NEW SOUTH WALES

Higher School Certificate

Mathematics Extension 2

Exercise 41/67

by James Coroneos*

1. What shape is each of the following?
  (a) the orbit of the earth around the sun.
  (b) the path followed by a projectile thrown through the air.
  (c) the orbit of Halley's comet.
  (d) the cross-section of the reflector from a car headlight.
  (e) the curve assumed by a piece of heavy rope hanging freely with both ends attached to a beam.

2. Find the lengths of the axes, the eccentricity, the coordinates of the foci, and the equations of the directrices and asymptotes (if any) of the following curves. Sketch each curve.
  (a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ **(b)** $9x^2 - 16y^2 = 1$ **(c)** $3x^2 + 4y^2 = 12$ **(d)** $5x^2 - 2y^2 = 20$

3. Find the equations of the following conics each with its centre at the origin. Sketch each curve.
  (a) an ellipse with axes 6 units and 4 units respectively.
  (b) a hyperbola with focus $(4, 0)$ and directrix $x = 2$.
  (c) an ellipse with focus $(4, 0)$ and eccentricity $\frac{1}{4}$.
  (d) an ellipse with focus $(0, 4)$ and directrix $y = 6$.
  (e) a hyperbola with transverse axis 8 units in length and focus at $(0, 6)$.

4. Find the equations of each of the following conics. Sketch each curve.
  (a) focus $(3, 5)$, directrix $2x + y - 4 = 0$, eccentricity $\frac{1}{3}$.
  (b) focus $(-1, 1)$, directrix $x + y + 1 = 0$, eccentricity $\frac{4}{3}$.
  (c) focus $(2, -1)$, directrix $x - y + 1 = 0$, eccentricity 1.

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5. P is any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with foci S, S', directrices d, d'. PM, PM' are the perpendiculars drawn from P to d, d' respectively. Using the fact that $|PS| = e|PM|$, prove that $|PS| + |PS'| = 2a$.

6. P is any point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with foci S, S', directrices d, d'. PM, PM' are the perpendiculars drawn from P to d, d' respectively. Prove that $\left||PS| - |PS'|\right| = 2a$.

7.
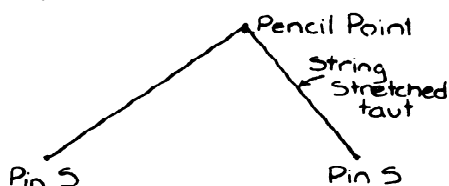
Find the distance between thye foci S, S' of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. If P is any point on the ellipse, what is the magnitude of $|PS| + |PS'|$? Using a scale of 1cm to the unit, perform the following construction. Mark points S, S' the correct distance apart. Cut a piece of string equal in length to $|PS| + |PS'|$. With drawing pins, fix one end of this string at S, the other at S'. Use a pencil point to stretch the string taut, keeping it in contact with the page. Now move the pencil so that the string is kept taut, thus tracing out the ellipse with the pencil point.

8. Write down the equations of the following conics. Sketch each conic.
   (a) circle, centre $(3, -2)$, radius 5.
   (b) parabola, vertex at $(-3, 1)$, focal length 2, axis parallel to y-axis.
   (c) ellipse, centre $(1, -2)$, axes of length 6, 4 units, major axis parallel to x-axis.
   (d) hyperbola, centre $(1, 2)$, focus at $(5, 2)$, eccentricity 2.
   (e) ellipse, extremities of major axis at $(2, 4)$, $(2, -2)$, extremities of minor axis at $(0, 1)$, $(4, 1)$. What are the coordinates of the foci?

9. What are the coordinates of the centre of each of the following conics? Name and sketch each curve.

    (a) $(x+3)^2 + (y-1)^2 = 4$

    (b) $(x-2)^2 = 4(y+3)$

    (c) $\frac{(x-1)^2}{4} + \frac{(y+4)^2}{9} = 1$

    (d) $4(x+3)^2 - 9y^2 = 36$

    (e) $2x^2 - 3y^2 + 4x + 12y - 11 = 0$

    (f) $25x^2 + 4y^2 + 100x - 8y + 4 = 0$

    (g) $2x^2 - 3y^2 + 4x + 18y - 19 = 0$