



2013
TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION
GIRRAWEE HIGH SCHOOL
MATHEMATICS EXTENSION 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board – approved calculators may be used
- A table of standard integrals is provided
- Show all necessary working in

Questions 11-14

Total marks - 70

Section 1 pages 2-3

10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section 2 pages 4 - 8

- Attempt Questions 11 - 14
- Allow about 1 hour 45 minutes for this section

SECTION 1

Multiple Choice (10 marks) Circle the correct answer on this question paper.

1. Which expression is a correct factorisation of $y^3 - 125$?

(A) $(y-5)(y^2 + 5y + 25)$ (C) $(y-5)(y^2 - 5y + 25)$

(B) $(y+5)(y^2 + 5y + 25)$ (D) $(y+5)(y^2 - 5y + 25)$

2. The point P divides the interval from $A(-5,6)$ to $B(-2,3)$ externally in the ratio $3 : 2$.

What is the y -coordinate of P ?

(A) -4 (B) -3 (C) 3 (D) 4

3. A polynomial equation has roots α, β and γ where $\alpha + \beta + \gamma = -2$, $\alpha\beta + \alpha\gamma + \beta\gamma = 3$ and $\alpha\beta\gamma = -4$. Which polynomial equation has the roots α, β and γ ?

(A) $x^3 + 2x^2 + 3x - 4 = 0$ (C) $x^3 - 2x^2 + 3x - 4 = 0$

(B) $x^3 - 2x^2 + 3x + 4 = 0$ (D) $x^3 + 2x^2 + 3x + 4 = 0$

4. $\frac{\sin x}{1 - \cos x}$ when expressed in terms of half-angle is equal to

(A) $\tan \frac{x}{2}$ (B) $-\tan \frac{x}{2}$ (C) $\cot \frac{x}{2}$ (D) $-\cot \frac{x}{2}$

5. Find the equation of the normal to the parabola $x = 6t, y = 3t^2$ at the point where $t = -2$.

(A) $x - 3y + 24 = 0$ (C) $2x + y + 12 = 0$

(B) $x - 2y + 36 = 0$ (D) $2x - y - 12 = 0$

6. In the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$, the constant term is

- (A) $^{-15}C_8$ (B) $^{15}C_8$ (C) $^{-16}C_8$ (D) $^{16}C_8$

7. The solutions of the equation $e^{2x} - 3e^x + 2 = 0$ are

- (A) 0,1 (B) 1,2 (C) $0, \log_e 2$ (D) $1, \log_e 2$

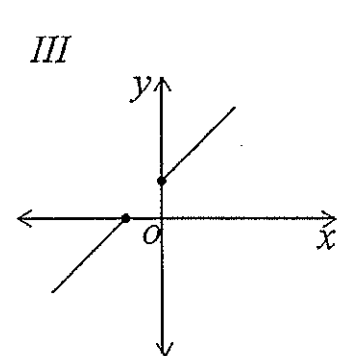
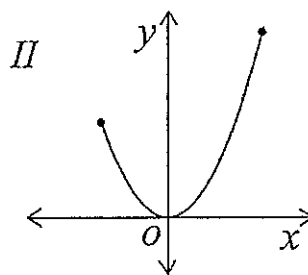
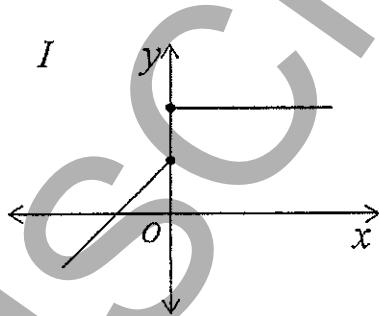
8. The value of k if $\int_0^1 \frac{dx}{x^2 + 3} = k\pi$ is

- (A) $6\sqrt{3}$ (B) $\frac{1}{6\sqrt{3}}$ (C) 6 (D) $\frac{1}{6}$

9. The velocity vm/s of a particle moving in simple harmonic motion along the x -axis

is given by $v^2 = -5 + 6x - x^2$, where x is in metres. The amplitude of the oscillation is (A) $2m$ (B) $3m$ (C) $4m$ (D) $5m$

