CATHOLIC SECONDARY SCHOOLS
ASSOCIATION OF NEW SOUTH WALES

2005
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1
Marking guidelines/ solutions

Please note: Mapping grid for this examination is on the last page of these Marking guidelines/solutions

2603- 2

Question 1

a. Outcomes Assessed: H5

Marking Guidelines

Criteria	Marks
• recognises that $\lim_{x\to 0}\frac{\sin 2x}{2x}=1$	1
• uses this result to show the required limit is $\frac{2}{5}$	1

Answer

$$\lim_{x\to 0}\frac{\sin 2x}{5x}=\lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{2}{5}=1\times\tfrac{2}{5}=\tfrac{2}{5}$$

b. Outcomes Assessed: i. PE3 ii. PE4

Marking Guidelines

Criteria	Marks
i. • uses the fact that 2 is a zero of $P(x)$ to write an equation in a and b	1
• uses the remainder theorem to write an equation in a and b	1
ii. • solves simultaneous equations to find the values of a and b	1

Answer

i. $P(2)=0 \Rightarrow 8+2a+b=0$ (1)
$P(-1)=-15 \Rightarrow -1-a+b=-15$ (2)

ii. (1) − (2) $\Rightarrow 9+3a = 15$
$\therefore a=2$
Substitution in (2) gives $b=-12$

c. Outcomes Assessed: i. H5 ii. P4

Marking Guidelines

Criteria	Marks
i. • differentiates and substitutes to find both gradients	1
ii. • obtains an expression in terms of e for the tangent of the angle	1
• evaluates this expression then finds the angle correct to the nearest degree	1

Answer

i. $y=e^x$
$\frac{dy}{dx}=e^x$

The tangent at $x=0$ has gradient $e^0=1$
The tangent at $x=1$ has gradient $e^1=e$

ii. $\tan\theta=\left|\frac{e-1}{1+e}\right|\approx 0\cdot 4621$ $\therefore$ required acute angle is $25°$ (to the nearest degree)

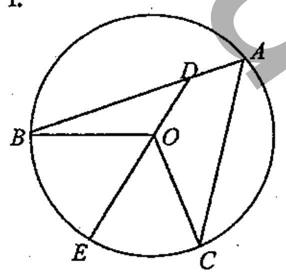
d. Outcomes Assessed: ii. **PE3** iii. **PE2, PE3**

Marking Guidelines

Criteria	Marks
ii. • quotes an appropriate angle property of cyclic quadrilateral $ADOC$	1
iii. • states $\angle COB = 2\angle CAB$	1
• deduces required result	1
• justifies these deductions by quoting an appropriate circle property	1

Answer

i.



ii. Exterior angle COE of cyclic quadrilateral $ADOC$ is equal to the interior opposite angle CAD.

iii. $\angle COB = 2\angle CAB$ (angle at centre is twice angle at circumference subtended by arc BC)

$\therefore \angle COB = 2\angle COE$ ($\angle CAB = \angle CAD = \angle COE$)

$\therefore \angle BOE = \angle COE$ and hence DOE bisects $\angle BOC$

Question 2

a. Outcomes Assessed: H3

Marking Guidelines

Criteria	Marks
• uses the change of base formula to express the logarithm to base e or 10	1
• evaluates the logarithm	1

Answer

$$\log_2 7 = \frac{\log_e 7}{\log_e 2} \approx 2\cdot 81 \text{ (to 2 decimal places)}$$

b. Outcomes Assessed: i. **H5** ii. **H5**

Marking Guidelines

Criteria	Marks
i. • simplifies the expression by taking a common denominator	1
• recognises the expression for $\tan 2x$	1
ii. • uses the result from i. to evaluate the expression in surd form.	1

Answer

i

$$\frac{1}{1-\tan x} - \frac{1}{1+\tan x} = \frac{1+\tan x-(1-\tan x)}{1-\tan^2 x}$$
$$= \frac{2\tan x}{1-\tan^2 x}$$
$$= \tan 2x$$

ii.

