Exercise 5

Extracts from

Taylors College COMPLEX NUMBERS Study Guide

+ ANSWERS

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De Moivre's Theorem

Let $z = r\text{cis}\theta$. Then $|z^n| =$ _______ $=$ _______ and $\arg z^n =$ _______ $=$ _______ so $z^n =$ _______. De Moivre's theorem states $(\text{cis}\theta)^n = \text{cis}n\theta$. Proof: $(e^{i\theta})^n = e^{in\theta}$.

1. Simplify **(i)** $(cis\theta)^2$ **(ii)** $(cis\frac{\pi}{5})^{10}$ **(iii)** $(\cos\theta - i\sin\theta)^{-1}$ **(iv)** $(1+i\sqrt{3})^3(\sqrt{3}-i)^4$ **(v)** $z_1^5 + z_2^3$ where $z_1 = -\sqrt{2} + \sqrt{2}i$ and $z_2 = 1 - \sqrt{3}i$
2. **(i)** Express $z = 1 + i$ in modulus-argument form.
(ii) Find $(1+i)^4$. Express your answer in the form $a + ib$.
(iii) Hence find $z^4 + \frac{1}{z^4}$.
3. Find the square roots of $1 + i\sqrt{3}$
5. **(ii)** If $\omega = \frac{1+i\sqrt{3}}{2}$ calculate ω^{10}.

ANSWERS

De Moivre's Theorem

$|z^n| = |z|^n = r^n$ and $\arg z^n = n \arg z = n\theta$ so $z^n = r^n\text{cis}n\theta$
1. **(i)** $cis2\theta$ **(ii)** 1 **(iii)** $cis\theta$ **(iv)** $64(1+\sqrt{3})$ **(v)** $8 - 16i$
2. **(i)** $\sqrt{2}cis\frac{\pi}{4}$ **(ii)** -4 **(iii)** $-4\frac{1}{4}$ **3.** $\pm\frac{1}{2}(\sqrt{6} + i\sqrt{2})$ **5.** **(ii)** $-\frac{1}{2}(1 + i\sqrt{3})$