10. Which of the following graphs represent function(s) whose inverse(s) are also functions?



- (A) I and II (C) III only
(B) I and III (D) II only

Question 11 (15 marks)**Marks**

- (a) Evaluate: $\int_0^1 \frac{1}{\sqrt{2-x^2}} dx$ 2
- (b) Evaluate $\int_{-1}^0 x\sqrt{1+x} dx$ using the substitution $u = 1+x$. 3
- (c) Solve $\frac{x}{x+4} > 2$ 3
- (d) The line $y = mx$ makes an angle of 45° with the line $y = 3x$. Find the values of m . 3
- (e) In how many ways can a jury of 7 people reach a majority decision? 1
- (f) Find the value of k if the roots of the equation $x^3 - 3x^2 - 6x + k = 0$ are in arithmetic progression. 3

Question 12 (15 marks)

(a) The temperature gauge of a car is giving an overheating warning and the car is stopped. At time t minutes, the temperature T of the liquid in the radiator decreases according to the equation $\frac{dT}{dt} = -k(T - 35)$, where k is a positive constant. The initial temperature of the liquid in the radiator is 95°C and it cools to 70°C after 10 minutes.

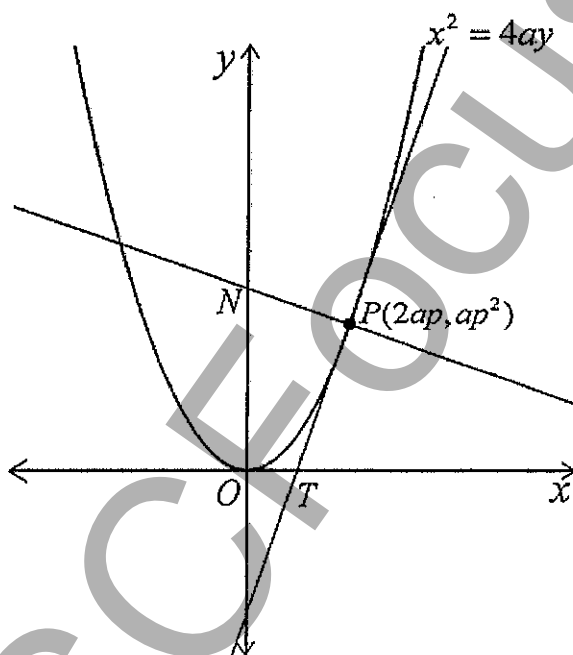
- (i) Show that $T = 35 + Ae^{-kt}$ is a solution of $\frac{dT}{dt} = -k(T - 35)$ 2
- (ii) Find the value of A . 1
- (iii) Find the value of k , correct to 3 decimal places. 1
- (iv) How long will it take for the temperature of the radiator to cool to 40°C ?
Give your answer correct to the nearest minute. 1

(b) (i) Use mathematical induction to prove that

$$\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right) \dots \left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n} \text{ for all integers } n \geq 2. \quad 3$$

(ii) Hence find the exact value of $\left(1 - \frac{1}{49}\right)\left(1 - \frac{1}{64}\right)\left(1 - \frac{1}{81}\right) \dots \left(1 - \frac{1}{144}\right).$ 2

(c) The diagram shows the graph of the parabola $x^2 = 4ay$. The tangent to the parabola at $P(2ap, ap^2)$ cuts the x -axis at T . The normal to the parabola at P cuts the y -axis at N .



(i) Find the coordinates of T , given that the equation of the tangent at P is $y = px - ap^2$ 1

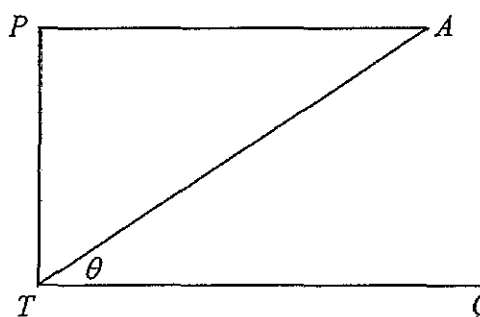
(ii) Show that the coordinates of N are $(0, a(p^2 + 2))$. 1

(iii) Let M be the midpoint of NT . Find the Cartesian equation of the locus of M and describe this locus. 3

Question 13 (15 marks)

(a) A person on horizontal ground is looking at an aeroplane A through a telescope T .

The aeroplane is approaching at a speed of 80 m/s at a constant altitude of 200 meters above the telescope. When the horizontal distance of the aeroplane from the telescope is x metres, the angle of elevation of the aeroplane is θ radians.



