

Question 1

a) $(3x-6) - (5-4x)$
 $= 3x-6-5+4x$
 $= 7x-11$

b) $\frac{23.1}{53.6 \times \sqrt{25.04}}$
 $= 0.08194821$
 $= 0.0820$
 (3 sig. fig.)

c) $\frac{d}{dx} \tan\left(\frac{1}{2}x\right) = \frac{1}{2} \sec^2\left(\frac{x}{2}\right)$

d) $2-3p < 7$
 $-3p < 5$
 $p > -\frac{5}{3}$

e) $126^\circ + \angle QRL = 180^\circ$
 $\angle QRL = 54^\circ$
 $y^\circ = 54^\circ + 90^\circ$
 $= 144^\circ$

f) $\frac{x-2}{2} + \frac{x+1}{5} = 2$
 $5(x-2) + 2(x+1) = 2 \times 2 \times 5$
 $5x-10+2x+2=20$
 $7x-8=20$
 $7x=28$
 $x=4$

Question 2

a) $\sqrt{27} - \sqrt{3} + \sqrt{18}$
 $= \sqrt{9 \times 3} - \sqrt{3} + \sqrt{9 \times 2}$
 $= 3\sqrt{3} - \sqrt{3} + 3\sqrt{2}$
 $= 2\sqrt{3} + 3\sqrt{2}$

b) $|x-1| = 2x-1$
 $x-1 = 2x-1$ or $-(x-1) = 2x-1$
 $0 = x$ or $-x+1 = 2x-1$
 $2 = 3x$
 $x = \frac{2}{3}$

c) i) Since L_1 is at 45° to the x-axis then
 $m = \tan 45^\circ = 1$

ii) $L_1: m=1, A(4,3)$
 $y-y_1 = m(x-x_1)$
 $y-3 = 1(x-4)$
 $y-3 = x-4$
 $\therefore 0 = x-y-1$

iii) sub $y=1$ into L_1
 $0 = x-1-1$
 $x=2$
 B is $(2,1)$

iv) if $L_1 \perp L_2$ and $m_1=1$
 $m_1 m_2 = -1$ gives $m_2 = -1$
 through $A(4,3)$
 $y-y_1 = m(x-x_1)$
 $y-3 = -1(x-4)$
 $y-3 = -x+4$
 $y-x = 7$

(Q2 contd...)

v) $\triangle ABC$ is isosceles if exactly 2 sides are equal.

$L_2: x+y-7=0$
 sub. $y=1$
 $\therefore x+1-7=0$
 $x=-6$
 C is $(-6,1)$

$A(4,3) \quad B(2,1) \quad C(-6,1)$
 $d_{AB} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
 $= \sqrt{(2-4)^2 + (1-3)^2}$
 $= \sqrt{4+4}$
 $= \sqrt{8}$

$d_{BC} = \sqrt{(-6-2)^2 + (1-1)^2}$
 $= \sqrt{8^2 + 0^2}$
 $= 8$

$d_{AC} = \sqrt{(4-6)^2 + (3-1)^2}$
 $= \sqrt{2^2 + 2^2}$
 $= \sqrt{8}$

$d_{AB} = d_{AC} \neq d_{BC}$
 isosceles $\triangle$

Question 3

a) i) $\frac{d}{dx} (4-3x)^4 = 6(4-3x)^3$

ii) $\frac{d}{dx} x^2 e^{2x} = u'v + uv'$
 $u = x^2 \quad v = e^{2x}$
 $u' = 2x \quad v' = 2e^{2x}$
 $= 2xe^{2x} + 2x^2 e^{2x}$

iii) $\frac{d}{dx} \frac{\sin 2x}{x} = \frac{u'v - uv'}{v^2}$
 $u = \sin 2x \quad v = x$
 $u' = 2 \cos 2x \quad v' = 1$
 $= \frac{2x \cos 2x - \sin 2x}{x^2}$

b) $0.34 = 0.3434343434$
 $= \frac{34}{100} + \frac{34}{100^2} + \frac{34}{100^3} + \dots$

i) $a = \frac{34}{100} \quad r = \frac{1}{100}$

ii) $S_{\infty} = \frac{a}{1-r}$
 $= \frac{34}{100} \div \left(1 - \frac{1}{100}\right)$
 $= \frac{34}{100} \times \frac{100}{99}$
 $0.34 = \frac{34}{99}$

