NEW SOUTH WALES

Higher School Certificate

Mathematics Extension 2

Exercise 3/67

by James Coroneos*

1. **(i)** If $(2+3i)(3-4i)=a+ib$, where a,b are real, find a and b, and hence the value of a^2+b^2.
(ii) If $x+iy=(3-i)^2+(2-3i)(3+2i)$ and x,y are real, find x and y.

2. Find real numbers x and y, such that

(i) $(2-i)x+(1+3i)y+14=0$ (Note $0=0+0i$)
(ii) $(x+iy)^2=3+4i$ **(iii)** $(2+i)x+(1-3i)(5+iy)+4-7i=0$
(iv) $\frac{2x}{1-i}-\frac{y}{2+i}=3i$ **(v)** $x+iy=\frac{3-2i}{3+2i}-\frac{2+3i}{2-3i}$
(vi) $\frac{iy}{1+ix}-\frac{3y+4i}{3x+y}=0$

3. Find real numbers x,y,r,θ (where $r>0$), such that $\frac{2i-3}{(1+i)^2}=x+iy=r(\cos\theta+i\sin\theta)$, where $0^\circ<\theta<90^\circ$

4. If $z=x+iy$, find x,y when **(i)** $\frac{z}{2+i}+\frac{25}{11i+2}=\frac{2z}{1+i}$ **(ii)** $\frac{2z}{1+i}-\frac{2z}{i}=\frac{5}{2+i}$

5. **(i)** If $(x+iy)(a+ib)=b+ia$ express x,y in terms of a,b
(ii) If $u+iv=\frac{2+i}{z+3}$, where $z=x+iy$, express u,v in terms of x,y.
(iii) If $\frac{1}{x+iy}=(a+ib)(c+id)$ where a,b,c,d,x,y are real, express x,y in terms of a,b,c,d.
(iv) If $\frac{1}{x^2+1}=\frac{Ax+B}{x+i}+\frac{Ci+D}{x-i}$, find the real numbers A,B,C,D.

6. If $\sqrt{x+iy}=a+ib$ where x,y,a,b are real and $a>0$, prove that $a^2-b^2=x$ and $2ab=y$. Hence express $\sqrt{5+12i}$ in the form $a+ib$.

*Other resources by James Coroneos are available. Write to P.O. Box 25, Rose Bay, NSW, 2029, Australia, for a catalogue. Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX.

7. Find the square roots of the following in the form $a+ib$, $a>0$:

(i) $21-20i$ **(ii)** $21+20i$ **(iii)** i **(iv)** $-11-60i$ **(v)** $4+3i$ **(vi)** $8+15i$
(vii) $\frac{1+i}{1-i}$ **(viii)** $-2i$

8. **(i)** Solve for all values of z, $z^2=-3-4i$

(ii) Express $\sqrt{8+6i}$ in the form $a+ib$ $(a>0)$, and hence solve the equation $z^2+2(1+2i)z-(11+2i)=0$. Express each of the roots in the form $x+iy$ (where x,y are real), and verify that the sum of the roots is $-2(1+2i)$ and the product is $-(11+2i)$.

9. Express the roots of the following equations in the form $a+ib$,

(i) $x^2-3x+10i=0$ **(ii)** $z^2-(1-i)z-2i=0$
(iii) $z^2-(1-i)z+7i-4=0$ **(iv)** $z^2-(1-i)z-6+17i=0$
(v) $x^2-4(1-i)x+15=0$ **(vi)** $x^2-(1+4i)x+(17i-1)=0$

10. **(i)** If $\xi+\eta i$ is a value of x for which $ax^2+bx+c=0$, where a,b,c are real, show that $2a\xi+b=0$ and $a\eta^2=a\xi^2+b\xi+c$, assuming $\eta\neq 0$.

(ii) If $\sqrt[3]{x+iy}=X+iY$, show that $4(X^2-Y^2)=\frac{x}{X}+\frac{y}{Y}$

11. **(i)** Prove that $\frac{(4+3i)\sqrt{3+4i}}{3+i}=\frac{5}{2}(1+i)$

(ii) Find x in the domain $0<x<\frac{\pi}{2}$, if $\frac{\sqrt{2}(\cos x-i\sin x)}{2+i}=\frac{1-3i}{5}$

12. **(i)** Show that $x^3-2x-4=(x-2)(x+1-i)(x+1+i)$, and hence find the roots of the cubic equation $x^3-2x-4=0$.

(ii) Divide x^3-2-2i by $x+1-i$, and hence prove that the 3 cube roots of $2+2i$ are $i-1$ and $\frac{1}{2}\{1-i\pm(\sqrt{3}+\sqrt{i})\}$.

13. Show that the number of days in the year n A.D. is $365+\frac{1}{4}\{1+(-1)^n\}\{1+i^n\}$ where n is not a multiple of 100 except for multiples of 400. [Hint, let $n=4k, 4k+1, 4k+2$ and $4k+3$ and discuss the results.]