(i) Show that $\theta = \tan^{-1} \frac{200}{x}$ 1

(ii) Show that $\frac{d\theta}{dt} = \frac{16000}{x^2 + 40000}$ 2

(iii) Find the rate at which θ is changing when $\theta = \frac{\pi}{4}$, (answer in degrees) 3

(b) By integrating the expansion of $(1-x)^n$ show that

$$1 - \frac{{}^nC_1}{2} + \frac{{}^nC_2}{3} - \dots + (-1)^n \frac{{}^nC_n}{n+1} = \frac{1}{n+1}$$
3

(c) (i) Show that the function $f(x) = x^2 - \log_e(x+3)$ has a zero between 1.1 and 1.3. 2

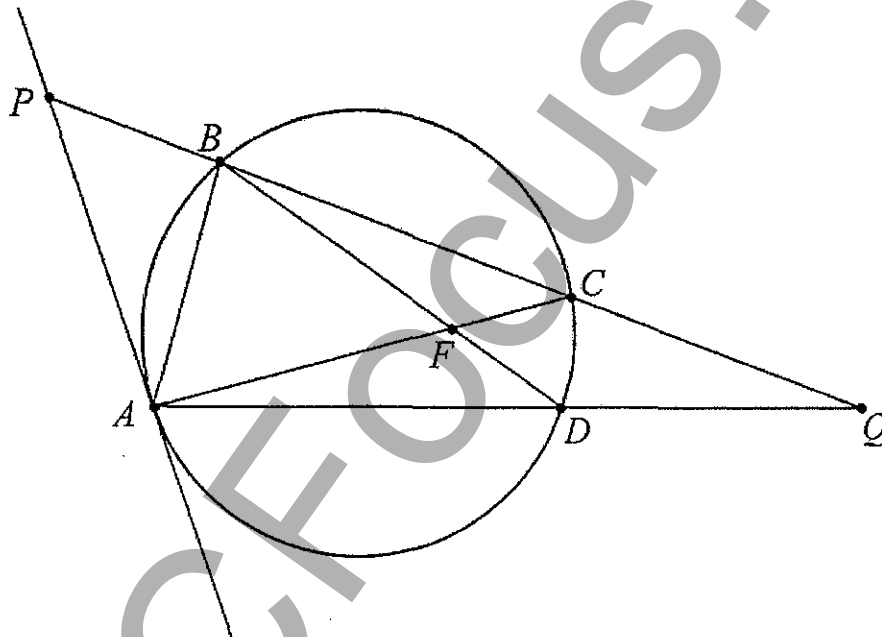
(ii) Use the method of halving the interval to find an approximation to this

zero of $f(x)$, correct to one decimal place. 2

- (d) The function $f(x) = \sin x - \frac{x}{3}$ has a zero near $x = 2.2$. Use one application of Newton's method to find a second approximation to the zero. Write your answer correct to 3 significant figures. 2

Question 14 (15 marks)

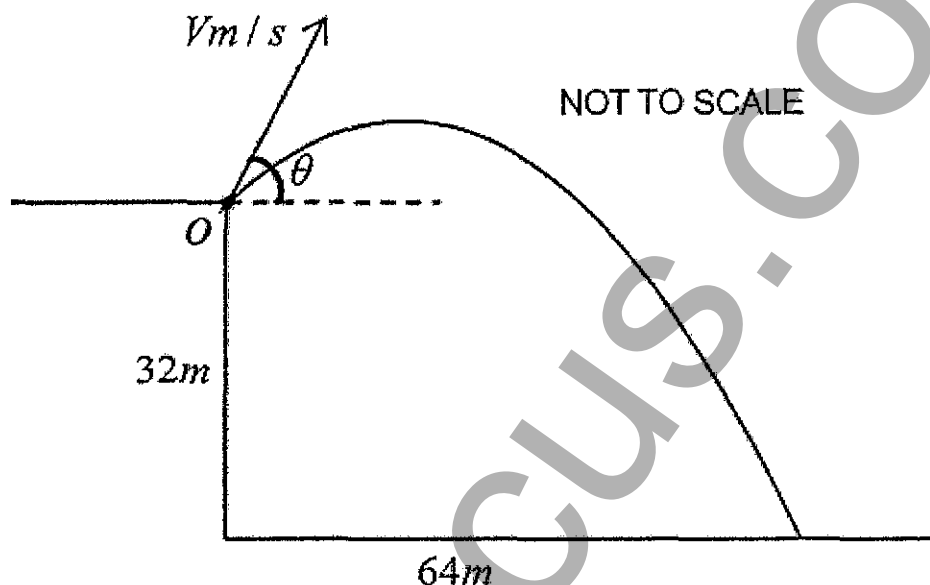
- (a) In the figure below, AP is a tangent to the circle at A . $PBCQ$ and ADQ are straight lines. Prove that $\angle PAB = \frac{1}{2}(\angle CFD + \angle CQD)$ 3



