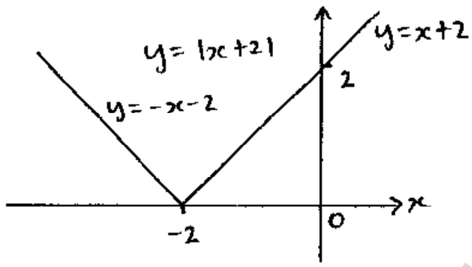
DIAGNOSTIC TEST 1

1.(a)

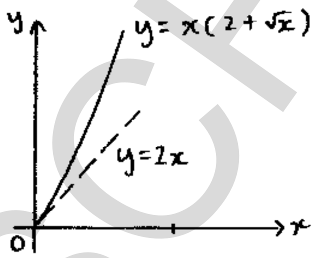


when $x \to -2^-$, $\frac{dy}{dx} = -1$

when $x \to -2^+$, $\frac{dy}{dx} = 1$

$\therefore$ The point $(-2, 0)$ is a critical point where $\frac{dy}{dx}$ is not defined.

(b)



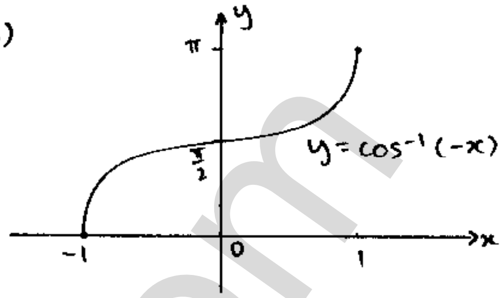
$y = 2x + x\sqrt{x}$ $\quad \therefore y = 2x + x^{3/2}$

$\therefore \frac{dy}{dx} = 2 + \frac{3}{2}x^{1/2}$

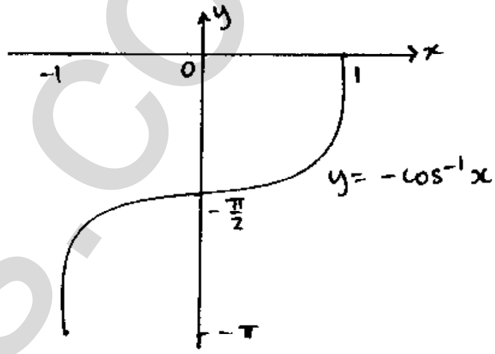
when $x \to 0^+$, $\frac{dy}{dx} = 2$

$\therefore (0,0)$ is a critical point where the tangent at this point is $y = 2x$.

2.(a)



(b)



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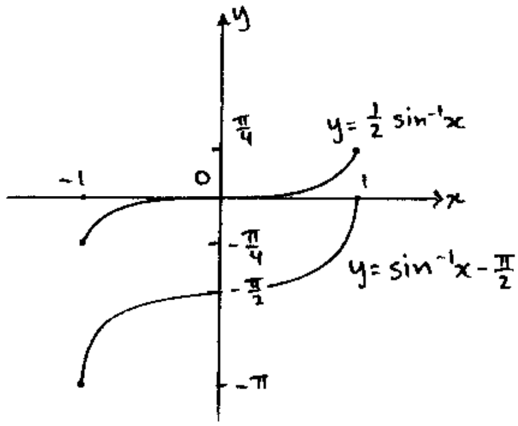

