YEAR 11

PRELIMINARY EXAM 2005

EXTENSION 1 MATHEMATICS

Time allowed : 2 hours
(plus 5 minutes reading time)

Directions to candidates

- Attempt all questions
- All questions are of equal value
- All necessary working should be shown in every question
- Board approved calculators may be used
- Start a new page for each question

Total marks

- 84 marks

Question 1 (12 marks) **Marks**

(a) Find the acute angle between the lines $y = 5x - 1$ and $3y - 6x - 1 = 0$.
Give your answer to the nearest minute. **3**

(b) Find the coordinates of the point P that divides the interval joining the points $A(-5,6)$ and $B(1,0)$ externally in the ratio 3:1. **3**

(c) Find the equation of a line through the point of intersection of the lines $5x - y - 3 = 0$ and $2x - y = 0$ and perpendicular to the line $y = \frac{1}{2}x + 4$. **3**

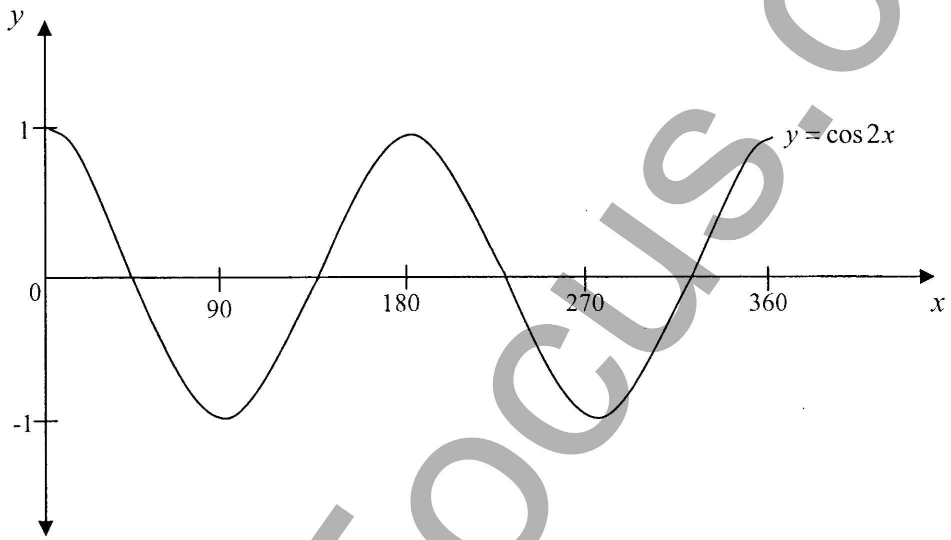
(d) Find the shortest distance between the parallel lines $3x - 2y + 1 = 0$ and $3x - 2y + 3 = 0$. Express your answer in surd form. **3**

Question 2 (12 marks) Start a new page. **Marks**

(a) Show that the exact value of $\sin 75^\circ$ is $\dfrac{\sqrt{6}+\sqrt{2}}{4}$. **2**

(b) Express $\sec x + \tan x$ in simplest form, in terms of t (where $t = \tan\dfrac{\theta}{2}$). **2**

(c) The graph shows the curve $y = \cos 2x$, for $0^\circ \le x \le 360^\circ$.



(i) Copy the diagram onto your page and sketch the curve $y = \sin x$ on the same set of axes. State the number of solutions of the equation $\cos 2x = \sin x$ for $0^\circ \le x \le 360^\circ$. **2**

(ii) Hence, or otherwise, find the solutions of the equation $\cos 2x = \sin x$ for $0^\circ \le x \le 360^\circ$. **3**

(d) Prove that $\dfrac{\sin 2\theta}{\sqrt{4-4\sin^2\theta}} = \sin\theta$ **3**

Question 3 (12 marks) Start a new page. **Marks**

(a) Find the values of p, q and r so that $2x^2 - 5x + 7 \equiv p(x-1)^2 + q(x-1) + r$. **3**

(b) If α and β are the roots of the equation $3x^2 - 15x + 7 = 0$, find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **1**

(c) The equation $x^2 - (1-2k)x + k + 3 = 0$ has consecutive roots. Find the value(s) of k. **3**

(d) Solve for x

$$\frac{2x+3}{x-2} \leq 1.$$ **3**

Question 4 (12 marks) Start a new page. **Marks**

(a) If $f(x) = \sqrt{x^2 + 4}$

(i) Find the domain of $f(x)$. **1**

(ii) Find the range of $f(x)$. **1**

(b) Consider the function $f(x) = \dfrac{x}{x^2 - 9}$.