- (b) The motion of a particle is given by $x = 10 + 8\sin 2t + 6\cos 2t$. Prove that the motion is simple harmonic. Write down the centre and period of the motion. 2
- (c) The rise and fall of the tide at a certain port may be considered to be simple harmonic.

The interval between successive high tides is 12 hours. The port entrance has a depth of 12 m at high tide and 4m at low tide. If the low tide occurs at noon on a certain day, find the earliest time thereafter that a ship drawing 9m can pass through the entrance. 2

- (d) A particle is projected with velocity $V \text{ m/s}$ at an angle θ above the horizontal from a point O on the edge of a vertical cliff 32 metres above a horizontal beach. The particle moves in a vertical plane under gravity, and 4 seconds later it hits the beach at a point 64 metres from the foot of the cliff. (Take $g = 10 \text{ m/s}^2$)



- (i) Use integration to show that after t seconds the horizontal displacement x metres and the vertical displacement y metres of the particle from O are given by
- $$x = (V \cos \theta)t \text{ and } y = (V \sin \theta)t - 5t^2 \text{ respectively.} \quad 2$$
- (ii) Write down two equations in V and θ then solve these equations to find the exact value of V and the value of θ in degrees correct to the nearest minute. 3
- (iii) Find the speed of impact with the beach correct to the nearest whole number and the angle of impact with the beach correct to the nearest minute. 3

END OF TEST

Year 12 Extension 1 Trial HSC 2013

Section 1 (10 marks)

1. A 2. B 3. D 4. C 5. B 6. D 7. C 8. B

9. A 10. C

Question 11 (15 marks)

$$(a) \int_0^1 \frac{1}{\sqrt{2-x^2}} dx$$

$$= \left[\sin^{-1} \frac{x}{\sqrt{2}} \right]_0^1$$

$$= \sin^{-1} \frac{1}{\sqrt{2}} - \sin^{-1} 0 \quad (2)$$

$$= \frac{\pi}{4}$$

$$(b) \tan 45 = \left| \frac{m-3}{1+3m} \right|$$

$$\left| \frac{m-3}{1+3m} \right| = 1 \quad (3)$$

$$\frac{m-3}{1+3m} = 1 \quad \frac{-(m-3)}{1+3m} = 1$$

$$m-3 = 1+3m \quad -m+3 = 1+3m$$

$$2m = -4 \quad 4m = 2$$

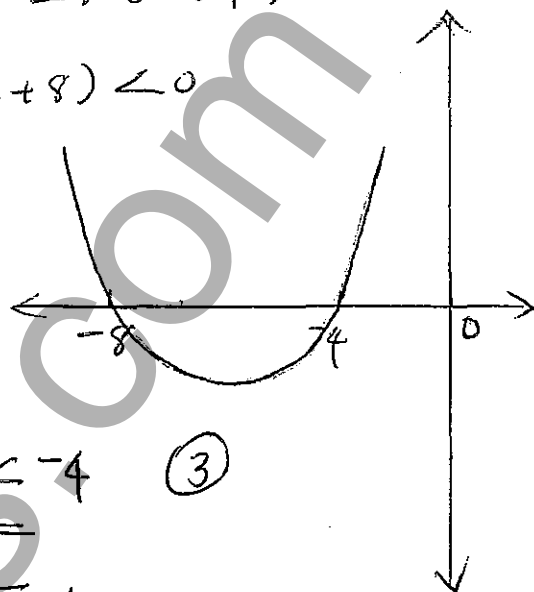
$$m = -2 \quad m = \frac{1}{2}$$

$$(c) \frac{x}{x+4} > 2$$

$$x(x+4) > 2(x+4)^2$$

$$2(x+4)^2 - x(x+4) < 0$$

$$(x+4)(x+8) < 0$$



$$\underline{-8 < x < -4} \quad (3)$$

$$(d) \int_{-1}^0 x \sqrt{1+x} dx$$

$$= \int_0^1 (u-1) \sqrt{u} du$$

$$= \int_0^1 (u^{\frac{3}{2}} - u^{\frac{1}{2}}) du$$

$$= \left[\frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}} \right]_0^1 \quad (3)$$