$$\frac{1}{1-\tan\frac{\pi}{6}} - \frac{1}{1+\tan\frac{\pi}{6}} = \tan\tfrac{\pi}{3}$$
$$= \sqrt{3}.$$

c. Outcomes Assessed: i. **P4** ii. **P4**

Marking Guidelines

Criteria	Marks
i. • finds equation for x	1
ii. • solves a quadratic equation to find two values for x (award 1 mark for correct process with a minor error in application)	2

Answer

i.

$$A(x, 10) \quad B(x^2, 6)$$

$$3 : -1$$

$$\left(\frac{3x^2 - x}{2}, \frac{18-10}{2}\right) \equiv P(5, 4)$$

$$\therefore \frac{3x^2 - x}{2} = 5$$

ii.

$$3x^2 - x = 10$$

$$3x^2 - x - 10 = 0$$

$$(3x+5)(x-2) = 0$$

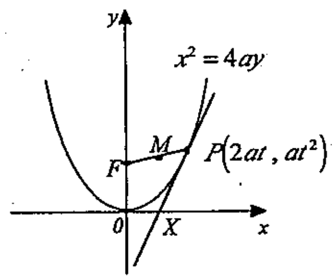
$$\therefore x = -\tfrac{5}{3} \text{ or } x = 2$$

d. Outcomes Assessed: i. **PE4** ii. **PE3**

Marking Guidelines

Criteria	Marks
i. • shows that the gradient of the tangent is t	1
• uses gradient t and coordinates of P to find the equation of the tangent	1
ii. • finds the x coordinate of X	1
• finds the x coordinate of M and deduces MX parallel to the y axis.	1

Answer



i. $x = 2at \qquad y = at^2$

$$\frac{dx}{dt} = 2a \qquad \frac{dy}{dt} = 2at$$

$$\therefore \frac{dy}{dx} = \frac{2at}{2a} = t$$

Tangent at P has gradient t and equation

$$y - at^2 = t(x - 2at)$$

$$y - at^2 = tx - 2at^2$$

$$tx - y - at^2 = 0$$

ii. $F(0, a) \quad \therefore \quad M\left(at, \frac{a(1+t^2)}{2}\right)$

At X, $y = 0 \Rightarrow tx - at^2 = 0 \quad \therefore x = at$

Since both M and X lie on the vertical line $x = at$, MX is parallel to the y axis.

Question 3

a. Outcomes Assessed: i. **H5** ii. **H5** iii. **HE4** iv. **HE4**

Marking Guidelines

Criteria	Marks
i. • shows first derivative positive for $x > 0$ and deduces f increasing	1
• shows second derivative negative and deduces curve concave down	1
ii. • finds gradient of tangent by evaluating first derivative at $x = 1$	1
• finds y coordinate when $x = 1$; deduces $y = x$ is equation of required tangent	1
iii. • uses an appropriate algebraic method to find the equation of the inverse function	1
iv. • shows curve $y = f(x)$ rising and concave down with negative y axis as asymptote	1
• shows curve $y = f^{-1}(x)$ as reflection in $y = x$ touching $y = f(x)$ at $(1, 1)$	1
• shows x intercept for $y = f(x)$ and y intercept for $y = f^{-1}(x)$	1

Answer

i. $f(x) = 1 + \ln x$ has domain $\{x : x > 0\}$.

$f'(x) = \frac{1}{x} > 0$ *for* $x > 0$ $\quad \therefore f$ is increasing for all x in its domain

$f''(x) = -\frac{1}{x^2} < 0$ *for all* x $\quad \therefore$ curve $y = f(x)$ is concave down for all x in the domain of f

ii. When $x = 1$, $y = f(1) = 1$.

$f'(1) = 1$ $\quad \therefore$ tangent at $(1, 1)$ has gradient 1 and equation $y = x$

iii. $y = 1 + \ln x$

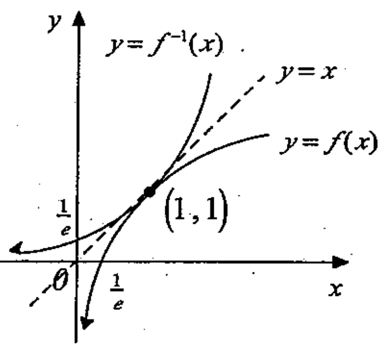
$y - 1 = \ln x$

$e^{y-1} = x$

Interchanging $x \leftrightarrow y$, inverse function has equation $y = e^{x-1}$.

