

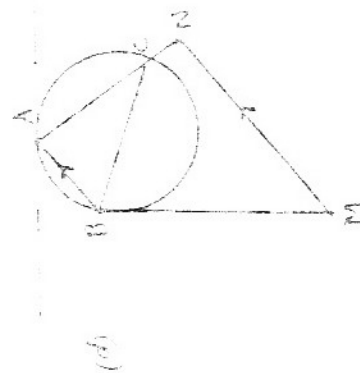
$$\begin{aligned} N(1) &= \lim_{n \rightarrow \infty} \frac{5(10^n) + 3}{2(10^n) + 1} = \lim_{n \rightarrow \infty} \frac{5 + 3(10^{-n})}{2 + 1(10^{-n})} \\ &= \frac{5+0}{2+0} \\ &= \frac{5}{2} \end{aligned}$$

$$\begin{aligned} (A_1, y_1) & (x_2, y_2) \\ A(-2, 5) & B(3, -1) \\ x &= \frac{mx_2 + nx_1}{m+n} \\ x &= \frac{2 \times 7 + 1 \times -2}{2+1} \\ x &= \frac{12}{3} \\ y &= \frac{2}{3} \\ y &= \frac{3}{3} \\ y &= (1, 1) \end{aligned}$$

$$\frac{x}{2} < 1-x < \frac{x}{2} \quad (5)$$

$$\begin{aligned} \frac{d}{dx} x x^2 &> (x-1) x^2 \\ \frac{d}{dx} x &> x^2 - x^2 \\ x^3 - x^2 - 2x &< 0 \\ x(x^2 - x - 2) &< 0 \\ x(x-2)(x+1) &< 0 \end{aligned}$$

$$\frac{2.4 \times 10^4 \times 2}{1}$$



EXTENSION 1 2003 CSSA

(ii) $\angle MOC = \angle BAC$
Angle in Alt. Segment Th.

(iii) $\angle MBC = \angle BAC$ (ii)

$$\angle BAC = 180^\circ - \angle ANB$$

(Cont. Ls Supp. BA||MN)

$\therefore \angle MNC$ is cyclic quad (opp. angles supp.)

Q2. (a) $y = (x^2 + 1)^5$

$$y' = 5(x^2 + 1)^4 \cdot 2x$$

$$= 10x(x^2+1)^4$$

$$y''' = 40x(x^2+1)^3 + 10(x^2+1)^4$$

$$= (x^2+1)^3 \cdot 80x^2 + (10x^2+10)(x^2+1)^3$$

$$y'' = (x^2 + 1)^3 (90x^2 + 10)$$

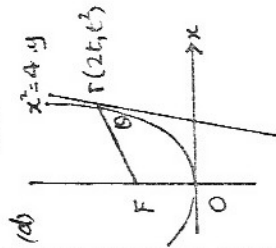
$$y'' = 10(x^2+1)^3(9x^2+1)$$

$$\begin{aligned} \sum_{n=2}^5 n C_2 &= {}^2C_2 + {}^3C_2 + {}^4C_2 + {}^5C_2 \\ (13) \quad &= 1 + 3 + 6 + 10 \\ &= 20 \end{aligned}$$

(c) (i) $(\sin A - \cos A)^2 = 1 - \sin 2A$

$$\begin{aligned} \text{RHS} &= 1 - \sin(A+A) \\ &= 1 - (\sin A \cos A + \sin A \cos A) \\ &= 1 - 2 \sin A \cos A \\ \text{LHS} &= \sin^2 A + \cos^2 A - 2 \sin A \cos A \\ &= 1 - 2 \sin A \cos A \\ &= \text{LHS} \end{aligned}$$

(e) (ii) $\sin 15^\circ - \cos 15^\circ \Rightarrow \sqrt{1 - \sin 30^\circ} (\sin 15^\circ - \cos 15^\circ)$
 $= \sqrt{1 - \frac{1}{2}} = \sqrt{\frac{1}{2}} \text{ or } \frac{\sqrt{2}}{2}$



$$(i) \quad \begin{aligned} x^2 &= 4 \\ y &= \frac{y^2}{4} \end{aligned}$$

$$m_T = y' = \frac{2x}{q_1}$$

$$\frac{x}{2}$$

$$m_T = \frac{2t}{2'}$$

$$m_T = t$$

$$m_{FT} = \frac{t^2 - 1}{2t}$$

$$= x(\text{shown})$$

$$\tan \theta = \frac{m_T - m_{FT}}{1 + m_T \cdot m_{FT}}$$

$$= \frac{t - \frac{t-1}{2t}}{1 + t \left(\frac{t-1}{2t} \right)}$$

$$\frac{2x^2 - x^2 + 1}{2t} = 1 + \frac{1}{2}t^2 - \frac{1}{2}$$

$$\frac{2t^2 - t^2 + 1}{2t + t^3 - t}$$

$$\frac{x^2 + 1}{x^3 + 1}$$

$$\frac{t^3 + 1}{t(t^2 + 1)}$$

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(also accept $\frac{1}{r}$)

