Exercise 2

Extracts from

Taylors College COMPLEX NUMBERS Study Guide

+ ANSWERS

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Quadratic Theory and Complex Numbers

If α and β are the roots of $ax^2 + bx + c = 0$ then
$\alpha + \beta =$ ______________
$\alpha\beta =$ ______________
$\alpha^2 + \beta^2 =$ ______________
$ax^2 + bx + c = a(x^2 - (\alpha + \beta)x + \alpha\beta) = 0$

1. Form the quadratic equation with roots **(i)** $5 + i$ and $-3 - i$ **(ii)** $5 + i$ and $\overline{5 + i}$

2. $3 + 4i$ is a root of the equation $z^2 + 5iz + k = 0$. Find k in two different ways.

3. $3 + 4i$ is a root of $z^2 + kz + 5i = 0$. Find k.

4. $z^2 - 4z + k = 0$ has one root α where $\Im(\alpha) = 1$. If k is real, find both roots of the equation and the value of k.

The Complex Number Plane - Argand Diagram

z and $\overline{z}$ are ______________ of each other in the _______ axis.

Radians

$180° =$ _______ radians = _______ rad = _______
$90° =$ _______ $360° =$ _______ $270° =$ _______ $45° =$ _______
$60° =$ _______ $30° =$ _______
$72° =$ _______

Modulus - Argument Form of z=x+iy

$|z| =$ distance from z to the origin.
$|z| =$ _______
$|z|^2 =$ _______

$\arg z = \theta \Rightarrow \tan\theta =$ _______
Now $x = |z|\cos\theta$ and $y =$ _______ so $z = x + iy =$ __________ $=$ __________
This is the modulus-argument (mod-arg) form of z.

Write the following complex numbers in mod-arg form.

1. $1+i$ **2.** $-1+i$ **3.** $-1-i$ **4.** $1-i$ **5.** $2i$ **6.** -2 **7.** $3+4i$ **8.** $\overline{3+4i}$
$\sin(-\theta) =$ _______ $\cos(-\theta) =$ _______

Modulus - Argument Relationships

Find the modulus and argument of

1. $\frac{(-4+i)(-2+3i)}{1-i}$ **2.** $\frac{5cis50^\circ 2cis70^\circ}{3cis20^\circ}$ **3.** $(1-\sqrt{3}i)^3(1+i)^4$
4. (1984 HSC) Calculate the modulus and argument of the product of the roots of $(5+3i)z^2 - (1-4i)(z + (8-2i) = 0$.

Answers

Quadratic Theory and Complex Numbers

z and $\overline{z}$ are complex conjugates of each other in the real axis.

Radians

$180^\circ = \pi$ radians $= \pi$ rad $= \pi^c = \pi$
$90^\circ = \frac{\pi}{2}$, $360^\circ = 2\pi$, $270^\circ = \frac{3\pi}{2}$, $45^\circ = \frac{\pi}{4}$
$60^\circ = \frac{\pi}{3}$, $30^\circ = \frac{\pi}{6}$, $72^\circ = \frac{\pi}{180^\circ} \cdot 72^\circ = \frac{\pi}{5}$

Modulus-Argument form of z=x+iy

$|z| = \sqrt{x^2+y^2}$, $|z|^2 = x^2+y^2$, $\tan\theta = \frac{y}{x}$, $y = |z|\sin\theta$, $z = |z|cis\theta$
1. $\sqrt{2}cis\frac{\pi}{4}$ **2.** $\sqrt{2}cis\frac{3\pi}{4}$ **3.** $\sqrt{2}cis\frac{-3\pi}{4}$ **4.** $\sqrt{2}cis\frac{-\pi}{2}$ **5.** $2cis\frac{\pi}{2}$ **6.** $2cis\pi$ **7.** $5cis\tan^{-1}\frac{4}{3}$
8. $5cis-\tan^{-1}\frac{4}{3}$

Modulus - Argument Relationships

1. $\sqrt{\frac{221}{2}}$; $\frac{9\pi}{4} - \tan^{-1}\frac{1}{4} - \tan^{-1}\frac{3}{2}$ **2.** $\frac{10}{3}$; 100° **3.** 32; 0 **4.** $\sqrt{2}$; $\tan^{-1}\frac{3}{5} - \tan^{-1}\frac{1}{4}$.