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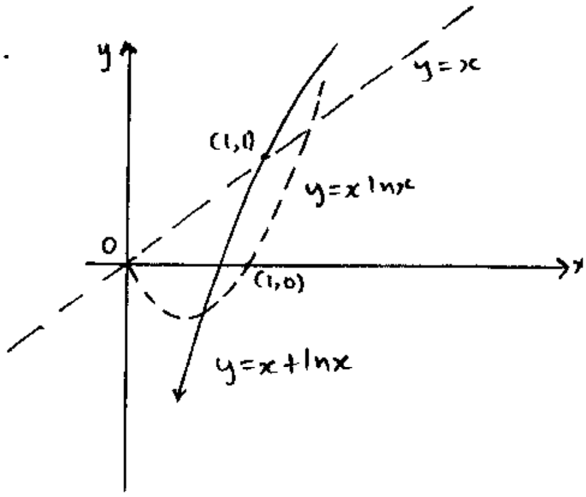
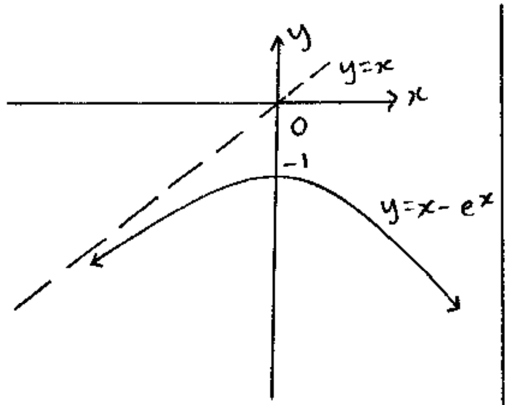
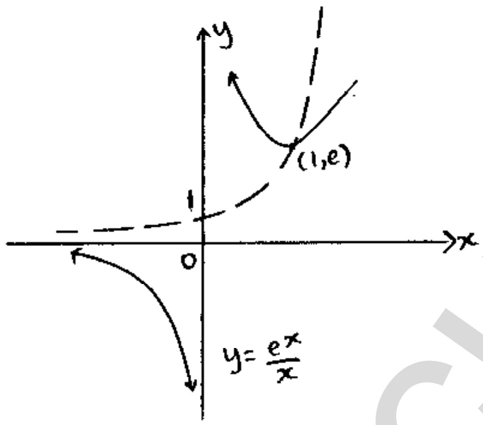


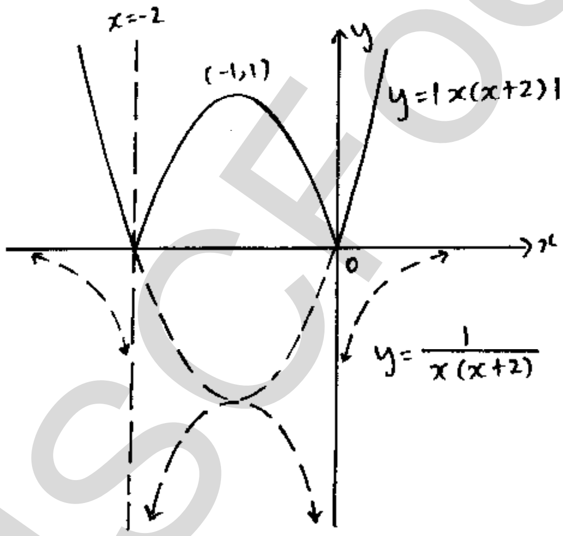
5.(a)



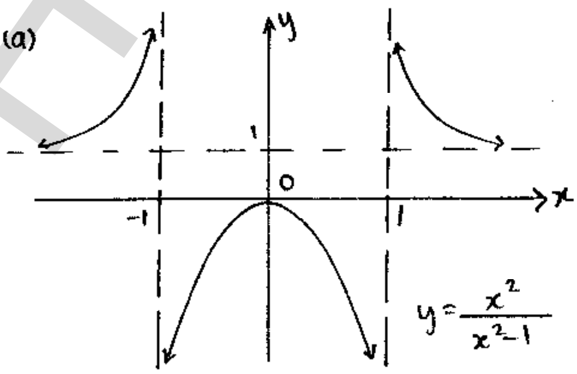
(b)



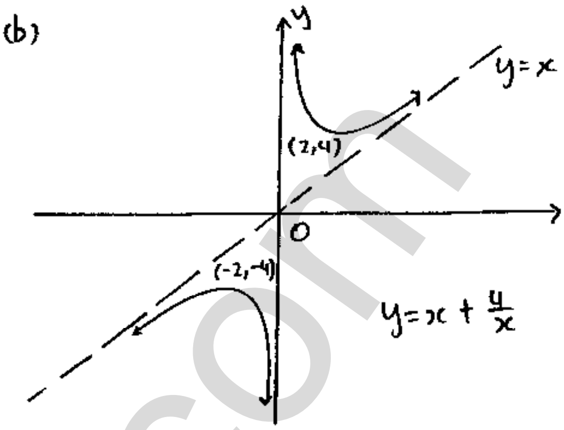
6.