(Q3 cont'd...)

c) $y = \ln x$
 $y' = \frac{1}{x}$

at $x=1$, $y' = \frac{1}{1} = 1$

$m_t = 1$

$m_n = -1$

at $x=1$, $y = \ln 1 = 0$

$y - y_1 = m(x - x_1)$
 $y - 0 = -1(x - 1)$

$y = -x + 1$

$\therefore x + y - 1 = 0$ is the normal.

Question 4

a) i) $b^2 = c^2 + a^2 - 2ac \cos B$
 $= 3.3^2 + 4.56^2 - 2 \times 3.3 \times 4.56 \times \cos 96.2^\circ$

$= 31.7497 - 20.798598$

$\therefore b = 3.30924914$

$= 3.31$ (2 d.p.)

(ii) $A = \frac{1}{2} ac \sin B$

$= \frac{1}{2} \times 4.56 \times 3.31 \times \sin 96.2^\circ$

$= 5.4697$

$\therefore A = 5m^2$ (to nearest m^2)

b) $y' = 3x^2 - 6x - 9$ (1, -2)

i) $y = \int 3x^2 - 6x - 9 dx$
 $= x^3 - 3x^2 - 9x + c$
 through (1, -2)

$\therefore -2 = (1)^3 - 3(1)^2 - 9(1) + c$
 $-2 = 1 - 3 - 9 + c$
 $\therefore c = 9$

$\therefore y = x^3 - 3x^2 - 9x + 9$ is the curve.

ii) stat. pts. when $y' = 0$
 $0 = 3x^2 - 6x - 9$

$0 = x^2 - 2x - 3$

$0 = (x - 3)(x + 1)$

$x = 3$ or $x = -1$

Sub. into $y = x^3 - 3x^2 - 9x + 9$

$\therefore (3, -18)$ and $(-1, 14)$

(Q4 cont'd...)

if $y' = 3x^2 - 6x - 9$
 then $y'' = 6x - 6$

at $x=3$, $y'' = 6 \times 3 - 6 = 12 > 0$

$\therefore (3, -18)$ is concave up min.

at $x=-1$, $y'' = 6 \times (-1) - 6 = -12 < 0$

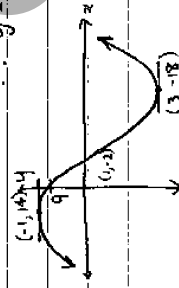
$\therefore (-1, 14)$ is concave down max.

(iii) inflexion pts. at $y'' = 0$
 $\therefore 0 = 6x - 6$
 $\therefore x = 1$

$y = x^3 - 3x^2 - 9x + 9$
 $= 1^3 - 3(1)^2 - 9(1) + 9 = -2$
 $\therefore$ inflex. pt. at (1, -2)

ie. curve changes from concave down to concave up at (1, -2).

(iv) y-intercept at $x=0$
 $y = 9$



as $x \rightarrow \infty$, $y \rightarrow \infty$
 as $x \rightarrow -\infty$, $y \rightarrow -\infty$

Question 5

a) 5 function values $\therefore 4$ strips

$h = \frac{b-a}{n}$

$= \frac{5-1}{4}$

$= 1$

$A \approx \frac{1}{5} (y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$
 $= \frac{1}{5} (0 + 4 \times 1.386 + 2 \times 3.2 + 4 \times 5.545 + 8.04)$
 $= 14.121$

$= 14.12$ units² (2 d.p.)

b) $y = 2x^2 - 4x + 1$

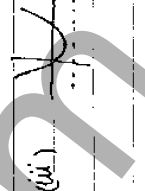
(i) vertex is on axis of symmetry or rearrange the equation.

