Question 1 (12 marks) **Marks**

a) Solve $\dfrac{2}{x-3} < 5$ and graph your solution on the number line. **3**

b) For the function $y = 3\sin^{-1}\dfrac{x}{2}$

(i) State the domain and range **2**

(ii) Sketch the graph of this function **1**

c) Evaluate $\displaystyle\int_0^3 \frac{dx}{x^2+9}$ **2**

d) Find the coefficient of the term in x^2 in the expansion of $(2x+\dfrac{1}{x^2})^{11}$ **4**

$n_1 = 2$

$n_2 = 3$

Question 2 (12 marks) **Start a new page.**

a) Find the acute angle, to the nearest minute, between the lines $y = 2x+1$ and $3x - y + 4 = 0$. **3**

$\dfrac{1}{1+(6)}$

b) Prove that $\dfrac{\sin\theta + \sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$ **3**

c) For $k = 0,1,2,3,4,\ldots\ldots,n$ **2**

P_k is defined as $P_k = \dbinom{n}{k} a^k (1-a)^{n-k}$ where a is real and $n > 0$.

Prove that $\displaystyle\sum_{k=0}^{n} P_k = 1$.

d) A particle is moving in simple harmonic motion with acceleration $\dfrac{d^2x}{dt^2} = -4x$ m/s^2.

The particle starts at the origin with a velocity of 3 m/s.

(i) Find the period of the motion. **2**

(ii) Find the amplitude. **2**

Question 3 (12 marks) **Start a new page.** **Marks**

a) The polynomial $P(x) = 2x^3 + 3x^2 - 36x - 37$ has a zero at $x = -1$.

$6x^2 + 6x - 36$

(i) Find all the roots of $P(x) = 0$. **4**

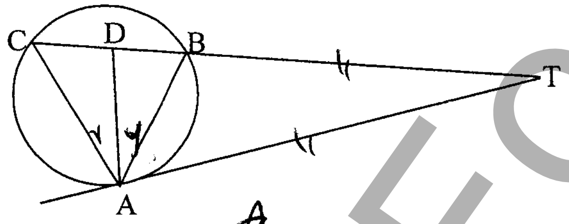
(ii) Apply Newton's Method once to find an approximate value, correct to 3 decimal places, for a root of $P(x)$, beginning with an initial approximation $x_1 = -5$. **4**

b) Solve the equation $2\ln(3x+1) - \ln(x+1) = \ln(7x+4)$. **4**

Question 4 (12 marks) **Start a new page.**

a) In what ratio does the point $P(3,5)$ internally divide the interval $A(-1,1)$ and $B(6,8)$? **2**

b) TA is a tangent. AD bisects $\angle BAC$. Prove that $TA = TD$ **4**



$x = y^2 - 6y = x - 11$

$y^2 - 6y + 3^2 = x - 11$

$(y-3)^2 = x - 2$

c) (i) Find the largest domain of positive real numbers for which the curve $y = x^2 - 6x + 11$ is a one-to-one function. **2**

$x > 3$

(ii) Hence, find its inverse function. **3**

(iii) What is the domain of the inverse function? **1**

Question 5 (12 marks) **Start a new page.** **Marks**

a) In how many different ways can the letters of the word COMMITTEE be arranged so that the vowels (a,e,i,o,u) are always together? **2**

b) Differentiate $\log_e\left(\sin^3 x\right)$ writing your answer in the simplest form. **2**

c) At any point on the curve $y = f(x)$, the gradient function is given by $\frac{dy}{dx} = \sin^2 x$. **4**

Find the value of $f(\frac{3\pi}{4}) - f(\frac{\pi}{4})$.

d) Use Mathematical Induction to show that $5^n + 2\left(11^n\right)$ is a multiple of 3 for all positive integers n. **4**

$u = x^2$

$du = 2x\,dx$

$x\,dx = du/2$

Question 6 (12 marks) **Start a new page.**

a) Use the substitution $u = x^2$ to find $\int_0^{\frac{1}{\sqrt{2}}} \frac{x}{\sqrt{1-x^4}}dx$.

$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$

$= \frac{1}{2}\sin^{-1} u + C$

b) A particle moves in a straight line. Initially it is 2m to the right of O. At time t seconds its displacement is x metres from a fixed point O on the line, its acceleration is a ms^{-2}, and its velocity is v m/s where v is given by $v = \frac{32}{x} - \frac{x}{2}$.

(i) Find an expression for a in terms of x. **3**

(ii) Show that $t = \int \frac{2x}{64-x^2}dx$, and hence show that $x^2 = 64 - 60e^{-t}$. **5**

$\frac{64-x^2}{2x}$

$\frac{4096 - 128x^2 + x^4}{8x^2}$

$4096x^{-2} - 128 + x^2$

Question 7 (12 marks) **Start a new page.** **Marks**

a) Prove that ${}^nC_0 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \ldots\ldots + \frac{1}{n+1}\,{}^nC_n = \frac{2^{n+1}-1}{n+1}$. **4**

b) A particle is projected from a point O with speed 50ms^{-1} at an angle of elevation θ and moves freely under gravity where $g = 10\,\text{ms}^{-2}$.

(i) Write down expressions for the horizontal and vertical displacements of the particle at time t seconds, referred to axes Ox and Oy. **2**

(ii) Hence show that the equation of the path of the projectile, given as a quadratic equation in $\tan\theta$ is **3**

$$x^2\tan^2\theta - 500x\tan\theta + \left(x^2 + 500y\right) = 0.$$

(iii) If the projectile passes through the point (X, X) and the roots of the equation of the path of the projectile are $\tan\alpha$ and $\tan\beta$, find expressions in terms of X for $\tan\alpha + \tan\beta$ and $\tan\alpha\tan\beta$ and hence show that $\alpha + \beta = \frac{3\pi}{4}$. **3**

END OF PAPER