$\therefore f^{-1}(x) = e^{x-1}$

iv.



At the x intercept of curve $y = f(x)$

$1 + \ln x = 0$

$\ln x = -1$

$x = e^{-1}$

Curves $y = f(x)$, $y = f^{-1}(x)$ are reflections of each other in the line $y = x$. Since this line is tangent to the curve $y = f(x)$ at the point $(1, 1)$, the curves touch at this point

b. Outcomes Assessed: i. HE4 ii. HE4

Marking Guidelines

Criteria	Marks
i. • uses the product rule to differentiate the first term	1
• differentiates $\sin^{-1} x$ and simplifies the expression for the derivative	1
ii. • writes an expression for the integral using the derivative in i.	1
• evaluates this expression in simplest exact form.	1

Answer

i.
$$\frac{d}{dx}\left(x\sqrt{1-x^2}+\sin^{-1}x\right)=\left\{1.\sqrt{1-x^2}+x.\tfrac{1}{2}(1-x^2)^{-\frac{1}{2}}.(-2x)\right\} + \frac{1}{\sqrt{1-x^2}}$$
$$=\sqrt{1-x^2} + (1-x^2)^{-\frac{1}{2}}\left(-x^2+1\right)$$
$$=\sqrt{1-x^2} + (1-x^2)^{\frac{1}{2}}$$
$$\therefore \frac{d}{dx}\left(x\sqrt{1-x^2}+\sin^{-1}x\right)=2\sqrt{1-x^2}$$

ii.
$$\left[x\sqrt{1-x^2}+\sin^{-1}x\right]_0^{\frac{1}{2}}=\int_0^{\frac{1}{2}} 2\sqrt{1-x^2}\ dx$$
$$\left(\tfrac{1}{2}\sqrt{1-\tfrac{1}{4}}-0\right)+\left(\sin^{-1}\tfrac{1}{2}-\sin^{-1}0\right)=2\int_0^{\frac{1}{2}}\sqrt{1-x^2}\ dx$$
$$\tfrac{\sqrt{3}}{4}+\tfrac{\pi}{6}-0=2\int_0^{\frac{1}{2}}\sqrt{1-x^2}\ dx$$
$$\therefore \int_0^{\frac{1}{2}}\sqrt{1-x^2}\ dx=\tfrac{\sqrt{3}}{8}+\tfrac{\pi}{12}$$

Question 4

a. Outcomes Assessed: i. PE3 ii. PE3

Marking Guidelines

Criteria	Marks
i. • shows that $f(x)=x^3-3x-3$ changes sign between $x=2$ and $x=3$	1
• notes that f is continuous to deduce $2<\alpha<3$	1
ii. • evaluates $f'(2)$	1
• applies Newton's method once with a first approximation $\alpha\approx 2$	1

Answer

i. Let $f(x)=x^3-3x-3$.
$f(x)$ is continuous,
$f(2)=-1<0$ and $f(3)=15>0$.
$\therefore 2<\alpha<3$

ii. $f'(x)=3x^2-3$
$$\alpha\approx 2-\frac{f(2)}{f'(2)}=2-\frac{-1}{9}$$
$\alpha\approx 2\cdot 1$

b. Outcomes Assessed: HE6

Marking Guidelines

Criteria	Marks
• expresses du in terms of dx	1
• finds limits for integral with respect to u	1
• finds primitive function as a function of u	1
• evaluates the integral in simplest exact form	1