(i) Determine whether the function is odd, even or neither. **1**

(ii) Find the coordinates of any intercepts. **1**

(iii) Find any vertical asymptotes. **1**

(iv) Calculate $\lim_{x \to \infty} \dfrac{x}{x^2 - 9}$. **2**

(v) Draw a neat sketch of the function, showing all essential features clearly. **2**

(c) (i) On the same axes, sketch the curves $y = x^2$ and $y = |x|$. **1**

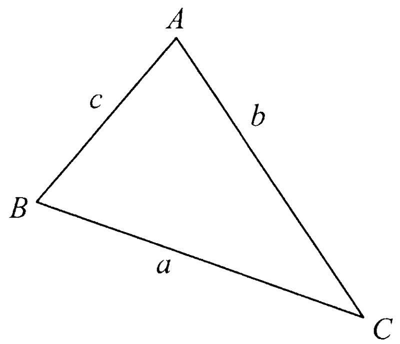
(ii) Hence, or otherwise, solve $x^2 < |x|$. **2**

Question 5 (12 marks) Start a new page. **Marks**

(a) (i) Express $3\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$, where R and α are constants. **1**

(ii) Hence find, correct to the nearest degree, the two angles between 0° and 360° that satisfy the equation $3\cos x - \sqrt{3}\sin x = -\sqrt{3}$. **2**

(b)

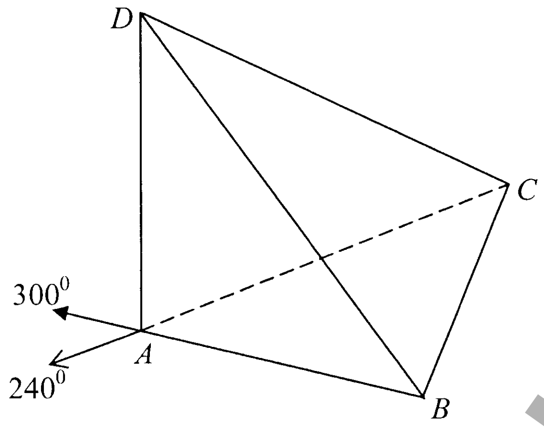


In triangle ABC, it is given that $3a=4b$.

(i) Use the sine rule to show that $\dfrac{\sin A}{\sin B} = \dfrac{4}{3}$. **2**

(ii) If angle A is double the size of angle B, find the value of $\cos B$. **2**

(c) The diagram below shows Barry standing at B on level ground, whilst Carmen is standing 2000 m away at C on the same level ground . They both take the bearing and elevation of a plane D at the same instant. Barry finds the bearing is 300^0 T and the angle of elevation 25^0, whilst Carmen finds the bearing to be 240^0 T and the angle of elevation 17^0.



(i) Copy the diagram onto your paper, showing all the information given. **1**

(ii) Find the size of $\angle BAC$. **1**

(iii) Show that if the height DA of the plane is h metres, then **3**

$$h = \frac{2000}{\sqrt{\left(\cot^2 25^0 + \cot^2 17^0 - 2\cot 25^0 \cot 17^0 \cos 60^0\right)}}$$

Question 6 (12 marks) Start a new page. **Marks**

(a) The graphs of $y = x$ and $y = x^3$ intersect at $x = -1$. Find the size of the acute angle between these curves at $x = -1$. **3**

(b) Find the equation of the tangent to the curve $y = 2x(x+1)$ at the point $(3, 24)$. **3**

(c) If $f(x) = 4x(3x^2 + 7)^5$, show that $f'(x) = 4(3x^2 + 7)^4(33x^2 + 7)$ **3**

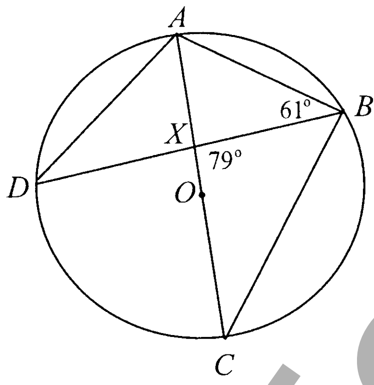
(d) Find the value of p so that the gradient of the normal to the curve $12y = x^2 - px + 4$, at $x = 1$, is 2. **3**

Question 7 (12 marks) Start a new page. **Marks**

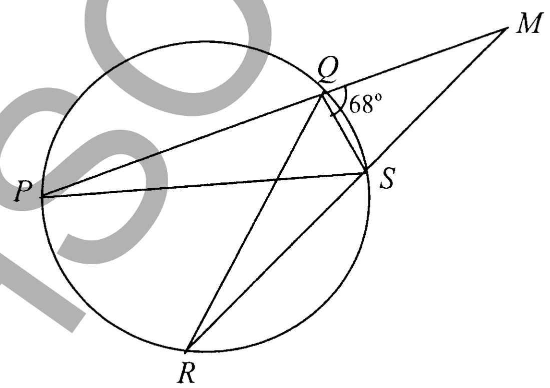
(a) Solve for x : $3^{2x} + 26(3)^{x-1} = 3$ **2**

(b) If the line $cx + dy + e = 0$ touches the parabola $x^2 = 4ay$, show that $ac^2 = ed$. **3**

(c) In the diagram, O is the centre of the circle and AC is a diameter. $\angle ABX = 61^\circ$ and $\angle BXC = 79^\circ$. Copy or trace this diagram and find the value of $\angle ADX$, giving reasons for your answer. **3**



(d) In the diagram, $MQ = MS$ and $\angle MQS = 68^\circ$. Copy or trace this diagram and prove that $MP = MR$. **4**



END OF TEST