Q3. (a) $f(x) = x - e^{-x}$

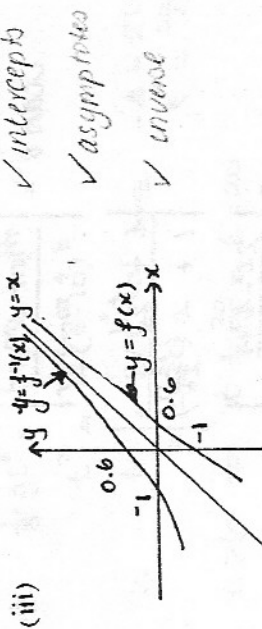
(i) $f'(x) = 1 + e^{-x} > 0 \forall x$
 $\therefore f$ is increasing

$f''(x) = -e^{-x} < 0 \forall x$
 $\therefore$ concave down

(ii) $x = 0.5 \quad f(0.5) = 0.5 - e^{-0.5}$
 $f'(0.5) = 1 + e^{-0.5}$

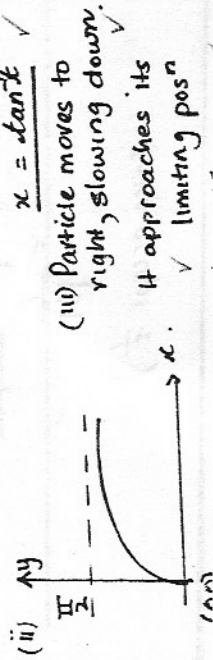
$\Rightarrow x \doteq 0.5 - \frac{0.5 - e^{-0.5}}{1 + e^{-0.5}}$

$x \doteq 0.56631 \dots$
 $x \doteq 0.6$ (1 DP)



(b) $v = \cos^2 x \quad x_0 = 0$

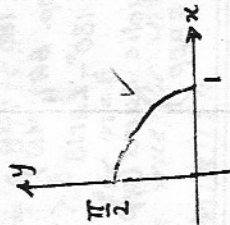
(i) $a = \frac{d}{dt} \left(\frac{1}{2} v^2 \right) \quad \frac{dx}{dt} = v$
 $a = \frac{d}{dt} \left(\frac{1}{2} \cos^4 x \right) \Rightarrow \frac{dx}{dt} = \cos^2 x$
 $a = -2 \cos^2 x \sin x$
 $\frac{dv}{dx} = \sec^2 x$
 $t = \tan x$
 $x = \tan^{-1} t$



(ii) Particle moves to right, slowing down.
 It approaches its limiting position.

Q4. (a) $f(x) = \cos^{-1} \sqrt{x}$

(i) D: $-1 \leq \sqrt{x} \leq 1 \quad R: x = 0 \quad f(0) = \frac{\pi}{2}$
 $0 \leq \sqrt{x} \leq 1 \quad x = 1 \quad f(1) = 0$
 $D: 0 \leq x \leq 1 \quad R: 0 \leq y \leq \frac{\pi}{2}$



(ii)

x	0	1/2	1
y	pi/2	pi/4	0

 $A \doteq \frac{1}{6} [y_0 + 4y_1 + y_2]$
 $= \frac{1}{6} \left(\frac{\pi}{2} + \pi + 0 \right)$
 $A = \frac{\pi}{4}$ units²

(iii) $A = \int_0^{\pi/2} \cos^2 y \, dy$
 $= \int_0^{\pi/2} \frac{1}{2} (\cos 2y + 1) \, dy$
 $= \frac{1}{2} y + \frac{1}{4} \sin 2y \Big|_0^{\pi/2}$
 $= \frac{\pi}{4} + \frac{1}{4} \sin \pi - 0 - \frac{1}{4} \sin 0$
 $A = \frac{\pi}{4}$ units²

(b) $\ddot{x} = -4(x-2) \quad t=0 \quad x=0$

(i) $\ddot{x} = \frac{d}{dt} \left(\frac{1}{2} v^2 \right)$
 $\therefore v^2 = 2 \int \ddot{x} \, dx$
 $= -8 \int x - 2 \, dx$
 $= -4x^2 + 16x + C$

$x=0, v=0 \Rightarrow C=0$
 $\therefore v^2 = -4x^2 + 16x$

(ii) $\ddot{x} = -(2)^2(x-2) \therefore n=2 \quad \text{Period } \frac{2\pi}{2} = \pi \text{ sec}$

