

1

(a) $\frac{1}{x-2} \geq 2 \quad x \neq 2$
 $(x-2) \geq 2(x-2)^2$

$(x-2) - 2(x-2)^2 \geq 0$

$(x-2)(1-2(x-2)) \geq 0$

$(x-2)(5-2x) \geq 0$

$\therefore$ Solution $2 < x \leq 2\frac{1}{2}$

b) $\lim_{h \rightarrow 0} \left(\frac{\cos 3h - 1}{h} \right) \left(\frac{\cos 3h + 1}{\cos 3h + 1} \right)$

$= \lim_{h \rightarrow 0} \frac{\cos 3h - 1}{h [\cos 3h + 1]}$

$= \lim_{h \rightarrow 0} \frac{-\sin^2 3h}{h [\cos 3h + 1]}$

$= \lim_{h \rightarrow 0} \frac{\sin 3h}{3h} \cdot \frac{-3 \sin 3h}{\cos 3h + 1}$

$= 1 \cdot \frac{0}{2}$

$= 0$

or

(i) $A(-1, 5) \quad B(3, -2)$

$P \equiv \left[\frac{3r-1}{r+1}, \frac{-2r+5}{r+1} \right]$

ii) $2 \left[\frac{3r-1}{r+1} \right] - 3 \left[\frac{-2r+5}{r+1} \right] + 4 = 0$

$6r-2 + 6r-15 + 4r+4 = 0$

$16r = 13$

$r = \frac{13}{16}$

(d) $\int_0^1 (x^2+1)^3 dx$
 $= \int_0^1 (x^6 + 3x^4 + 3x^2 + 1) dx$
 $= \left[\frac{x^7}{7} + \frac{3x^5}{5} + x^3 + x \right]_0^1$
 $= 2 \frac{26}{35}$

2 $\frac{dT}{dt} = -k(T-T_0)$

$T = T_0 + A e^{-kt}$

$T_0 = 22^\circ$ And $t=0 \quad T=55$

$\therefore 55 = 22 + A e^0$
 $A = 33 \Rightarrow T = 22 + 33 e^{-kt}$

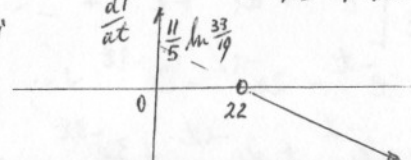
And $41 = 22 + 33 e^{-10k}$
 $19 = 33 e^{-10k}$
 $k = \frac{1}{10} \ln \frac{33}{19}$

$\therefore T = 22 + 33 e^{-\frac{t}{10} \ln \frac{33}{19}}$

(i) $t = 25$
 $T = 22 + 33 e^{-\frac{25}{10} \ln \frac{33}{19}}$
 $= 30.3^\circ C$

(ii) $25 = 22 + 33 e^{-\frac{t}{10} \ln \frac{33}{19}}$
 $\frac{3}{33} = e^{-\frac{t}{10} \ln \frac{33}{19}}$
 $-\frac{t}{10} \ln \frac{33}{19} = -\ln 11$
 $t = \frac{10 \ln 11}{\ln \frac{33}{19}}$
 $= 43.4 \text{ mins}$

iii) $\frac{dT}{dt}$ graph



2(b) $x = 5 \sin 3t - 7 \cos 3t$
 $\dot{x} = 15 \cos 3t + 21 \sin 3t$
 $\ddot{x} = -45 \sin 3t + 63 \cos 3t$
 $= -9 [5 \sin 3t - 7 \cos 3t]$

$\ddot{x} = -9x$

which is of the form $\ddot{x} = -n^2(x-b)$
 $n=3 \quad b=0$

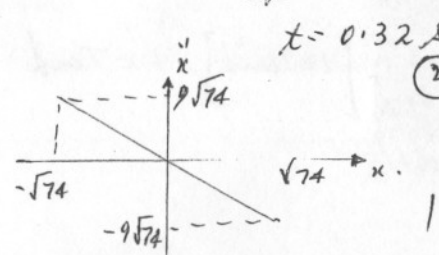
$\therefore$ motion SHM.

ii) Max displacement $= \sqrt{5^2 + 7^2}$
 $= \sqrt{25 + 49}$
 $= \sqrt{74} \text{ units.}$