Answer

$u = \sin^2 x$

$du = 2\sin x\cos x\, dx$

$du = \sin 2x\, dx$

$x = \frac{\pi}{4} \Rightarrow u = \frac{1}{2}$

$x = \frac{\pi}{3} \Rightarrow u = \frac{3}{4}$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{1-\sin^2 x}\ \sin 2x\ dx = \int_{\frac{1}{2}}^{\frac{3}{4}} \frac{1}{1-u}\ du$$
$$= \left[-\ln(1-u)\right]_{\frac{1}{2}}^{\frac{3}{4}}$$
$$= -\ln\tfrac{1}{4} + \ln\tfrac{1}{2}$$
$$= \ln(\tfrac{1}{2} \div \tfrac{1}{4})$$
$$= \ln 2$$

c. Outcomes Assessed: i. HE5 ii. HE5

Marking Guidelines

Criteria	Marks
i. • finds an expression for $\frac{1}{2}v^2$ by integration of a with respect to x	1
• evaluates the constant of integration and obtains v^2 as a function of x	1
ii. • uses an algebraic method to deduce $x^2 \le 10$ since $v^2 \ge 0$	1
• writes an inequality for x.	1

Answer

i. $a = 8x - 2x^3$

$\frac{d}{dx}(\frac{1}{2}v^2) = 8x - 2x^3$

$\frac{1}{2}v^2 = 4x^2 - \frac{1}{2}x^4 + c$

$\left.\begin{matrix} x=2 \\ v=6 \end{matrix}\right\} \Rightarrow \begin{matrix} 18 = 16 - 8 + c \\ \therefore c = 10 \end{matrix}$

$\therefore \frac{1}{2}v^2 = 4x^2 - \frac{1}{2}x^4 + 10$

$v^2 = 20 + 8x^2 - x^4$

ii. $v^2 = (10 - x^2)(2 + x^2)$

But $v^2 \ge 0$, $\therefore x^2 \le 10$

Hence $-\sqrt{10} \le x \le \sqrt{10}$

Question 5

a. Outcomes Assessed: i. PE3 ii. HE3

Marking Guidelines

Criteria	Marks
i. • counts how many ways two even are chosen from four, and one odd from five	1
• completes the factors in numerator and denominator to evaluate the probability	1
ii. • counts how many ways two even are chosen from four, and one odd from five	1
• completes the factors in numerator and denominator to evaluate the probability	1

Answer

In each case there are three orders of E, E, O, and the odd number can be selected in 5 ways.

i. $P(exactly\ 2\ even) = \dfrac{3\times5\times4\times3}{9\times8\times7} = \frac{5}{14}$

(Select two evens in order in 4×3 ways
Select three balls in order in $9\times8\times7$ ways)

ii. $P(exactly\ 2\ even) = \dfrac{3\times5\times4^2}{9^3} = \frac{80}{243}$

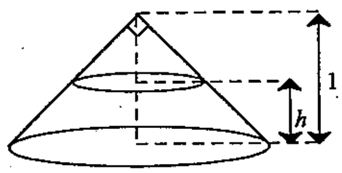
(Select two evens in order in 4×4 ways
Select three balls in order in $9\times9\times9$ ways)

b. Outcomes Assessed: i. **P4** ii. **HE5**

Marking Guidelines

Criteria	Marks
i. • finds the radius and volume of the conical vessel	1
• considers the volume of the conical empty space to express V in terms of h.	1
ii. • expresses $\dfrac{dV}{dt}$ in terms of h and $\dfrac{dh}{dt}$	1
• calculates the rate of increase of the depth by substitution in this expression	1

Answer



i. The conical vessel has height 1 m, radius 1 m and volume $\frac{\pi}{3}\ \text{m}^3$.
When the depth of the water is h, the empty space is a similar cone, the ratio of heights being $(1-h):1$.

$\therefore V = \frac{\pi}{3}\left\{1-(1-h)^3\right\} = \frac{\pi}{3}\left(h^3 - 3h^2 + 3h\right)$

ii.