$\therefore 2x^2 - 4x = y - 1$
 $x^2 - 2x = \frac{1}{2}y - \frac{1}{2}$
 $x^2 - 2x + 1 = \frac{1}{2}y - \frac{1}{2} + 1$
 $(x - 1)^2 = \frac{1}{2}y + \frac{1}{2}$
 $(x - 1)^2 = \frac{1}{2}(y + 1)$

$\therefore$ vertex is at (1, -1)

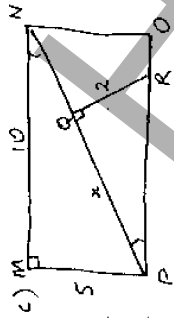
(ii) $4a = \frac{1}{2}$

$\therefore a = \frac{1}{8}$



$y = -1 - \frac{1}{8}$
 $= -\frac{9}{8}$

(Q5 cont'd...)



i) aim: $\triangle PQR \parallel \triangle NMP$
 $\widehat{MNP} = 90^\circ$ (vertex angle of a rectangle)
 $\widehat{MNP} = \widehat{PQR}$
 $MN \parallel PQ$ (opp sides of a rectangle)
 $\widehat{MNP} = \widehat{PQR}$ (alternate $\angle$'s, $MN \parallel PQ$)
 $\triangle PQR \parallel \triangle NMP$
 (2 corresp. $\angle$'s equal)

ii) $\frac{MN}{MP} = \frac{QP}{QR}$ (corresp. sides of similar Δ 's)
 $\frac{10}{5} = \frac{QP}{4}$
 $QP = 4 \text{ cm}$
 $PQ = 4 \text{ cm}$

Question 6

a) $y = \sin x$ looks like
 $\therefore y = \sin 2x$ has period $\frac{2\pi}{2} = \pi$



ii) there are 4 solutions since they meet (are equal) 4 times on the graph.

b) i) $x = -5$ at $t = 0$
 ii) $x = 0$ when $t = 1, 6, 10$ sec
 iii) $v = \frac{dx}{dt}$ at rest when $\frac{dx}{dt} = 0$
 i.e. $v = 0$ when $t = 4, 8$ sec.

iv) $a = \frac{d^2x}{dt^2}$ i.e. $a = 0$ at the inflexion point
 $\therefore a = 0$ at $t = 5$ seconds

v) from $t = 0$ to 3
 it travelled from $x = -5$ to 1
 i.e. 9 metres

from $t = 3$ to $t = 8$
 it travelled from $x = 3$ to -1
 i.e. 4 metres

from $t = 8$ to 10 it travelled 1m.
 $\therefore$ total distance = $9 + 4 + 1$

(Q6 cont'd...)

c) \$60,000
 $r = \frac{1}{2}\%$ per month
 $n = 5 \times 12 = 60$ months
 $M =$ monthly repayment

i) After 1 month Bruce owes = $\$60,000 \times (1 + \frac{0.5}{100}) - M$
 $A_1 = \$60,300 - M$

ii) After 2 months Bruce owes $A_2 = A_1 \times 1.005 - M$
 $= (\$60,300 - M) \times 1.005 - M$

$\therefore A_2 = \$60,000 \times 1.005^2 - 1.005M - M$
 $A_3 = A_2 \times 1.005 - M$
 $= (\$60,000 \times 1.005^2 - 1.005M - M) \times 1.005 - M$

$A_3 = \$60,000 \times 1.005^3 - 1.005^2M - 1.005M - M$
 $= \$[60,000(1.005)^3 - M(1.005^2 + 1.005 + 1)]$

iii) $A_n = \$60,000 \times 1.005^n - M(1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1})$
 but $n = 60$ months
 and $A_{60} = 0$ left to repay

$\therefore 0 = \$60,000 \times 1.005^{60} - M(1 + 1.005 + \dots + 1.005^{59})$
 $M = \frac{\$60,000 \times 1.005^{60}}{1 + 1.005 + \dots + 1.005^{59}}$

(Q6 cont'd...)

