle is moving in a straight line. At time t seconds it has displacement x metres (where $0 \leq x < \frac{\pi}{2}$) from a fixed point O on the line, velocity $v \text{ ms}^{-1}$ given by $v = \cos^2 x$ and acceleration $a \text{ ms}^{-2}$. The particle starts at O .
- Find expressions for a in terms of x and for x in terms of t . 2
 - Sketch the graph of x against t . 1
 - Describe the motion of the particle from its initial position to its limiting position. 2

Question 4

Begin a new page.

- (a) Consider the function $f(x) = \cos^{-1} \sqrt{x}$.
- (i) Find the domain and the range of the function and sketch the graph of $y = f(x)$. 3
 - (ii) Use Simpson's Rule with three function values to find an approximation to the area bounded by the curve $y = f(x)$ and the coordinate axes. 2
 - (iii) Use integration to find the exact area bounded by the curve $y = f(x)$ and the coordinate axes. 2
- (b) A particle moving in a straight line is performing Simple Harmonic Motion. At time t seconds it has displacement x metres from a fixed point O on the line, velocity $v \text{ ms}^{-1}$ and acceleration $\ddot{x} \text{ ms}^{-2}$ given by $\ddot{x} = -4(x - 2)$. The particle is at rest at the fixed point O .
- (i) Show that $v^2 = -4x^2 + 16x$. 2
 - (ii) Find the period and amplitude of the motion. 2
 - (iii) Find the distance travelled by the particle in the first minute of its motion, giving the answer correct to the nearest metre. 1

Question 5

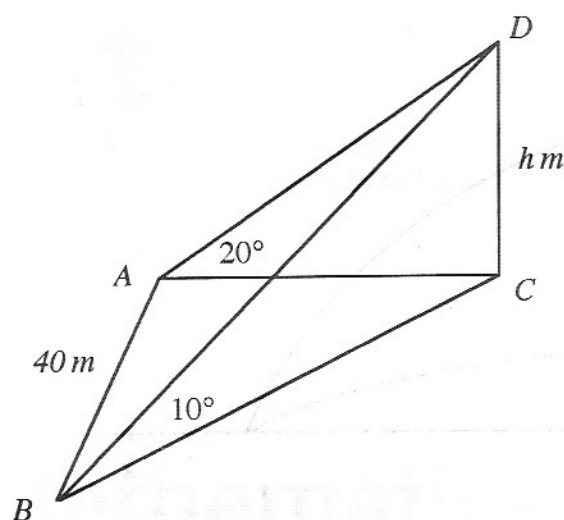
Begin a new page.

- (a) In the expansion of $(1 + ax)^9$ the coefficient of x^5 is twice the coefficient of x^6 . Find the value of the constant a . 4
- (b) Evaluate $\int_1^{49} \frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}} dx$ using the substitution $u = 1 + \sqrt{x}$, expressing the answer in the form \sqrt{n} for some positive integer n . 4
- (c) A container with capacity A litres is being filled with water. After t minutes the volume V litres of water in the container is given by $V = A(1 - e^{-kt})$ for some constant $k > 0$.
- (i) Show that $\frac{dV}{dt} = k(A - V)$. 2
 - (ii) If one quarter of the container is filled in the first two minutes find what fraction of the container is filled in the next two minutes. 2

Question 6

Begin a new page.

(a)



A vertical flagpole CD of height h metres stands with its base C on horizontal ground. A is a point on the ground due West of C and B is a point on the ground 40 metres due South of A . From A and B the angles of elevation of the top D of the flagpole are 20° and 10° respectively. Find the height of the flagpole correct to the nearest metre.

4

(b) A die is biased so that in any single throw the probability of an odd score is p where p is a constant such that $0 < p < 1$, $p \neq 0.5$.

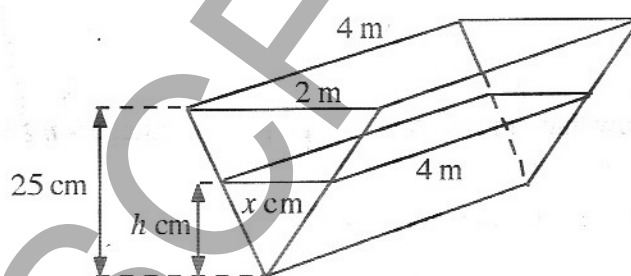
(i) Show that in six throws of the die the probability of at most one even score is $6p^5 - 5p^6$.

2

(ii) Find the probability that in six throws of the die the product of the scores is even.

2

(c)



An open flat topped water trough in the shape of a triangular prism is being emptied through a hole in its base at a constant rate of 16 litres per second. Its top measures 2 metres by 4 metres and its triangular end has a vertical height of 25 centimetres. When the water depth is h centimetres the water surface measures x centimetres by 4 metres.

(i) Show that when the water depth is h centimetres the volume $V \text{ cm}^3$ of water in the trough is given by $V = 1600h^2$.

2

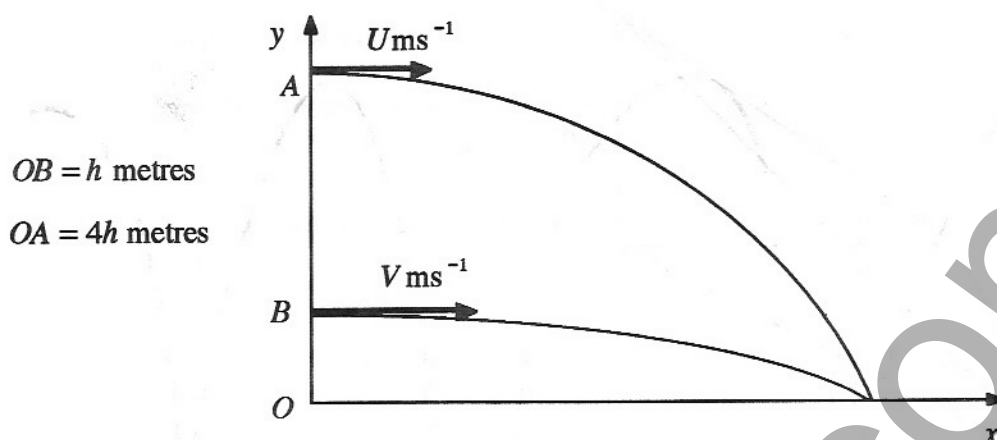
(ii) Find the rate at which the depth of water is changing when $h = 10 \text{ cm}$.

2

Question 7

Begin a new page.

(a)



A vertical building stands with its base O on horizontal ground. A and B are two points on the building vertically above each other such that A is $4h$ metres above O and B is h metres above O . A particle is projected horizontally with speed $U \text{ ms}^{-1}$ from A and 10 seconds later a second particle is projected horizontally with speed $V \text{ ms}^{-1}$ from B . The two particles hit the ground at the same point and at the same time.

- (i) Write down expressions for the horizontal and the vertical displacements relative to O of each particle t seconds after the first particle is projected. 2
- (ii) Find the time of flight of each particle. 3
- (iii) Show that $V = 2U$. 1

(b)

- (i) Use Mathematical Induction to show that $\ln(n!) > n$ for all positive integers $n \geq 6$. 3
- (ii) Hence show that $\frac{1}{n!} < \frac{1}{e^n}$ for all positive integers $n \geq 6$. 1
- (iii) Hence show that $\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \dots < \frac{103}{60} + \frac{1}{e^5(e-1)}$. 2