$$= \frac{2}{5} - \frac{2}{3} = \underline{\underline{-\frac{4}{15}}}$$

(e)

$${}^7C_7 + {}^7C_6 + {}^7C_5 + {}^7C_4 \quad (1)$$

$$= \underline{\underline{64}}$$

$$(f) x^3 - 3x^2 - 6x + k = 0$$

Let the roots be $\alpha-d, \alpha, \alpha+d$

Sum of roots

$$3\alpha = 3$$

$$\alpha = 1$$

Sum of roots taken two at a time

$$(\alpha-d)\alpha + (\alpha-d)(\alpha+d) + \alpha(\alpha+d)$$

$$3\alpha^2 - d^2 = -6$$

$$d^2 = 9 \quad (3)$$

Product of roots

$$\alpha(\alpha-d)(\alpha+d) = \alpha(\alpha^2 - d^2)$$

$$\alpha(\alpha^2 - d^2) = -k$$

$$1(1-9) = -k$$

$$\underline{k = 8}$$

Question 12 (15 marks)

$$(a) (i) T = 35 + Ae^{-kt}$$

$$\frac{dT}{dt} = Ae^{-kt} \times -k$$

$$= -kAe^{-kt}$$

$$= -k(T-35) \quad (2)$$

$$(\because T-35 = Ae^{-kt})$$

Hence $T = 35 + Ae^{-kt}$ is a solution of $\frac{dT}{dt} = -k(T-35)$

(ii) When $t=0, T=95$ page 2

$$95 = 35 + A \quad (1)$$

$$A = 60$$

(iii)

When $t=10, T=70$

$$70 = 35 + 60e^{-10k}$$

$$60e^{-10k} = 35$$

$$e^{-10k} = \frac{35}{60} = \frac{7}{12}$$

$$-10k = \ln\left(\frac{7}{12}\right) \quad (1)$$

$$k = \frac{-1}{10} \ln\left(\frac{7}{12}\right) = 0.054$$

(iv)

$T=40, t=?$

$$40 = 35 + 60e^{-kt}$$

$$60e^{-kt} = 5$$

$$e^{-kt} = \frac{5}{60} = \frac{1}{12}$$

$$-kt = \ln\left(\frac{1}{12}\right)$$

$$t = \frac{-1}{k} \ln\left(\frac{1}{12}\right) \quad (1)$$

$$= 46.02$$

The temperature will fall to 40°C after 46 minutes

(b) When $n=2$, LHS = $1 - \frac{1}{4} = \frac{3}{4}$

RHS = $\frac{2+1}{2 \times 2} = \frac{3}{4}$

LHS = RHS

$\therefore$ the result is true when $n=2$

Assume the result is true for $n=k$

$$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \left(1 - \frac{1}{4^2}\right) \dots \left(1 - \frac{1}{k^2}\right) = \frac{k+1}{2k} \quad \text{--- (1)}$$

Aim: To prove that it is true for $n=k+1$

$$\text{i.e. } \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \dots \left(1 - \frac{1}{k^2}\right) \left(1 - \frac{1}{(k+1)^2}\right) = \frac{k+2}{2(k+1)} \quad \text{--- (2)}$$

Proof:

$$\begin{aligned} &\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \dots \left(1 - \frac{1}{k^2}\right) \left(1 - \frac{1}{(k+1)^2}\right) \\ &= \frac{k+1}{2k} \left(1 - \frac{1}{(k+1)^2}\right) \quad (\text{by assumption (1)}) \end{aligned}$$

$$= \frac{(k+1)}{2k} \left(\frac{k^2 + 2k + 1 - 1}{(k+1)^2} \right)$$

$$= \frac{(k+1) k (k+2)}{2k (k+1)^2}$$

$$= \frac{k+2}{2(k+1)} = \text{RHS of (2)}$$

Hence by the principle of mathematical induction, the result is true for $n \geq 2$.

