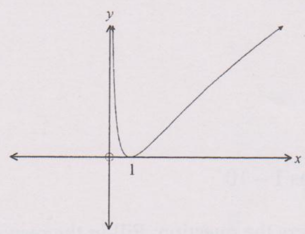
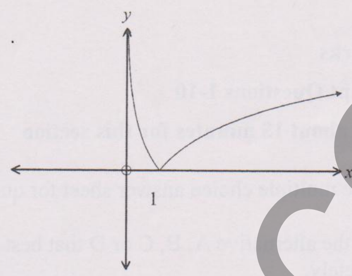
Western Region Trial 2012 Mathematics Extension 2

1. Which graph best represents $y = |\ln x|$?

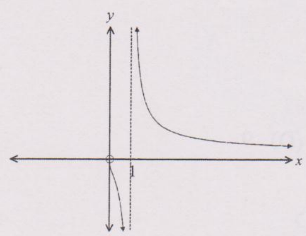
(A)



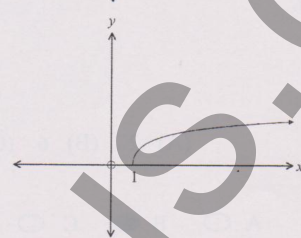
(B)



(C)



(D)



2. A particle is moving in a circular path of radius r, with a constant angular speed ω. The normal component of the acceleration is:

(A) ω

(B) $r\omega$

(C) $r\omega^2$

(D) $(r\omega)^2$

3. The modulus and principle argument of $-2i$ are given by:

(A) $|z| = -2$ and $arg(z) = -\frac{\pi}{2}$.

(B) $|z| = -2$ and $arg(z) = \frac{\pi}{2}$.

(C) $|z| = 2$ and $arg(z) = \frac{\pi}{2}$.

(D) $|z| = 2$ and $arg(z) = -\frac{\pi}{2}$.

4. $\displaystyle\int \frac{2x\,dx}{(x+1)(x+3)} = ?$

(A) $\displaystyle\int \frac{3}{x+3} - \frac{1}{x+1}\,dx$

(B) $\displaystyle\int \frac{1}{x+3} - \frac{3}{x+1}\,dx$

(C) $\displaystyle\int \frac{3}{x+3} + \frac{1}{x+1}\,dx$

(D) $\displaystyle\int \frac{1}{x+3} + \frac{1}{x+1}\,dx$

5. The curve represented by the equation $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is an ellipse with:

(A) eccentricity given by $e = \frac{3}{5}$ and foci at $(\pm 3, 0)$.

(B) eccentricity given by $e = \frac{5}{3}$ and foci at $(\pm 5, 0)$.

(C) eccentricity given by $e = \frac{3}{5}$ and foci at $(\pm 5, 0)$.

(D) eccentricity given by $e = \frac{4}{5}$ and foci at $(\pm 3, 0)$.

6. The polynomial $4x^3 - 27x + k = 0$ has a double root. The possible values of k are:

(A) $k = \pm\frac{3}{2}$

(B) $k = \pm\frac{27}{4}$

(C) $k = \pm 9$

(D) $k = \pm 27$

7. Given that $a > b > 0$, which of the following is not necessarily true?

(A) $a^2 + b^2 > 2ab$

(B) $a^2 + b^2 > 4ab$

(C) $(a+b)^2 > 4ab$

(D) $a + b > 2\sqrt{ab}$

8. The circle $x^2 + y^2 = 4$ is rotated about the line $x = 3$. Using the method of cylindrical shells, the volume could be calculated by:

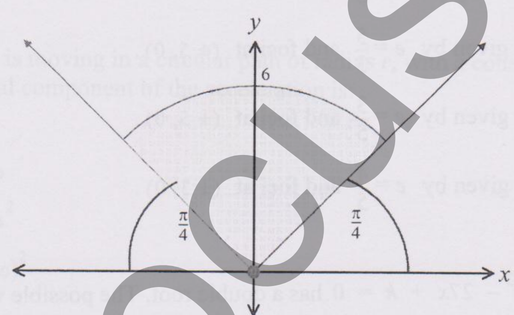
(A) $V = 2\pi\int_{-2}^{2} (3-x)\sqrt{4-x^2}\ dx$.

(B) $V = 4\pi\int_{0}^{2} (4-x)\sqrt{3-x^2}\ dx$.

(C) $V = 4\pi\int_{-2}^{2} (3-x)\sqrt{4-x^2}\ dx$.

(D) $V = 2\pi\int_{0}^{2} (3-x)\sqrt{4-x^2}\ dx$.

9. The shaded region of the Argand plane represents the points where which of the following inequalities hold simultaneously.