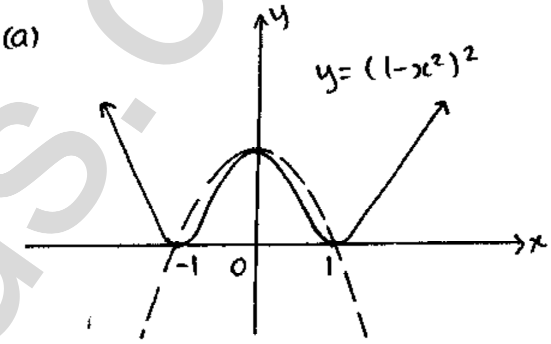
7.(a)



(b)

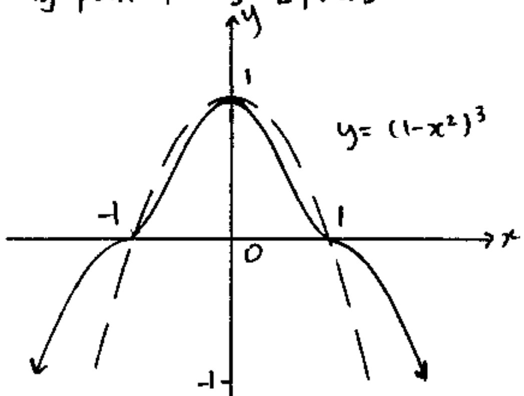


8.(a)



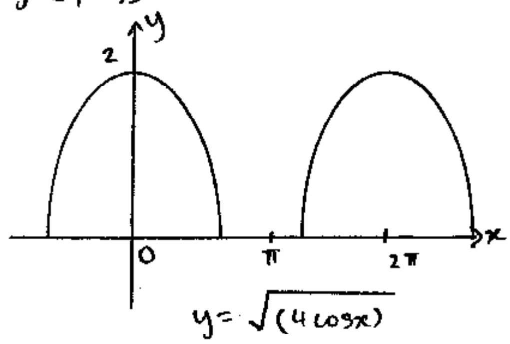
When squaring a function $f(x)$, every intersection with x-axis becomes a minimum turning point for $y=[f(x)]^2$

(b)

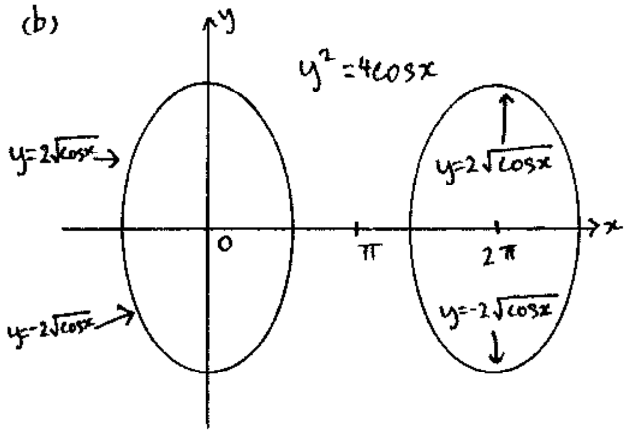


When cubing $y=f(x)$, every intersection with y-axis becomes a horizontal point of inflexion for $y=[f(x)]^3$.