Max velocity $= \sqrt{15^2 + 21^2}$
 $= 3\sqrt{74} \text{ m/s}$

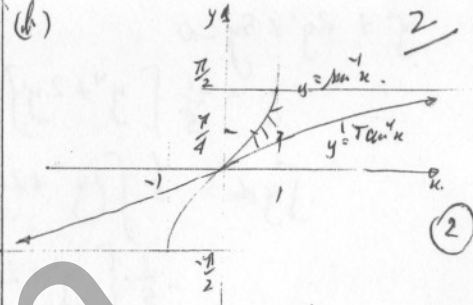
iii) $x=0 \quad 5 \sin 3t - 7 \cos 3t = 0$
 $\tan 3t = \frac{7}{5}$

$3t = \tan^{-1} \frac{7}{5}$
 $t = 0.32 \text{ s}$



iv)

3 (a) $\frac{d}{dx} \cos^{-1} \left(\frac{1}{x} \right) = \frac{1}{x^2} \cdot \frac{-1}{\sqrt{1 - \frac{1}{x^2}}}$
 $= \frac{-\sqrt{x^2}}{x^2 \sqrt{x^2 - 1}}$
 $= \frac{-|x|}{x^2 \sqrt{x^2 - 1}}$
 $= \frac{-1}{|x| \sqrt{x^2 - 1}}$



(b) $\int_0^{\pi/4} \tan^{-1} x dx = \frac{\pi}{4} - \int_0^{\pi/4} \tan y dy$
 $= \frac{\pi}{4} - \left[\frac{\sin y}{\cos y} \right]_0^{\pi/4}$
 $= \frac{\pi}{4} + \left[\ln \cos y \right]_0^{\pi/4}$
 $= \frac{\pi}{4} + \ln \left(\frac{\sqrt{2}}{2} \right)$
 $= \frac{\pi}{4} - \frac{1}{2} \ln 2$

ii) Region $\int_0^{\pi/4} \sin^{-1} x dx$
 $= \frac{\pi}{2} - 1 - \left[\frac{\pi}{4} - \frac{1}{2} \ln 2 \right]$
 $= \frac{\pi}{4} - 1 + \frac{1}{2} \ln 2$

(c) (i) $y = e^{-x} \sin 2x$
 $\frac{dy}{dx} = -e^{-x} \sin 2x + 2e^{-x} \cos 2x$
 $= e^{-x} [2 \cos 2x - \sin 2x]$
 $\frac{d^2 y}{dx^2} = -e^{-x} [2 \cos 2x - \sin 2x] + e^{-x} [-4 \sin 2x - 2 \cos 2x]$
 $= e^{-x} [-3 \sin 2x - 4 \cos 2x]$
 $\therefore y'' + 2y' + 5y = e^{-x} [-3 \sin 2x - 4 \cos 2x + 4 \cos 2x - 2 \sin 2x + 5 \sin 2x]$
 $= 0$

$$y'' + 2y' + 5y = 0$$

$$y = -\frac{1}{5} [y'' + 2y']$$

$$\int y dx = -\frac{1}{5} \int (y'' + 2y') dx$$

$$= -\frac{1}{5} [y' + 2y] + C$$

$$\int e^{-k} \sin 2k dx = -\frac{1}{5} \left[e^{-k} (2 \cos 2k - \sin 2k) + 2e^{-k} \sin 2k \right] + C$$

$$= -\frac{e^{-k}}{5} [2 \cos 2k + \sin 2k] + C \quad (1)$$

$$4(a)(i) \quad x = 30t \cos \theta$$

$$y = -5t^2 + 30t \sin \theta$$

$$t = \frac{x}{30 \cos \theta}$$

$$y = -5 \left(\frac{x}{30 \cos \theta} \right)^2 + 30 \sin \theta \frac{x}{30 \cos \theta}$$

$$y = \frac{-x^2}{180} \sec^2 \theta + x \tan \theta$$

$$\text{or } y = \frac{-x^2}{180} [1 + \tan^2 \theta] + x \tan \theta \quad (1)$$

$$i) \quad \theta = 45^\circ$$

$$y = 8$$

$$x = d + 15$$

$$\therefore 8 = \frac{-x^2}{180} (1 + 1) + x$$

$$x^2 - 90x + 720 = 0$$

$$x = \frac{90 \pm \sqrt{90^2 - 4 \times 720}}{2}$$

$$d + 15 = 81.12 \quad \text{or} \quad 17.75$$

$$d = 66.12 \text{ m} \quad \text{Shortest distance.}$$

(2)