$1 + 1.005 + \dots + 1.005^{59}$
 is a G.S. with $a = 1, r = 1.005$,
 $\therefore S_{60} = \frac{a(r^n - 1)}{r - 1}$
 $= \frac{1(1.005^{60} - 1)}{1.005 - 1}$

$\therefore M = \frac{\$60,000 \times 1.005^{60}}{\frac{1.005^{60} - 1}{0.005}}$

$M = \$1,159.97$
 (to nearest cent)

Question 7

a) $x^2 + (k+3)x - k = 0$

$\Delta = b^2 - 4ac$

$= (k+3)^2 - 4 \times 1 \times (-k)$
 $= k^2 + 10k + 9$

real roots when $\Delta \geq 0$

$k^2 + 10k + 9 \geq 0$

$(k+9)(k+1) \geq 0$

$k \leq -9$ or $k \geq -1$

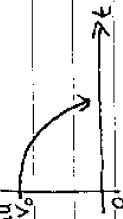


$k \leq -9$ or $k \geq -1$

b) i) V is dec. at an inc. rate

i) $\frac{dV}{dt} < 0$ and $\frac{d^2V}{dt^2} < 0$

ii) V_{full}



c) $x = 5 + \ln(1+2t)$

i) $V = \frac{dx}{dt}$

$= \frac{d}{dt} (5 + \ln(1+2t))$

$= \frac{2}{1+2t}$

$a = \frac{dV}{dt}$

$= \frac{d}{dt} \frac{2}{1+2t}$

$= -\frac{2 \times 2}{(1+2t)^2} = -4$

ii) $x = 10$

$\therefore 10 = 5 + \ln(1+2t)$

$5 = \ln(1+2t)$

$e^5 = 1+2t$

$t = \frac{e^5 - 1}{2}$

≈ 73.7 seconds

iii) $V = \frac{2}{1+2t}$

$= \frac{2}{1+e^5 - 1}$

$= \frac{2}{e^5}$

≈ 0.013475894

$a = -4$

$1+e^5 - 1$

$= -\frac{4}{e^5}$

$\approx -1.82 \times 10^{-4}$

iv) since $V = \frac{2}{1+2t}$ and $t \geq 0$

then $V \geq 0$ for all t

similarly $a \geq 0$ for all t

always moving in a

positive direction

$\therefore$ it does not change

direction

Question 8

a) 15, 18, 21, 24

A.S. $d = 21 - 18$ $a = 15$

$= 18 - 15$ $a = 52$

$= 3$

i) $u_n = a + (n-1)d$

$u_{52} = 15 + 51 \times 3$

$= 168$ cars

ii) $S_n = \frac{n}{2}(a+L)$

$= \frac{52}{2}(15 + 168)$

$= 4758$ cars

b) $\int \frac{x}{x^2+1} dx = \left[\frac{1}{2} \ln(x^2+1) \right]'$

$= \frac{1}{2} \ln(3^2+1) - \frac{1}{2} \ln(1^2+1)$

$= \frac{1}{2} \ln 10 - \frac{1}{2} \ln 2$

$= 0.804718956$

c) $y = \tan x$, $y = 2 \sin x$

i) $\tan x = \tan \frac{\pi}{3}$

$= \sqrt{3}$

$2 \sin x = 2 \times \sin \frac{\pi}{3}$

$= 2 \times \frac{\sqrt{3}}{2}$

$= \sqrt{3}$

$\therefore \tan x = 2 \sin x$

for $x = \frac{\pi}{3}$ and $y = \sqrt{3}$

ii) $\frac{d}{dx} (\ln \cos x) = \frac{1}{\cos x} \cdot \frac{-\sin x}{\cos x}$

$= -\sin x$

$= -\frac{\sin x}{\cos x}$

$= -\tan x$

(28 cont'd...)

iii) Area $= \int_0^{\frac{\pi}{2}} \tan x dx = \int_0^{\frac{\pi}{2}} 2 \sin x$

$\therefore A = \left[-\ln \cos x \right]_0^{\frac{\pi}{2}} = \left[-2 \cos x \right]_0^{\frac{\pi}{2}}$

$\therefore A = \left(-\ln \cos \frac{\pi}{2} - \ln \cos 0 \right)$

$= \left(-2 \cos \frac{\pi}{2} - 2 \cos 0 \right)$

$= -\ln \frac{1}{2} + \ln 1 + 2 \times \frac{1}{2} + 2 \times 1$

$= -\ln \frac{1}{2} + 0 + 1 + 2$

$= 3 - \ln \frac{1}{2}$

(Q9 cont'd...)