$v=0 \quad -4x^2 + 16x = 0$
 $-4x(x-4) = 0$
 $x = 0, 4$
 Centre $x=2 \therefore$ Amplitude 2

(iii) $t = \pi \quad x = 8 \therefore 1 \text{ sec}$
 $\frac{8}{\pi} \text{ metres}$
 $1 \text{ min } \frac{8}{\pi} \times 60 = 153 \text{ m}$

Q5. (a) $(1+ax)^9 = 1 + {}^9C_1(ax) + \dots + {}^9C_5(ax)^5 + {}^9C_6(ax)^6 + \dots + (ax)^9$
 ${}^9C_5 a^5 = 2^9 {}^9C_6 a^6$
 $126a^5 = 168a^6$
 $a = \frac{126}{168}$
 $a = \frac{3}{4}$

(b) $\int_1^{49} \frac{1}{\sqrt{x} \sqrt{1+\sqrt{x}}} \, dx = I$

$u = 1 + \sqrt{x} \quad I = \int \frac{2 \, du}{\sqrt{u} \cdot \sqrt{u}} = \int \frac{2 \, du}{u}$
 $\frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow \frac{dx}{du} = 2\sqrt{x}$
 $2 \, du = \frac{dx}{\sqrt{x}} \Rightarrow \frac{dx}{\sqrt{x}} = 2 \, du$
 $x = 49 \quad u = 8 \quad \int_2^8 \frac{2 \, du}{u} = 2 \left[\ln u \right]_2^8 = 2 \ln 4 = 4 \ln 2$
 $x = 1 \quad u = 2 \quad = 2 \ln 2 = \ln 4$

10 Cont.

(c) $V = A(1 - e^{-kt})$ $k > 0$

(i) $\frac{dV}{dt} = \frac{d}{dt} (A - Ae^{-kt})$

$= Ake^{-kt}$

$A - V = A - (A - Ae^{-kt})$

$= Ae^{-kt}$

$\therefore \frac{dV}{dt} = k(A - V)$

(ii) $t = 2$ $V = \frac{A}{4}$

$\frac{A}{4} = A(1 - e^{-2k})$

$1 - e^{-2k} = \frac{3}{4}$

$e^{-2k} = -\frac{3}{4}$

$e^{-2k} = \frac{3}{4}$ ①

$t = 4$ $V = A(1 - e^{-4t})$

$= A(1 - (e^{-2k})^2)$

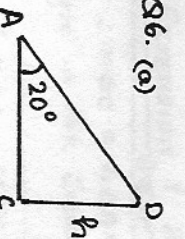
$= A(1 - (\frac{3}{4})^2)$ ②

$= \frac{7A}{16}$

$\therefore \frac{7A}{16} - \frac{4A}{16} \Rightarrow \frac{3}{16}$ of container is filled

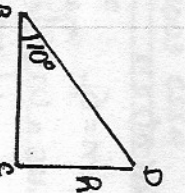
in next 2 minutes.

Q6. (a)



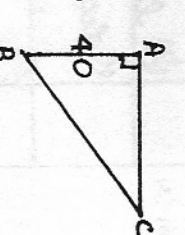
$\tan 20^\circ = \frac{h}{AC}$

$AC = \frac{h}{\tan 20^\circ}$



$\tan 10^\circ = \frac{h}{BC}$

$BC = \frac{h}{\tan 10^\circ}$



$AC^2 + AB^2 = BC^2$ (Pyth.)

$(\frac{h^2}{\tan^2 20^\circ}) + 40^2 = \frac{h^2}{\tan^2 10^\circ}$

$\Rightarrow \frac{h^2 + 1600 \tan^2 20^\circ}{\tan^2 20^\circ} = \frac{h^2}{\tan^2 10^\circ}$

$h^2 + \tan^2 20^\circ = h^2 \tan^2 10^\circ + 1600 \tan^2 20^\circ \tan^2 10^\circ$

$h^2 = \frac{1600 \tan^2 20^\circ \tan^2 10^\circ}{\tan^2 20^\circ - \tan^2 10^\circ}$

$h = 8.06 \dots$

(b) $P(0) = p$

$0 < p < 1$ $p \neq 0.5$

ix Six throws:

$P(\text{At Most 1E}) = P(50 \text{ or } 60)$

$= {}^6C_5 p^5 (1-p) + {}^6C_6 p^6$

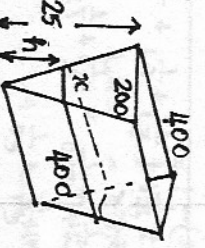
$= 6p^5 (1-p) + p^6$

$= 6p^5 - 5p^6$

(ii) $P(\text{Product Even}) = 1 - P(\text{Product Odd})$

$= 1 - p^6$

(c)