$V = \frac{\pi}{3}(h^3 - 3h^2 + 3h)$

$\dfrac{dV}{dt} = \frac{\pi}{3}(3h^2 - 6h + 3)\dfrac{dh}{dt}$

$0\cdot1 = \pi(h^2 - 2h + 1)\dfrac{dh}{dt}$

When $h = 0\cdot5$, $\quad 0\cdot1 = \pi\times0\cdot25\times\dfrac{dh}{dt}$

$\therefore \dfrac{dh}{dt} = \dfrac{2}{5\pi} \approx 0\cdot13$

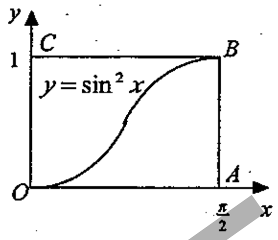
The depth is increasing at a rate $\frac{2}{5\pi} \approx 0\cdot13$ m per minute.

c. Outcomes Assessed: **H8**

Marking Guidelines

Criteria	Marks
• writes an integral to give the area under the curve; states the area of the rectangle	1
• writes $\sin^2 x$ in terms of $\cos 2x$	1
• finds the primitive function	1
• substitutes to evaluate the definite integral	1

Answer



$$\int_0^{\frac{\pi}{2}} \sin^2 x \; dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (1-\cos 2x) \; dx$$
$$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$$
$$= \frac{1}{2}\left\{\left(\frac{\pi}{2}-0\right)-\frac{1}{2}\left(\sin\pi - \sin 0\right)\right\}$$
$$= \frac{\pi}{4}$$
$$= \frac{\pi}{2} \div 2$$

Area of rectangle $OABC$ is $\frac{\pi}{2}$ sq. units
Area under curve is given by

Hence curve divides rectangle $OABC$ into two regions of area $\frac{\pi}{4}$ sq. units.

Question 6

a. Outcomes Assessed: i. **HE3** ii. **HE3** iii. **HE3** iv. **HE3, HE7**

Marking Guidelines

Criteria	Marks
i. • finds the value of R	1
• finds the value of α	1
ii. • states the amplitude	1
• states the period	1
iii. • finds initial values of x and v and deduces particle initially travelling towards O.	1
• finds initial value of a and deduces particle is speeding up.	1
iv. • writes a trigonometric equation for t.	1
• finds the required value of t.	1

Answer

i. $x = \cos 2t - \sin 2t$

$$x = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos 2t - \frac{1}{\sqrt{2}}\sin 2t\right)$$
$$= \sqrt{2}\left(\cos\frac{\pi}{4}\cos 2t - \sin\frac{\pi}{4}\sin 2t\right)$$
$$= \sqrt{2}\cos\left(2t+\frac{\pi}{4}\right)$$

ii. Amplitude is $\sqrt{2}$ m
Period is π seconds

iii. $v = -2\sqrt{2}\sin\left(2t+\frac{\pi}{4}\right)$

$a = -4\sqrt{2}\cos\left(2t+\frac{\pi}{4}\right)$

When $t=0$, $v=-2$, $a=-4$, $x=1$

Initial velocity is 2 ms^{-1} to the left
Initial acceleration is 4 ms^{-2} to the left
Particle is initially 1 m to the right of O.
Initially particle is moving towards O and speeding up.

iv. On first return to its initial position

$$\sqrt{2}\cos\left(2t+\frac{\pi}{4}\right) = 1$$
$$\cos\left(2t+\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$
$$2t+\frac{\pi}{4} = \frac{\pi}{4}, \frac{7\pi}{4}, \ldots$$
$$2t = 0, \frac{3\pi}{2}, \ldots$$

Hence particle first returns to its starting point after $\frac{3\pi}{4}$ s.

b. Outcomes Assessed: HE4

Marking Guidelines

Criteria	Marks
• establishes the truth of $S(1)$	1
• substitutes for sum of first k terms in expression for LHS of $S(k+1)$ if $S(k)$ is true	1
• uses the sum of an A.P. to simplify $(k+1)$th term of expression for LHS of $S(k+1)$	1
¥ rearranges expression to show if $S(k)$ is true then $S(k+1)$ is true	1

Answer

Let $S(n)$ be the statement $\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+n}=\frac{2n}{n+1}$, $n=1,2,3,\ldots$

Consider $S(1)$: $LHS=\frac{1}{1}=1$; $RHS=\frac{2\times 1}{1+1}=1$. Hence $S(1)$ is true.