(A) $|z| \leq 6$ and $-\frac{\pi}{4} \leq arg(z) \leq \frac{\pi}{4}$

(B) $|z| \leq 6$ and $\frac{\pi}{4} \leq arg(z) \leq \frac{3\pi}{4}$

(C) $|z| \geq \sqrt{6}$ and $-\frac{\pi}{4} \leq arg(z) \leq \frac{\pi}{4}$

(D) $|z| \geq \sqrt{6}$ and $\frac{\pi}{4} \leq arg(z) \leq \frac{3\pi}{4}$

10. Using the recurrence relation $u_n = \int tan^n x \, dx = \frac{tan^{n-1} x}{n-1} - u_{n-2}$,

$\int tan^6 x \, dx = ?$

(A) $\frac{tan^4 x}{4} - \frac{tan^2 x}{2} + x + c$

(B) $\frac{tan^5 x}{5} - \frac{tan^3 x}{3} + tanx + c$

(C) $\frac{tan^6 x}{6} - \frac{tan^4 x}{4} + \frac{tan^2 x}{2} + c$

(D) $\frac{tan^5 x}{5} - \frac{tan^3 x}{3} + tanx - x + c$

End Section I

Section II

Total Marks (90)
Attempt Questions 11 – 16.
Allow about 2 hours&45 minutes for this section.

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page. Do not write on the back of sheets.
All necessary working should be shown in every question.

Question 11 (15 marks) Start a new sheet of writing paper. **Marks**

(a) (i) Find real numbers a and b such that $\dfrac{7x}{x^2+x-12} \equiv \dfrac{a}{x+4} + \dfrac{b}{x-3}$. **2**

(ii) Hence find $\displaystyle\int \frac{7x\,dx}{x^2+x-12}$ **1**

(b) Find $\displaystyle\int cos^5 x\,dx$. **3**

(c) If $z_1 = 3 - 2i$ and $z_2 = 3 - 4i$, find in the form $x + iy$:

(i) $z_1 - z_2$ **1**

(ii) $(z_2)^2$ **1**

(iii) $\dfrac{z_2}{z_1}$ **1**

(iv) $z_1 . \overline{z_2}$ **1**

(d) Factorise $x^4 + x^2 - 12$ completely over the field of :

(i) Rational numbers. **1**

(ii) Real numbers. **1**

(iii) Complex Numbers. **1**

(e) Given that α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$, find the equation whose roots are α^2, β^2 and γ^2. **2**

End of Question 11

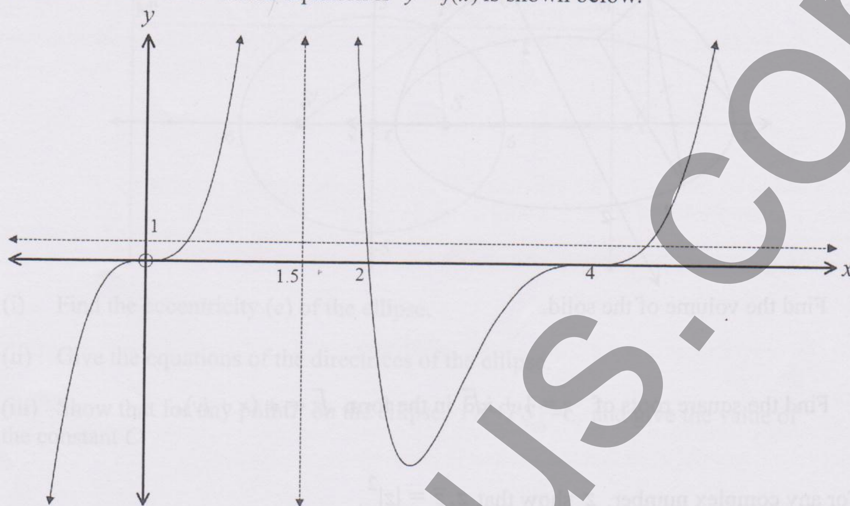
Question 12 (15 marks) Start a new sheet of writing paper. **Marks**

(a) Prove by induction that 4

$$1^3 - 2^3 + 3^3 - 4^3 + \ldots\ldots - (n-1)^3 + n^3 = \frac{(2n-1)(n+1)^2}{4}$$

where n is a positive odd integer.

(b) A sketch of the curve whose equation is $y = f(x)$ is shown below.



Draw neat, half page sketches of:

(i) $y = |f(x)|$ 2

(ii) $y = (f(x))^2$ 2

(iii) $y = \dfrac{1}{f(x)}$ 2

(c) A particle of mass m, is released to fall vertically under gravity in a medium where the resistance is k times the square of the velocity (v) of the particle.