3.

$$A(20, 8)$$

$$8 = -\frac{400}{180} [1 + \tan^2 \theta] + 20 \tan \theta$$

$$72 = -20 [1 + \tan^2 \theta] + 180 \tan \theta$$

$$20 \tan^2 \theta - 180 \tan \theta + 92 = 0$$

$$5 \tan^2 \theta - 45 \tan \theta + 23 = 0$$

$$\tan \theta = \frac{45 \pm \sqrt{45^2 - 4 \times 5 \times 23}}{2 \times 5}$$

$$= 0.544 \quad \text{or} \quad 8.46$$

$$\text{Angle elevation } \theta = 28^\circ 33' \quad \text{as } 0 \leq \theta \leq 45^\circ. \quad (2)$$

$$b) \quad \int \frac{4x-7}{2x^2+1} dx = \int \left(\frac{4x}{2x^2+1} - \frac{7}{2x^2+1} \right) dx$$

$$= \int \left(\frac{4x}{2x^2+1} - \frac{7}{2} \cdot \frac{1}{x^2 + \frac{1}{2}} \right) dx$$

$$= \ln(2x^2+1) - \frac{7}{2} \cdot \frac{1}{\sqrt{\frac{1}{2}}} \tan^{-1} \sqrt{\frac{1}{2}} + C$$

$$= \ln(2x^2+1) - \frac{7}{\sqrt{2}} \tan^{-1}(x\sqrt{2}) + C.$$

(2)

$$c) (i) \quad e^{-t} + e^{-2t} + e^{-3t} + \dots = \frac{e^{-t}}{1 - e^{-t}}$$

$$\text{as } |e^{-t}| < 1 \text{ for } t > 0.$$

$$= \frac{1}{e^t - 1}$$

(1)

ii)

$$\text{Now } \frac{d}{dt} [e^{-t} + e^{-2t} + e^{-3t} + \dots] = \frac{d}{dt} (e^t - 1)^{-1}$$

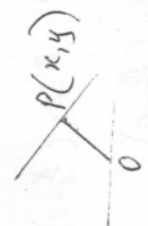
$$= -e^{-t} - 2e^{-2t} - 3e^{-3t} + \dots = - (e^t - 1)^{-2} \cdot e^t$$

$$\therefore e^{-t} + 2e^{-2t} + 3e^{-3t} + \dots = \frac{e^t}{(e^t - 1)^2}$$

(1)

4.

5.



(d) $y = \sqrt{r^2 - x^2}$

Gradient $m = \frac{dy}{dx} = \frac{-x}{\sqrt{r^2 - x^2}}$

Tangent

Gradient of $y = \frac{y - 0}{x - 0}$

$= \frac{y}{x} = \frac{\sqrt{r^2 - x^2}}{x}$

$m_1 = \frac{-x}{\sqrt{r^2 - x^2}}$

$\therefore m_1 \times m_2 = \frac{-x}{\sqrt{r^2 - x^2}} \cdot \frac{\sqrt{r^2 - x^2}}{x} = -1$

$\therefore$ Tangent $\perp$ radius.

5. $(a+b)^n = \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r$

$T_{n+1} = \binom{n}{r} (4x)^{n-r} 5^r$

$T_n = \binom{n}{r-1} (4x)^{n-r+1} 5^{r-1}$

$\frac{T_{n+1}}{T_n} = \frac{11!}{r!(11-r)!} \frac{(r-1)!(12-r)!}{11!} = \frac{12-r}{r} \cdot \frac{5}{4} \cdot \frac{1}{x}$

For largest coefficient $S(12-r) \geq 4r$

$60 - 5r \geq 4r$

$9r \leq 60$

$r \leq 7$

$\therefore$ Largest coefficient $\binom{11}{6} 4^5 5^6$

$T_8 < T_7$

(b) (i) NO motion does not oscillate.

