Projectile Motion

Question 10.
A large vertical wall stand on horizontal ground. The nozzle of a water hose is positioned at a point *C* on the ground at a distance *c* from the wall and the water jet can be pointed in any direction from *C*. Also the water issues from the nozzle with speed *V* . (Air resistance may be neglected and the constant *g* denotes the acceleration due to gravity.)
(i) Prove that the jet can reach the wall above ground level if and only if $V > \sqrt{(gc)}$.
(ii) If $V = 2\sqrt{(gc)}$ prove that the portion of the wall that can be reached by the jet is a parabolic segment of height $(15c)/8$ and area $(5\sqrt{15}(c^2))/2$.

10.
A gun fires a shot from O with initial speed V at an angle α with the horizontal.
If the acceleration due to gravity is constant (= g) prove that the shot describes a parabola of focal length $V^2 \cos^2 \alpha /(2g)$.
If the initial speed V is fixed but the direction of firing can be varied prove that the region of vulnerability (i.e. the set of points that can be hit) consists of points within and on the paraboloid whose equation (referred to a cartesian x, y, z-frame with origin at O and z-axis vertically upwards) is
$$x^2 + y^2 + (2V^2/g)z = V^4/g^2.$$

Question 3. Prove that the range on a horizontal plane of a particle projected upwards at anangle *α* to the plane with velocity *V* metres per second is *V* 2 sin 2*α g* metres, wher *g* metres per second per second is the acceleration due to gravity. A garden sprinkler sprays water symmetrically about its vertical axis at a constant speed of *V* metres per second. The initial direction of the spray varies continuously between angles of 15 and 60 to the horizontal.
Prove that, from a .xed position *O* on level ground, the sprinkler will wet the surface of an annular region with centre *O* and with internal and external radii $V^2/2g$ metres and V^2/g respectively.
Deduce that by locating the sprinkler appropriately relative to a rectangular garden bed of size 6 metres by 3 metres, the entire bed may be watered provided that
$$\frac{V^2}{2g} \geq 1 + \sqrt{7}.$$

Question 4.
From an origin on a horizontal plane, a particle is projected upwards with speed *V* and angle of elevation *α*. Show that the equation of the particle's path is
$$y = x \tan \alpha - \frac{gx^2 \sec^2 \alpha}{2V^2}.$$
where *x, y* are horizontal, vertical co-ordinates in the plane of the path of the particle and *g* is the acceleration due to gravity. Hence, or otherwise, deduce that the range on the horizontal plane is $V^2 \sin 2\alpha/g$.
A vertical wall is a distance d from the origin, and the plane of the wall is perpendicular to the plane of the particle's trajectory.
i) Show that if $d < V^2/g$, the particle will strike the wall provided that
$\beta < \alpha < \frac{\pi}{2} - \beta,$, where beta= $\beta = \frac{1}{2}\sin^{-1}(gd/V^2).$
ii) Show also that the maximum height the particle can reach on the wall is:

$$\frac{V^4 - g^2 d^2}{2gV^2}.$$

Question 5

A ball is dropped from a height of 49 meters above the ground. The height of the ball at time t is $h(t)=49-4.9t^2$ meters. A light which is also 49 meters above the ground is 10 meters to the left of the ball's original position. As the ball drops, the shadow of the ball caused by the light moves across the ground. How fast is the shadow moving one second after the ball is dropped?