(i) Show that the distance fallen (x) in terms of v is: 3

$$x = \frac{1}{2k} ln\left(\frac{g}{g - kv^2}\right)$$

(ii) Show that the velocity in terms of x is: 2

$$v = \sqrt{\frac{g}{k}\left(1 - e^{-2kx}\right)}$$

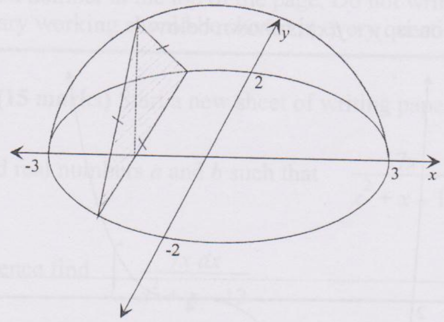
End of Question 12

Question 13 (15 marks) Start a new sheet of writing paper. **Marks**

(a) A solid has as its base, the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. **3**

Cross sections taken perpendicular to the x axis are equilateral triangles.

A typical cross section is shown on the diagram.



Find the volume of the solid.

(b) Find the square roots of $z = 1 + i\sqrt{3}$ in the form $\sqrt{z} = \pm(x + iy)$. **3**

(c) For any complex number, z show that $z.\overline{z} = |z|^2$. **2**

(d) (i) Write $z = \sqrt{3} + i$ in the form $r(\cos\theta + i\sin\theta)$. **1**

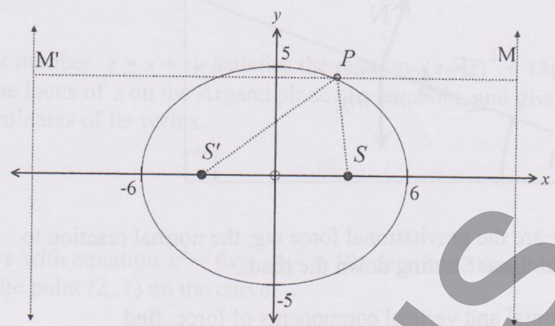
(ii) Hence, or otherwise, find z^5 in the form $x + iy$. **2**

Question 13 continues on the next page

Question 13 continued. **Marks**

(e) The ellipse $\frac{x^2}{36}+\frac{y^2}{25}=1$ is shown below along with its foci and directrices.

By definition, for any point P on the ellipse, $\frac{SP}{PM}=e$.



(i) Find the eccentricity (e) of the ellipse. **1**

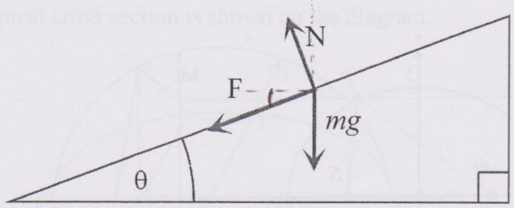
(ii) Give the equations of the directrices of the ellipse. **1**

(iii) Show that for any point P on the ellipse, $PS+PS'=C$, and give the value of the constant C. **2**

End of Question 13

Question 14 (15 marks) Start a new sheet of writing paper. **Marks**

(a) In a V8 supercar race, there is a circular bend or radius r, where the road is banked at an angle of θ to reduce the tendency to slide sideways when rounding the bend. Consider one of the V8 cars to be a point of mass m. Itistravelling at a constant speed v around the bend. The width of the road on the bend is d.



The forces acting on the car are the gravitational force mg, the normal reaction to the road N, and the frictional force F acting down the road.

(i) By resolving the horizontal and vertical components of force, find expressions for $N \sin\theta$ and $N \cos\theta$. **3**

(ii) Give an expression for the constant speed which will produce no friction against the road. **2**

(iii) Show that when the car is travelling at a constant speed which does produce friction against the road that the friction is given by the expression; **3**

$$F = \frac{mv^2}{r}\cos\theta - mg\sin\theta$$

(b) Find $\int e^x \sin x \, dx$ by the method of integration by parts. **3**

(c) (i) Show that the points P $\left(cp, \frac{c}{p}\right)$ and Q $\left(cq, \frac{c}{q}\right)$ lie on the hyperbola $xy = c^2$. **1**

(ii) Show that the equation of the chord PQ is $x + pqy = c(p + q)$. **2**

(iii) By considering the equation of the chord (and corresponding secant) as Q approaches P, determine the equation of the tangent at P. **1**

End of Question 14

Question 15 (12 marks) Start a new sheet of writing paper.

(a) Use the method of cylindrical shells to find the volume formed when the area bounded by $y = x^2 - 4x^4$ and the x axis is rotated about the y axis.

(b) Find the fourth roots of $2 + 2\sqrt{3}\, i$.

(c) The complex number $z = x + iy$ satisfies the relation $(z - \overline{z})^2 + 18(z + \overline{z}) = 36$. Show that the locus of z on the Argand plane is a parabola, and give its focal lengt and the coordinates of its vertex.

(d) For the curve with equation $x^2 + 6xy - 4y^2 = 10$, determine the gradient of the tangent at the point (2, 1) on the curve.