If $S(k)$ is true : $\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+k}=\frac{2k}{k+1}$ **

Consider $S(k+1)$:

$$LHS=\frac{1}{1}+\frac{1}{1+2}+\frac{1}{1+2+3}+\ldots+\frac{1}{1+2+3+\ldots+k}+\frac{1}{1+2+3+\ldots+(k+1)}$$

$$=\frac{2k}{k+1}+\frac{1}{1+2+3+\ldots+(k+1)} \qquad \textit{if } S(k) \textit{ is true, using**}$$

$$=\frac{2k}{k+1}+\frac{2}{(k+1)\{1+(k+1)\}} \qquad \textit{sum of } (k+1) \textit{ terms of A.P: } a=1,\ d=1$$

$$=\frac{2}{(k+1)}\left\{k+\frac{1}{k+2}\right\}$$

$$=\frac{2(k^2+2k+1)}{(k+1)(k+2)}$$

$$=\frac{2(k+1)^2}{(k+1)(k+2)}$$

$$=\frac{2(k+1)}{\{(k+1)+1\}}$$

$$=RHS$$

Hence if $S(k)$ is true, then $S(k+1)$ is true. But $S(1)$ is true, hence $S(2)$ is true, and then $S(3)$ is true and so on. Therefore by Mathematical Induction, $S(n)$ is true for all positive integers n.

Question 7

a. Outcomes Assessed: i. HE3 ii. HE3

Marking Guidelines

Criteria	Marks
i. • writes expression for x	1
• writes expression for y	1
ii. • substitutes for t to get y in terms of x, $\tan\theta$, $\cos\theta$	1
• uses trig identities to obtain required expression for y	1

Answer

i. $x = Vt\cos\theta$

$y = Vt\sin\theta - \frac{1}{2}gt^2$

ii. $y = Vt\cos\theta\tan\theta - \frac{1}{2}g\left(\frac{x}{V\cos\theta}\right)^2$

$$y = x\tan\theta - \frac{gx^2}{2V^2}\sec^2\theta$$

$$= x\tan\theta - \frac{gx^2}{2V^2}\left(1+\tan^2\theta\right)$$

b. Outcomes Assessed: i. HE3 ii. HE3

Marking Guidelines

Criteria	Marks
i. • substitutes values into equation of path of projectile from (a)	1
• solves resulting quadratic for $\tan\theta$ and evaluates $\tan\alpha$, $\tan\beta$	1
ii. • uses the horizontal displacements at collision to obtain equation for T	1
• evaluates T in simplest surd form	1

Answer

i. If a particle fired at an angle θ above the horizontal with a speed of $60\,\text{ms}^{-1}$ passes through the point where $x=240$, $y=80$ then, using the result in (a) ii.,

$$240\tan\theta - \frac{10\times 240^2}{2\times 60^2}\left(1+\tan^2\theta\right) = 80$$

$$3\tan\theta - \left(1+\tan^2\theta\right) = 1$$

$$\tan^2\theta - 3\tan\theta + 2 = 0$$

$$\left(\tan\theta - 1\right)\left(\tan\theta - 2\right) = 0$$

$$\therefore \tan\theta = 1 \;\; or \;\; \tan\theta = 2$$

If particles fired at angles α, β collide where $x=240$, $y=80$, then α, β are roots of this equation in θ.
Since $\beta < \alpha$, $\tan\alpha = 2$ and $\tan\beta = 1$.

ii. Let the particles collide t seconds after the second particle is fired. Then, using expression for x from (a) i.,

$$240 = 60(t+T)\cos\alpha \Rightarrow 4\sec\alpha = t+T$$

$$240 = 60t\cos\beta \Rightarrow 4\sec\beta = t$$

$$\therefore T = 4(\sec\alpha - \sec\beta)$$

$$= 4\left(\sqrt{5}-\sqrt{2}\right)$$

c. Outcomes Assessed: PE3, H5, HE1

Marking Guidelines

Criteria	Marks
• finds an expression for S as the sum of a G.P	1
• uses the quadratic equation to express x^{2n} in the form $(x+1)^n$	1
• uses the binomial theorem to expand $(x+1)^n - 1$	1
• uses the quadratic expression to show $x-1=\frac{1}{x}$, then obtains required expression for S.	1

