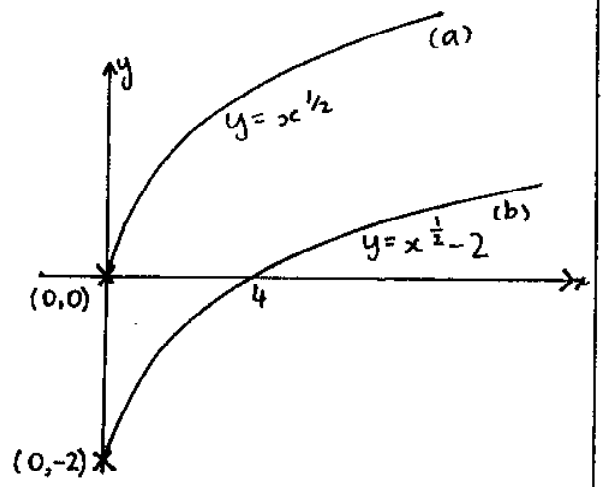
4U GRAPHS — D & G - ARNOLD

Solutions by Sami El Hourri

EXERCISE: 1·1

1.



(a) $\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

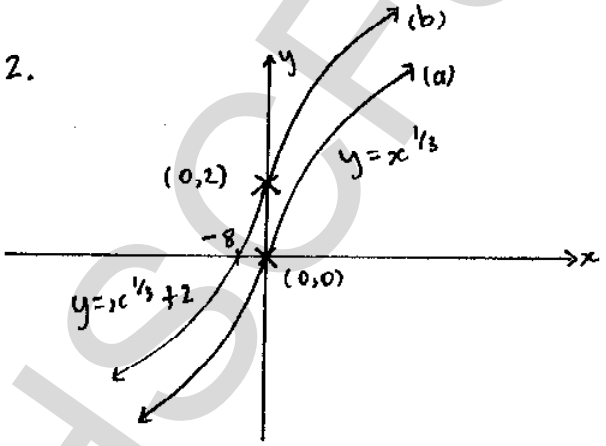
when $x \to 0^+$, $\frac{dy}{dx} \to +\infty$

$\therefore \frac{dy}{dx}$ is not defined.

$\therefore$ Tangent at $(0,0)$ is vertical.

(b) Similarly, the tangent is vertical at $(0,-2)$.

2.



(a) $\frac{dy}{dx} = \frac{1}{3}x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

when $x \to 0^+$, $\frac{dy}{dx} \to +\infty$

when $x \to 0^-$, $\frac{dy}{dx} \to -\infty$

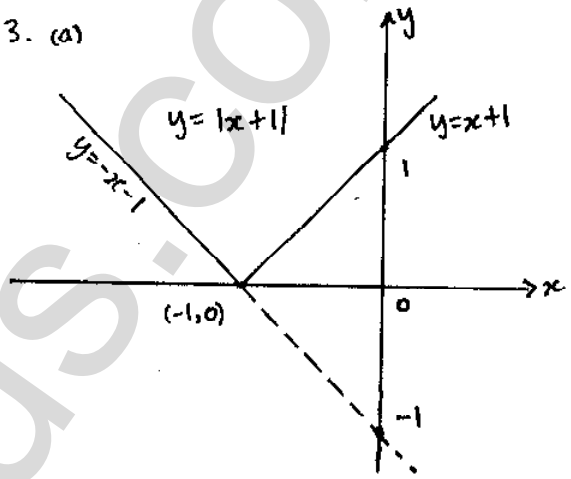
$\therefore \frac{dy}{dx}$ is not defined at $(0,0)$.

$\therefore$ Tangent at $(0,0)$ is vertical, also The tangent at $(0,2)$ is vertical.

[$(0,2)$ is a critical point]

(b) Similarly, the tangent is vertical at $(0,0)$ [critical point]

3. (a)

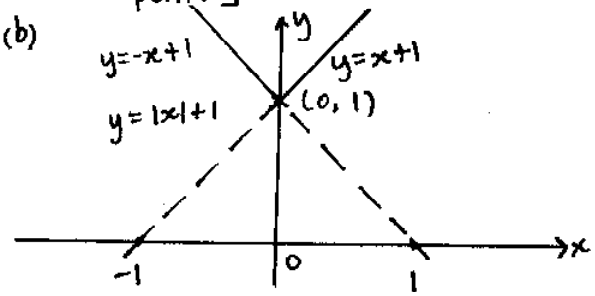


$\frac{dy}{dx} = -1$ when $x \to -1^-$

$\frac{dy}{dx} = 1$ when $x \to -1^+$

$(-1,0)$ is an angular point [critical point]

(b)



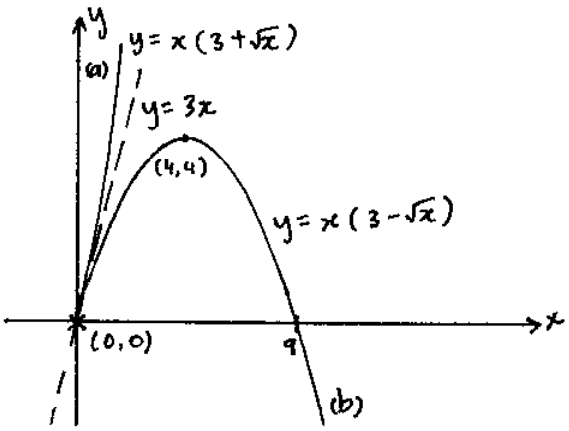
$\frac{dy}{dx} = -1$ when $x \to 0^-$

$\frac{dy}{dx} = 1$ when $x \to 0^+$

$(0,1)$ is an angular point

$\therefore (0,1)$ is a critical point, where $\frac{dy}{dx}$ is not defined.