Question 9

a) $r = 10\text{cm}$ $\theta = \frac{3\pi}{8}$

$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 10^2 \times \frac{3\pi}{8}$

$= \frac{75\pi}{2} \text{ cm}^2$

b) $d_{1990} = \$50,000$

$d_{1991} = \$50,000 \times 75\%$

$= \$37,500$

$d_{1992} = d_{1991} \times 75\%$

etc.

i) $A = \$50,000$ G.S.

$r = 0.75$

$n = 6$

$u_n = ar^{n-1}$

$u_6 = 50,000 \times (0.75)^5$

$d_{1995} = \$11,865.23$

ii) $S_{\infty} = \frac{a}{1-r}$

$= \frac{\$50,000}{1-0.75}$

$= \$200,000$

c) i) Perimeter = rectangle + circle

$L = 2 \times \text{length} + 2 \times \text{width} + \text{circle}$

$= 2 \times y + 2 \times 2x + \frac{1}{2} r \theta$

$= 2y + 4x + \frac{1}{4} \times \pi \times 2$

$= 2y + 4x + \frac{\pi}{2}$

but Area = length $\times$ width

$10000 = y \times 2x$

$y = \frac{10,000}{2x}$

Question 10

a) i) $-1 < r < 1$

where $S_{\infty} = \frac{a}{1-r}$

ii) $S_{\infty} = a$

$1-r$

$\frac{1}{1-w} = \frac{a}{1-\frac{w}{2}}$

$\frac{1}{1-w} = \frac{a}{\frac{2-w}{2}}$

$\therefore a = \frac{1 \times \frac{2-w}{2}}{1-w}$

$= \frac{1-w}{2(1-w)}$

$= \frac{1}{2}$

$\ln \frac{1}{2} = \ln e^{-k \cdot 30}$

$\ln \frac{1}{2} = -k \cdot 30$

$\therefore k = \ln \frac{1}{2} / -30$

iii) $S = 1590 \times S_0$

$= \frac{3}{20} S_0$

$\frac{3}{20} S_0 = S_0 e^{\frac{30 \ln \frac{1}{2}}{20}}$

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(Q10 cont'd...)

$\therefore V \approx 5.7596 \text{ units}^3$

ii) 1 Litre $\approx 1000 \text{ cm}^3$

$1000 \div 5.76 \approx 173.6$ icecreams

$\therefore 173$ icecreams can be made

c) i) $S = S_0 e^{-kt}$

$\frac{dS}{dt} = -k S_0 e^{-kt}$

$= -k S_0 e^{-kt}$

$= -k S$

ii) $t = 30, S = \frac{1}{2} S_0$

$\therefore \frac{1}{2} S_0 = S_0 e^{-k \cdot 30}$

$\frac{1}{2} = e^{-k \cdot 30}$

$\ln \frac{1}{2} = \ln e^{-k \cdot 30}$

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(Q9 cont'd...)

hence, $L = \frac{10000}{x} + 4x + \frac{\pi x}{2}$

ii) L is a minimum

when $L' = 0$

$L = \frac{10000}{x} + 4x + \frac{\pi x}{2}$

$L' = -\frac{10000}{x^2} + 4 + \frac{\pi}{2}$

$\therefore 0 = -\frac{10000}{x^2} + 4 + \frac{\pi}{2}$

$\frac{10000}{x^2} = 4 + \frac{\pi}{2}$

$\therefore x^2 = \frac{10000}{4 + \frac{\pi}{2}}$

$\therefore x = \sqrt{\frac{10000}{4 + \frac{\pi}{2}}}$

$\approx 42.4 \text{ m}$

$L'' = \frac{20000}{x^3}$

when $x = 42.4 \text{ m}$

$L'' > 0$

$\therefore U$ is minimum

(Q9 cont'd...)

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