Answer

S is the sum of $2n$ terms of a G.P. $\therefore\ S=\dfrac{x^{2n}-1}{x-1}$

$$\begin{aligned} x^2-x-1&=0\\ x^2&=x+1\\ x^{2n}&=(x+1)^n\\ &=\sum_{r=0}^{n}{}^nC_r\,x^r\\ x^{2n}-1&=\sum_{r=1}^{n}{}^nC_r\,x^r \end{aligned}$$

$$\begin{aligned} x^2-x-1&=0\\ x^2-x&=1\\ x-1&=\frac{1}{x}\\ \frac{1}{x-1}&=x \end{aligned}$$

$$\begin{aligned} \therefore S&=\frac{1}{x-1}\left(x^{2n}-1\right)\\ &=x\,.\sum_{r=1}^{n}{}^nC_r\,x^r\\ &=\sum_{r=1}^{n}{}^nC_r\,x^{r+1} \end{aligned}$$

CSSA Trial HSC 2005 Mathematics Extension 1 Mapping Grid

Question	Marks	Syllabus Content	Syllabus Outcomes	Targeted Performance Bands
1(a)	2	Trigonometric functions	H5	E2 — E3
1(b)(i)	2	Polynomials	PE3	E2 — E3
1(b)(ii)	1	Basic arithmetic and algebra	P4	E2 — E3
1(c)(i)	1	Exponential and logarithmic functions	H5	E2 — E3
1(c)(ii)	2	Angle between two lines	P4	E2 — E3
1(d)(ii)	1	Circle geometry	PE3	E2 — E3
1(d)(iii)	3	Circle geometry	PE2 , PE3	E2 — E3
2(a)	2	Exponential and logarithmic functions	H3	E2 — E3
2(b)(i)	2	Further trigonometry	H5	E2 — E3
2(b)(ii)	1	Further trigonometry	H5	E2 — E3
2(c)(i)	1	Division of an interval in a given ratio	P4	E2 — E3
2(c)(ii)	2	Basic arithmetic and algebra	P4	E2 — E3
2(d)(i)	2	Parametric representation	PE4	E2 — E3
2(d)(ii)	2	Parametric representation	PE3	E2 — E3
3(a)(i)	2	Exponential and logarithmic functions	H5	E2 — E3
3(a)(ii)	2	Exponential and logarithmic functions	H5	E2 — E3
3(a)(iii)	1	Inverse functions	HE4	E2 — E3
3(a)(iv)	3	Inverse functions	HE4	E2 — E3
3(b)(i)	2	Inverse trigonometric functions	HE4	E2 — E3
3(b)(ii)	2	Inverse trigonometric functions	HE4	E2 — E3
4(a)(i)	2	Iterative methods	PE3	E2 — E3
4(a)(ii)	2	Iterative methods	PE3	E2 — E3
4(b)	4	Methods of integration	HE6	E2 — E3
4(c)(i)	2	Velocity and acceleration as a function of x	HE5	E3 — E4
4(c)(ii)	2	Velocity and acceleration as a function of x	HE5	E3 — E4
5(a)(i)	2	Further probability	PE3	E3 — E4
5(a)(ii)	2	Further probability	HE3	E3 — E4
5(b)(i)	2	Basic arithmetic and algebra	P4	E2 — E3
5(b)(ii)	2	Applications of calculus to the physical world	HE5	E3 — E4
5(c)	4	Primitive of $\sin^2 x$	H8	E2 — E3
6(a)(i)	2	Simple harmonic motion	HE3	E3 — E4
6(a)(ii)	2	Simple harmonic motion	HE3	E3 — E4
6(a)(iii)	2	Simple harmonic motion	HE3	E3 — E4
6(a)(iv)	2	Simple harmonic motion	HE3 , HE7	E3 — E4
6(b)	4	Induction	HE2	E3 — E4
7(a)(i)	2	Projectile motion	HE3	E3 — E4
7(a)(ii)	2	Projectile motion	HE3	E3 — E4
7(b)(i)	2	Projectile motion	HE3	E3 — E4
7(b)(ii)	2	Projectile motion	HE3	E3 — E4
7(c)	4	Binomial theorem	PE3 . H5 . HE1	E3 — E4

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