$$(ii) \left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \dots \left(1 - \frac{1}{6^2}\right) \left(1 - \frac{1}{7^2}\right) \dots \left(1 - \frac{1}{12^2}\right) = \frac{13}{24}$$

$$\left(1 - \frac{1}{2^2}\right) \left(1 - \frac{1}{3^2}\right) \dots \left(1 - \frac{1}{6^2}\right) = \frac{7}{12}$$

$$\left(1 - \frac{1}{4^2}\right) \left(1 - \frac{1}{6^2}\right) \dots \left(1 - \frac{1}{14^2}\right) = \frac{13}{24} \div \frac{7}{12} \quad (2)$$

$$= \frac{13}{14}$$

(iv) Equation of normal

$$y - ap^2 = -\frac{1}{p}(x - 2ap)$$

Substitute $x=0$

(1)

$$y - ap^2 = 2a$$

$$y = ap^2 + 2a = a(p^2 + 2)$$

$$\underline{N(0, a(p^2 + 2))}$$

$$(iii) M = \left(\frac{ap}{2}, \frac{a(p^2 + 2)}{2} \right)$$

$$x = \frac{ap}{2} \Rightarrow p = \frac{2x}{a}$$

$$y = \frac{a(p^2 + 2)}{2} \quad (2)$$

substitute $p = \frac{2x}{a}$ in

$$y = \frac{a \left(\frac{4x^2}{a^2} + 2 \right)}{2} = \frac{a \left(\frac{4x^2 + 2a^2}{a^2} \right)}{2}$$

$$y = \frac{4x^2 + 2a^2}{a} \times \frac{1}{2} = \frac{2x^2 + a^2}{a}$$

(c) (i)

$$y = px - ap^2$$

Substitute $y=0$

$$px - ap^2 = 0$$

$$px = ap^2$$

$$x = ap$$

(1)

$$\underline{T(ap, 0)}$$

$$\begin{aligned}
 2x^2 + a^2 &= ay \\
 2x^2 &= ay - a^2 \\
 &= a(y - a) \\
 x^2 &= \frac{a}{2}(y - a) \quad (3)
 \end{aligned}$$

This is a parabola with vertex $(0, a)$ and focal length $\frac{a}{8}$ units.

Question 13 (15 marks)

(a) (i)

$$\angle TAP = \theta \text{ (alternate } \angle s, FA \parallel TO)$$

$$\tan \theta = \frac{200}{x}$$

$$\theta = \tan^{-1} \frac{200}{x} \quad (1)$$

$$(ii) \frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt}$$

$$= \frac{1}{1 + \left(\frac{200}{x}\right)^2} \times \frac{-200}{x^2} \times -80$$

$$= \frac{x^2}{x^2 + 40000} \times \frac{16000}{x^2}$$

$$= \frac{16000}{x^2 + 40000} \quad (2)$$

(iii) when $\theta = \frac{\pi}{4}$,

$$TP = AP \quad \therefore x = 200 \quad (3)$$

$$\frac{d\theta}{dt} = \frac{16000}{(200)^2 + 40000} = 0.2 \text{ radians/s}$$

Hence θ is increasing at $11^\circ/s$

(b)

$$(1-x)^n = 1 - nC_1x + nC_2x^2 - nC_3x^3 + \dots + (-1)^n nC_n x^n$$

$$\int_0^1 (1-x)^n dx = \int_0^1 (1 - nC_1x + \dots + (-1)^n nC_n x^n) dx$$

$$= \left[x - nC_1 \frac{x^2}{2} + nC_2 \frac{x^3}{3} - \dots - (-1)^n nC_n \frac{x^{n+1}}{n+1} \right]_0^1$$