(ii) $\frac{d}{dx} \left(\frac{v^2}{2} \right) = \frac{d}{dx} \left(\frac{v^2}{2} \right) \cdot \frac{dv}{dx}$

$= \frac{2v}{2} \cdot \frac{dv}{dx}$

$= v \frac{dv}{dx}$

$= \frac{d}{dt} \cdot \frac{dv}{dx}$

$= \frac{dv}{dt}$

$= \ddot{x}$

(iii) $\ddot{x} = -625x$

$\frac{d}{dx} \left(\frac{v^2}{2} \right) = -625x$

$\frac{v^2}{2} = -625 \frac{x^2}{2} + C$

$0 = -625 \frac{x^2}{2} + C$

$C = +625 \frac{x^2}{2}$

$\therefore \frac{v^2}{2} = 625 \frac{x^2}{2} - 625 \frac{x^2}{2}$

$v^2 = 625(1 - x^2)$

$v = 25 \sqrt{1 - x^2} \quad v > 0$

(iv) At Surface $x = 0 \quad \therefore v = 25 \text{ m/s}$

(v) $\frac{d^2 x}{dt^2} = -g$

$\frac{d}{dx} \left(\frac{v^2}{2} \right) = -10$

7. (ii) i) Let $\angle AOB = 2$ arc $\angle DOC = \theta$
 Arc length = $r \cdot 2 + r \cdot \theta$
 $= r(2 + \theta)$

But $\angle ADE = \frac{1}{2}$ (Angle at centre is twice angle at circumference standing on the same arc)
 Similarly $\angle DEC = \frac{\theta}{2}$

But $\angle DEC = \angle CAD + \angle ADE$
 $\theta = \frac{1}{2} + \frac{\theta}{2}$

OR $2\theta = 2\theta$
 $\therefore$ Arc length $AB + CD = r \times 2\theta = 2r\theta$

(ii) $\frac{BC}{OB} = \sin \frac{1}{2} \angle BOC$ [An interval drawn perpendicular to chord bisects the chord]

$BC = 2r \times \sin \frac{\theta}{3}$
 $= 2r \cdot \frac{\sqrt{3}}{2}$
 $= \sqrt{3}r$

Now $\angle AOD + \angle AOB + \angle DOC = \angle BOC$
 $\angle AOD + 2 + \theta = 2\frac{\theta}{3}$

$\angle AOD = 2\frac{\theta}{3} - 2\theta$

$\therefore \frac{AD}{AO} = \sin \frac{1}{2} \angle AOD$
 $AD = 2r \sin \left(\frac{\theta}{3} - \theta \right)$

$\therefore$ Perimeter $ABCD = 2r\theta + \sqrt{3}r + 2r \sin \left(\frac{\theta}{3} - \theta \right)$
 $= r \left[2\theta + \sqrt{3} + 2 \sin \left(\frac{\theta}{3} - \theta \right) \right]$

$\frac{dP}{d\theta} = r \left[2 - 2 \cos \left(\frac{\theta}{3} - \theta \right) \right]$

$\frac{d^2P}{d\theta^2} = 2r \sin \left(\frac{\theta}{3} - \theta \right)$

For maximum perimeter $\frac{dP}{d\theta} = 0$

$r \left[2 - 2 \cos \left(\frac{\theta}{3} - \theta \right) \right] = 0$

$\theta \neq 0 \quad \cos \left(\frac{\theta}{3} - \theta \right) = 1$

$\frac{\theta}{3} - \theta = 0 \quad \text{for } 0 < \theta < \frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{3}$

For nature test $\frac{d^2P}{d\theta^2}$ for concavity

at $\theta = \frac{\pi}{3} \quad \frac{d^2P}{d\theta^2} = r \times 0 = 0$

Test gradients:

θ	1	$\frac{\pi}{3}$	1.1
$\frac{dP}{d\theta}$	$2r \times 0$	0	$2r \times 0$
	-	-	+

gradients same sign

$\therefore$ Inflection point at $\theta = \frac{\pi}{3}$ and monotonic increasing, continuous

for $0 < \theta < \frac{\pi}{2}$

$\therefore$ Maximum perimeter at end points of domain
 at $\theta = \frac{\pi}{2}$