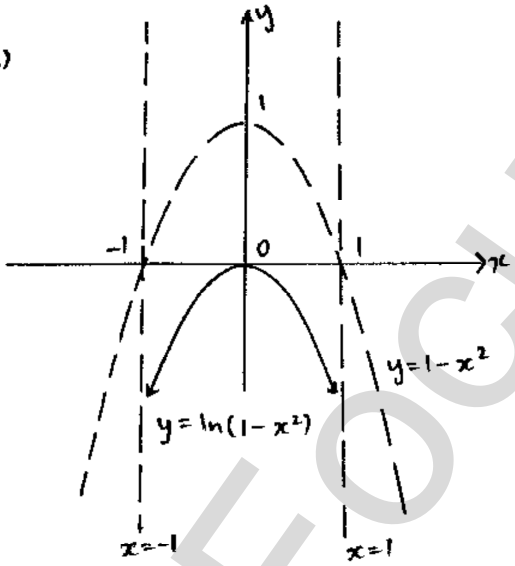
9.(a)



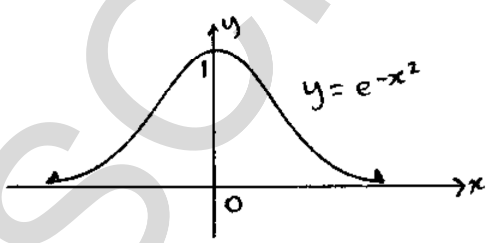
(b)



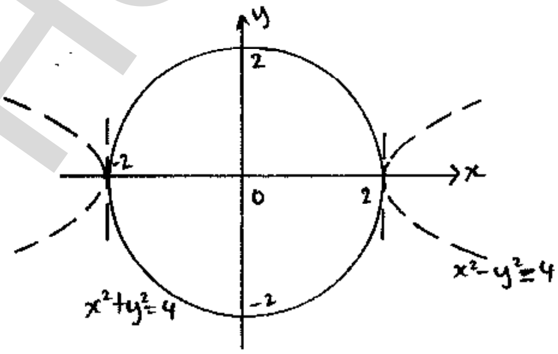
10. (a)



(b)

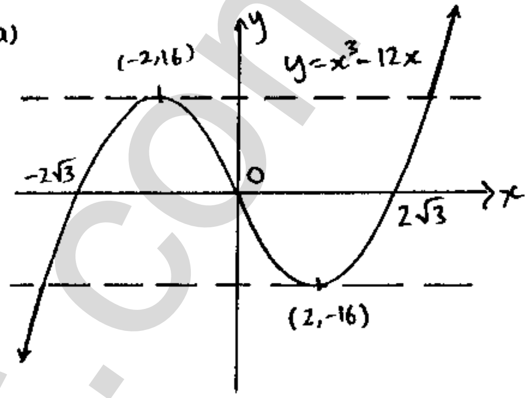


11.



$x^2+y^2=4$ is the equation of a circle and $x^2-y^2=4$ is the equation of a hyperbola.

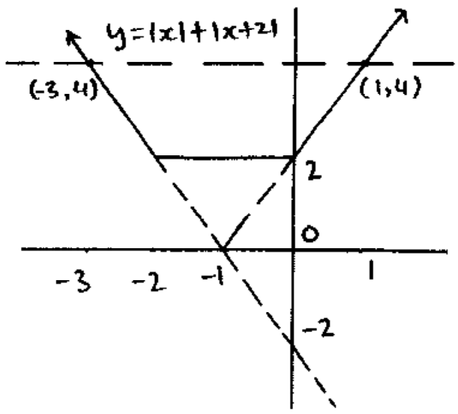
12. (a)



$x^3-12x+k=0 \quad \therefore x^3-12x=-k$

$\therefore$ The equation will have one root when $y=-k$ intersects the curve $y=x^3-12x$ only once. From the graph we can see that for $k>16$ or $k<-16$, this equation has only root.

(b) $y=\begin{cases}-2x-2 & \text{for } x<-2\\ 2 & \text{for } -2\le x\le 0\\ 2x+2 & \text{for } x>0\end{cases}$



$|x|+|x+2|=4 \quad$ for $x<-2$

$\therefore -2x-2=4 \quad \therefore x=-3$

$\therefore (-3,4)$ is a solution.

for $-2\le x\le 0 \quad \therefore 2=4$ invalid for $x>0$ $\therefore 2x+2=4$ $\therefore x=1$

$\therefore (1,4)$ is the other solution

from the graph we can see that $|x|+|x+2|>4$ when $x<-3$ or $x>1$.