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$$(b) (1-x)^n = 1 - nC_1 x + nC_2 x^2 - nC_3 x^3 + \dots + (-1)^n nC_n x^n$$

$$\int_0^1 (1-x)^n dx = \int_0^1 (1 - nC_1 x + nC_2 x^2 - nC_3 x^3 + \dots + (-1)^n nC_n x^n) dx$$

$$\left[\frac{(1-x)^{n+1}}{n+1} \right]_0^1 = \left[x - nC_1 \frac{x^2}{2} + nC_2 \frac{x^3}{3} - nC_3 \frac{x^4}{4} + \dots + (-1)^n nC_n \frac{x^{n+1}}{n+1} \right]_0^1$$

$$\frac{-1}{n+1} \left[(1-x)^{n+1} \right]_0^1 = \left[x - nC_1 \frac{x^2}{2} + nC_2 \frac{x^3}{3} - nC_3 \frac{x^4}{4} + \dots + (-1)^n nC_n \frac{x^{n+1}}{n+1} \right]_0^1$$

$$\frac{-1}{n+1} (0-1) = 1 - \frac{nC_1}{2} + \frac{nC_2}{3} - \frac{nC_3}{4} + \dots + (-1)^n \frac{nC_n}{n+1} \quad (3)$$

$$1 - \frac{nC_1}{2} + \frac{nC_2}{3} - \frac{nC_3}{4} + \dots + (-1)^n \frac{nC_n}{n+1} = \frac{1}{n+1}$$

$$(c) (i) f(1.1) = 1.1^2 - \log_e(4.1) = -0.201$$

$$f(1.3) = 1.3^2 - \log_e(4.3) = 0.231 \quad (2)$$

Since $f(x)$ changes sign between $x=1.1$ and $x=1.3$, there is a zero of $f(x) = x^2 - \log_e(x+3)$ between 1.1 and 1.3.

$$(ii) \begin{array}{ccc} & + & \\ \hline 1.1 & 1.2 & 1.3 \end{array}$$

$$f(1.2) = 1.2^2 - \log_e(4.2) = 0.005 \quad (2)$$

$$\frac{1.1+1.2}{2} = 1.15$$

To one decimal place $x=1.2$ is the zero of $f(x)$.

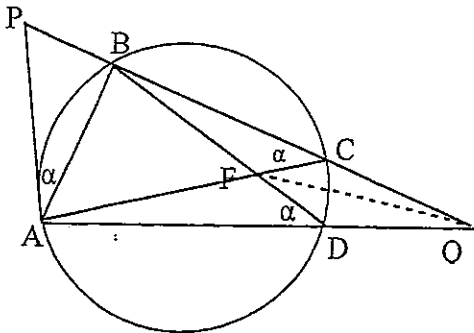
$$(d) f(x) = \sin x - \frac{x}{3}$$

$$f'(x) = \cos x - \frac{1}{3}$$

$$a_1 = 2.2 - \frac{f(2.2)}{f'(2.2)} \quad (2)$$

$$= 2.28$$

Question 14 (15 marks)



$\angle PAB = \angle ACB$ (angle between tangent and chord is equal to angle in the alternate segment)

$\angle ACB = \angle ADB$ (angles at the circumference standing on the same arc)

$\angle BCA = \angle CQF + \angle CFQ$
(exterior $\angle$ of $\triangle FQC$)

$\angle ADF = \angle DQF + \angle DFQ$
(exterior $\angle$ of $\triangle FQD$)

$$\begin{aligned} \angle BCA + \angle ADF &= \angle CQF + \angle CFQ + \angle DQF + \angle DFQ \\ &= \angle CQF + \angle DQF + \angle CFQ + \angle DFQ \end{aligned}$$

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$$= \angle CQF + \angle DQF + \angle CFQ + \angle DFQ$$

$$= \angle CQD + \angle CFD$$

$$2\alpha = \angle CQD + \angle CFD \quad (3)$$

$$\alpha = \frac{1}{2}(\angle CQD + \angle CFD)$$

$$(b) x = 10 + 8\sin 2t + 6\cos 2t$$

$$\dot{x} = 16\cos 2t - 12\sin 2t$$

$$\ddot{x} = -32\sin 2t - 24\cos 2t$$

$$= -4(8\sin 2t + 6\cos 2t)$$

$$= -4(x - 10) \quad (2)$$

Centre $\underline{x = 10}$, $n = 2$

Period $T = \frac{2\pi}{n} = \underline{\underline{\pi \text{ seconds}}}$

(c) Period = 12 hrs

12-00pm

4m

8m

12m

Amplitude = 4m

$$T = \frac{2\pi}{n} \quad n = \frac{2\pi}{T} = \frac{2\pi}{12} = \frac{\pi}{6}$$