4.



(a) $\frac{dy}{dx} = 3+\sqrt{x} + (\frac{1}{2}x^{-\frac{1}{2}})x$

$= 3+\sqrt{x} + \frac{1}{2\sqrt{x}} \times x$

$= 3+\sqrt{x} + \frac{x}{2\sqrt{x}} = \frac{6\sqrt{x}+3x}{2\sqrt{x}}$

$\therefore \frac{dy}{dx} = \frac{6\sqrt{x}+3x}{2\sqrt{x}}$ when $x \to 0^+$

$\therefore \frac{dy}{dx}$ is not defined.

$\therefore$ Tangent at $(0,0)$ is vertical.

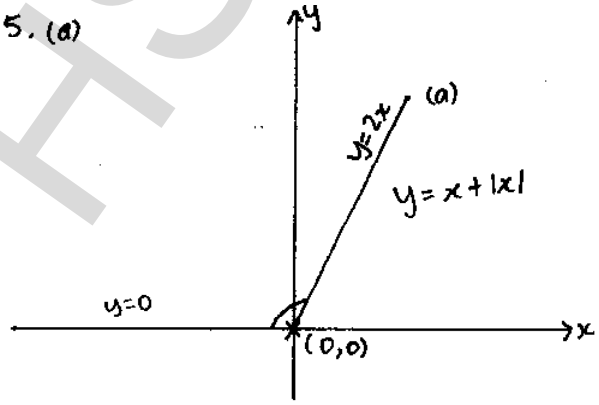
(b) $\frac{dy}{dx} = 3-\sqrt{x} + (-\frac{1}{2\sqrt{x}})x$

$= 3-\sqrt{x} - \frac{x}{2\sqrt{x}} = \frac{6\sqrt{x}-3x}{2\sqrt{x}}$

$\therefore \frac{dy}{dx} = \frac{6\sqrt{x}-3x}{2\sqrt{x}}$ when $x \to 0^+$

$\therefore$ Tangent at $(0,0)$ is vertical.

5. (a)

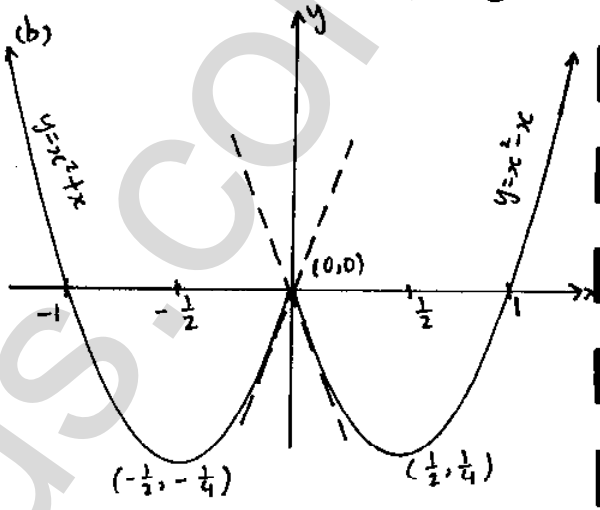


$\frac{dy}{dx} = 2$ when $x \to 0^+$

$\frac{dy}{dx} = 0$ when $x \to 0^-$

$\therefore (0,0)$ is an angular point.
[$(0,0)$ is a critical point]

(b)

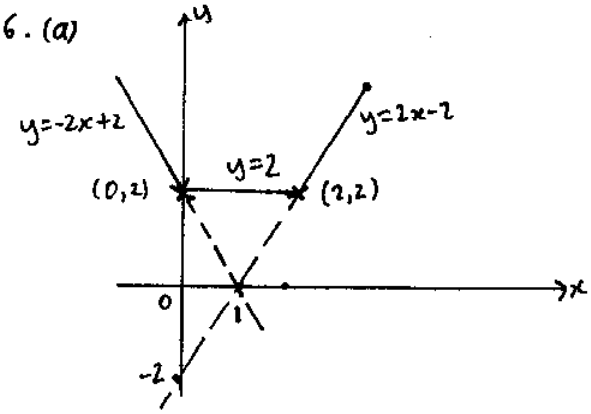


$\frac{dy}{dx} = 2x+1$ when $x \to 0^-$

$\frac{dy}{dx} = 2x-1$ when $x \to 0^+$

$\therefore$ Tangent at $(0,0)$ is vertical.

6. (a)

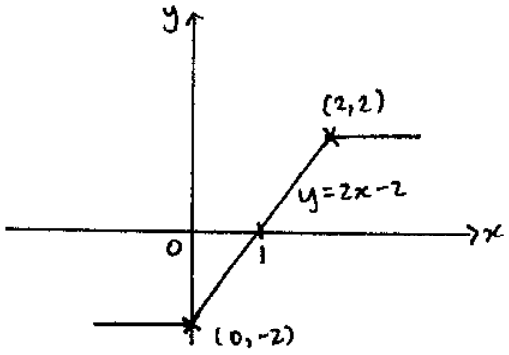


$\frac{dy}{dx} = -2$ when $x \to 0^-$

$\frac{dy}{dx} = 0$ when $x \to 0^+$

The gradient at 0^- is -2.
The gradient at 0^+ is 0.

(b)



$\frac{dy}{dx} = 0$ when $x \to 2^+$

$\frac{dy}{dx} = 2$ when $x \to 0^+$

$\frac{dy}{dx} = 0$ when $x \to 2^-$

At the points $(0,-2)$ and $(2,2)$, $\frac{dy}{dx}$ is not defined since at each of them the derivative to the left is not equal to the derivative to the right. These 2 critical points can be also called angular points.