$V = 16 \text{ L/sec}$

(Note: all measurements changed to cm)

(i) By similar as:

$\frac{x}{200} = \frac{h}{25}$

$25x = 200h$

$x = 8h$

$V = \frac{1}{2} \times x \times h \times 400$

$= \frac{1}{2} \times 8h^2 \times 400$

$V = 1600h^2$

(ii) $\frac{dh}{dt} = ?$ when $h = 10$

$\frac{dV}{dt} = \frac{dh}{dt} \cdot \frac{dV}{dh}$ ①

$\frac{dV}{dt} = 1600 \text{ cm}^3/\text{sec}$

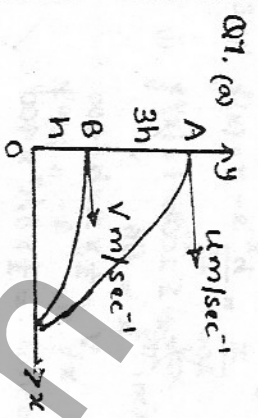
$\frac{dV}{dh} = 3200h \Rightarrow \frac{dh}{dt} = \frac{1}{3200h}$

$h = 10$ $\frac{dh}{dt} = \frac{1}{32000}$

$\therefore \frac{dh}{dt} = \frac{-16000}{32000}$

$= -\frac{1}{2}$

$\therefore$ Water level falling 0.5 cm/sec



(i) $x(A) = ut$ ✓ $x(B) = v(t-10)$ ✓
 $y(A) = 4h - \frac{1}{2}gt^2$ ✓ $y(B) = h - \frac{1}{2}g(t-10)^2$ ✓

(ii) When particle hits ground:
 $y(A) = y(B) = 0$

$\therefore 4h = \frac{1}{2}gt^2$ $h = \frac{1}{2}g(t-10)^2$
 $h = \frac{1}{8}gt^2$

$\rightarrow \frac{1}{8}gt^2 = \frac{1}{2}g(t-10)^2$ ✓
 $t^2 = 4(t-10)^2$

$\therefore t = 2(t-10)$ or $-t = 2(t-10)$
 $t = 20$ $t = \frac{20}{3}$

$t > 10 \Rightarrow t = \frac{20}{3}$ $t = 20$

• Particle A takes 20 sec ✓
 & Particle B takes 10 sec ✓

(iii) On impact $x(A) = x(B)$

$\therefore 20v = v(20-10)$ ✓
 $20v = 10v$ ✓
 $v = 20$ ✓

(b) $\ln(n!) > n$ $n \geq 6$
 (i) let $n = 6$

LHS = $\ln(6!)$
 $= \ln 720$
 $\approx 6.57925 \dots$
 > 6 ✓

$\therefore$ true for $n = 6$
 Assume true for $n = k$, $k \geq 6$

$\ln(k!) > k$

To prove true for $n = k+1$:

$\ln(k+1)! = \ln[k!(k+1)]$
 $= \ln(k!) + \ln(k+1)$ ✓
 $> \ln(k!) + k$

$\ln e = 1$ for $k \geq 6$ $k+1 > e$

$\ln(k+1) > \ln e$ ✓

$\therefore \ln(k+1) > 1$

$\therefore \ln[k!(k+1)] \geq 1+k$

Since shown true for $n = 6$ & $n = k+1$,
 we assume true for $n = k$ then
 it must hold true for all k .

(ii) $\ln(n!) > n$ $n \geq 6$

$n! > e^n$ ✓

$\therefore \frac{1}{n!} < \frac{1}{e^n}$ $n \geq 6$

(iii) $\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \dots$
 $< 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{6!} + \frac{1}{7!} + \dots$
 $< 1 + \frac{43}{60} + \frac{1}{e^6} + \frac{1}{e^7} + \frac{1}{e^8} + \dots$

$< \frac{103}{60} + \frac{1}{e^6} (1 + \frac{1}{e} + \frac{1}{e^2} + \dots)$
 $GPA = 1$ $r = \frac{1}{e}$
 $< \frac{103}{60} + \frac{1}{e^6} \left(\frac{1}{1 - \frac{1}{e}} \right)$
 $< \frac{103}{60} + \frac{\frac{1}{e^6}}{1 - \frac{1}{e}}$ ✓
 $< \frac{103}{60} + \frac{1}{e^6 - e^5}$
 $< \frac{103}{60} + \frac{1}{e^5(e-1)}$