Let 12-00pm denote $t = 0$

when $t = 0$, $x = -4$

$$-4 = 4\cos \alpha$$

$$\cos \alpha = -1 \quad \therefore \alpha = \pi$$

$$x = 4\cos\left(\frac{\pi}{6}t + \pi\right)$$

Let at time t_1 , the depth of water be q m. Then $x = 1$

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$$1 = 4 \cos\left(\frac{\pi}{6} t_1 + \pi\right)$$

$$\cos\left(\frac{\pi}{6} t_1 + \pi\right) = \frac{1}{4}$$

$$\frac{\pi}{6} t_1 + \pi = \cos^{-1}\left(\frac{1}{4}\right), 2\pi - \cos^{-1}\left(\frac{1}{4}\right)$$

$$\frac{\pi}{6} t_1 = \cos^{-1}\left(\frac{1}{4}\right) - \pi, \pi - \cos^{-1}\left(\frac{1}{4}\right)$$

$$t_1 = \frac{6}{\pi} \left(\cos^{-1}\left(\frac{1}{4}\right) - \pi\right), \frac{6}{\pi} \left(\pi - \cos^{-1}\left(\frac{1}{4}\right)\right)$$

$$= \frac{6}{\pi} \cos^{-1}\left(\frac{1}{4}\right) - 6, 6 - \frac{6}{\pi} \cos^{-1}\left(\frac{1}{4}\right) \quad (2)$$

$$= 3.4826 \text{ hr}$$

$$= 3 \text{ hr } 29 \text{ minutes}$$

$$t_1 = 12:00 \text{ pm} + 3 \text{ hr } 29 \text{ min}$$

$$= \underline{\underline{3:29 \text{ pm}}}$$

$$(d) (i) \ddot{x} = 0 \quad \text{--- (1)} \quad \ddot{y} = -g \quad \text{--- (2)}$$

$$\frac{dx}{dt} = A \quad \text{--- (3)} \quad \frac{dy}{dt} = -gt + B \quad \text{--- (4)}$$

where A and B are constants of integration

$$\text{when } t=0, \frac{dx}{dt} = v \cos \theta, \frac{dy}{dt} = v \sin \theta$$

$$v \cos \theta = A \quad v \sin \theta = B$$

$$\frac{dx}{dt} = v \cos \theta \quad \text{--- (5)} \quad \frac{dy}{dt} = -gt + v \sin \theta \quad \text{--- (6)}$$

$$x = (v \cos \theta) t + C \quad \text{--- (7)} \quad y = -\frac{g t^2}{2} + (v \sin \theta) t + D \quad \text{--- (8)}$$

where C and D are constants of integration.

when $t=0$, $x=0$ and $y=0$

$$0 = 0 + C \therefore C = 0$$

$$0 = 0 + 0 + D \therefore D = 0 \quad (2)$$

$$x = (V \cos \alpha) t \quad (9)$$

$$y = (V \sin \alpha) t - 5t^2 \quad (10)$$

(ii) when $t = 4$, $x = 64$ and $y = -32$

$$64 = (V \cos \alpha) 4$$

$$-32 = (V \sin \alpha) \times 4 - 5 \times 16$$

$$64 = 4V \cos \alpha$$

$$-32 = 4V \sin \alpha - 80$$

$$V \cos \alpha = 16 \quad (11)$$

$$V \sin \alpha = 12 \quad (12)$$

Squaring and adding (11) and (12) (3)

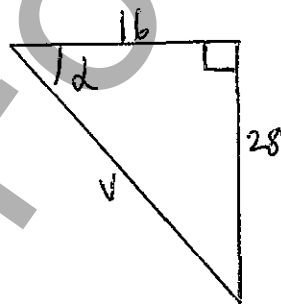
$$V^2 (\sin^2 \alpha + \cos^2 \alpha) = 144 + 256$$

$$= 400$$

$$V = 20 \text{ m/s}$$

$$\sin \alpha = \frac{12}{20} = \frac{3}{5}$$

$$\alpha = 36^\circ 52'$$



(iii) when $t = 4$,

$$x = V \cos \alpha = 16$$

$$y = V \sin \alpha - gt$$

$$= 12 - 40$$

$$= -28$$

$$V^2 = 16^2 + 28^2$$

$$= 1040$$

(3)

$$V = 32.25 \text{ m/s}$$

$$\tan \alpha = \frac{28}{16} = \frac{7}{4}$$

$$\alpha = 60^\circ 15'$$

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