2012
TRIAL HSC
EXAMINATION

Mathematics
Extension 1

SOLUTIONS

Trial HSC Examination - Mathematics Extension 1
Multiple Choice Answer Sheet

Name _____

Completely fill the response oval representing the most correct answer.

1.	A ○	B ○	C ●	D ○
2.	A ○	B ●	C ○	D ○
3.	A ●	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ●
5.	A ●	B ○	C ○	D ○
6.	A ●	B ○	C ○	D ○
7.	A ○	B ●	C ○	D ○
8.	A ○	B ○	C ○	D ●
9.	A ○	B ●	C ○	D ○
10.	A ○	B ○	C ●	D ○

Question 11	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)	$2x+3y=8$ $\quad x-2y=-5$ $3y=8-2x$ $\quad 2y=x+5$ $y=\frac{8}{3}-\frac{2x}{3}$ $\quad y=\frac{x}{2}+\frac{5}{2}$ $m_1=-\frac{2}{3}$ $\quad m_2=\frac{1}{2}$ $\tan\alpha=\left\lvert\frac{m_1-m_2}{1+m_1m_2}\right\rvert$ $\tan\alpha=\left\lvert\frac{-\frac{2}{3}-\frac{1}{2}}{1+-\frac{2}{3}\times\frac{1}{2}}\right\rvert$ $\tan\alpha=\left\lvert\frac{-\frac{7}{6}}{\frac{2}{3}}\right\rvert$ $\tan\alpha=\frac{7}{4}$ $\alpha=60°$ So the obtuse angle is $180°-60°=120°$	2	1 for the gradients 1 for correct angle
b)i)	$x=5\sin\theta$.....(1) $\quad y=5\cos\theta+1$.....(2) $x=5\sin\theta$.....(*squaring* x) $x^2=25\sin^2\theta$ $x^2=25(1-\cos^2\theta)$ $x^2=25-25\cos^2\theta$ $25\cos^2\theta=25-x^2$.....(3) $y=5\cos\theta+1$.....(*squaring* y) $y^2=25\cos^2\theta+10\cos\theta+1$.....(4) From (1) $\cos\theta=\frac{y-1}{5}$.....(5) *sub*(3)&(5)into(4) $y^2=25-x^2+10\left[\frac{y-1}{5}\right]+1$ $y^2=25-x^2+2y-2+1$ $y^2+x^2-2y-24=0$	2	Alternate solution by making sin θ and cos θ the subject and using $\sin^2\theta+\cos^2\theta=1$ 1 for doing some substitution of x into y or into $\sin^2\theta+\cos^2\theta=1$ 1 for correctly getting the equation

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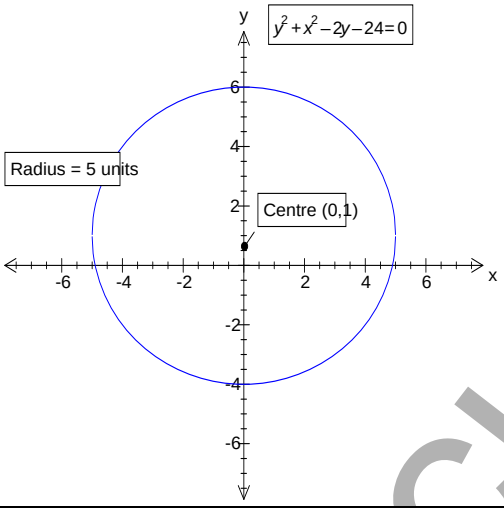
Part	**Solution**	**Marks**	**Comment**
ii)	$y^2 + x^2 - 2y - 24 = 0$ $y^2 + x^2 - 2y + 1 = 24 + 1$ $x^2 + (y-1)^2 = 25$ *Circle* *Centre*$(0,1)$ *Radius* $= 5$*units* 	2	1 for realising it is a circle and completing the square 1 for correct graph with centre & radius
c)i)	P(correct) = 0.25 P(incorrect) = 0.75 ${}^{10}_{10}C(0.25)^{10}(0.75)^{1}$ $= \dfrac{1}{1048576}$	1	
ii)	${}^{10}_{2}C(0.25)^{2}(0.75)^{8}$ $= 0.282$	1	
iii)	${}^{10}_{8}C(0.25)^{8}(0.75)^{2} + {}^{10}_{9}C(0.25)^{9}(0.75)^{1} + {}^{10}_{10}C(0.25)^{10}(0.75)^{0}$ $= 4.15\times10^{-4}$	2	

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Part	**Solution**	**Marks**	**Comment**
d)	$\int 3x\sqrt{4-x}dx$ $u = 4-x \qquad x = 4-u$ $du = -1dx$ $\int 3x\sqrt{4-x}dx$ $= 3\int x\sqrt{4-x}dx$ $= -3\int (4-u)\sqrt{u}du$ $= -3\int (4-u)u^{\frac{1}{2}}du$ $= -3\int 4u^{\frac{1}{2}} - u^{\frac{3}{2}}du$ $= -3\left[\frac{8}{3}u^{\frac{3}{2}} - \frac{2}{5}u^{\frac{5}{2}}\right] + C$ $= -8\sqrt{(4-x)^3} + \frac{6\sqrt{(4-x)^5}}{5} + C$	3	1 1 1
e)	$\left(3+\frac{1}{x}\right)^2 + 4\left(3+\frac{1}{x}\right) - 21 = 0$ $Let\ y = \left(3+\frac{1}{x}\right)$ $y^2 + 4y - 21 = 0$ $(y+7)(y-3) = 0$ $y = -7\ or\ 3$ $\therefore 3+\frac{1}{x} = -7 \qquad \text{or} \qquad 3+\frac{1}{x} = 3$ $\frac{1}{x} = -10 \qquad \frac{1}{x} = 0$ $x = -\frac{1}{10} \qquad$ No Solution So $x = -\frac{1}{10}$ is the only solution.	2	1 for using a substitution and finding values 1 for correct solution after resubstitution
		/15	

Question 12	**Trial HSC Examination - Mathematics Extension 1**	**2012**
Part	**Solution**	**Marks**	**Comment**
a)	$4^n > 1+3n \quad \text{for} \quad n>1$ *ie* $4^n - 3n - 1 > 0$ Step 1 Show true for $n>1$ *ie* $n=2$ $4^2 - 3\times 2 - 1 > 0$ $9 > 0 \therefore$ *true* Step 2 Assume true for $n=k$ $4^k - 3k - 1 > 0$ Step 3 Prove true for $n=k+1$ $4^{k+1} - 3(k+1) - 1 > 0$ $4\times 4^k - 3k - 3 - 1 > 0$ $4\times 4^k - 3k - 4 > 0$ $4(4^k - 3k - 1) + 9k > 0$ since $4^k - 3k - 1 > 0$ and $9k > 0$ Hence if the statement is true for $n=k$, then it is also true for $n=k+1$ The statement is true for $n=2$ and so it is true for $n=3$ and so on. Hence true for all $n>1$	3	1 mark for steps 1 and 2 2 marks for step 3, including conclusion.
b)i)	P(success) = (0.65) P(notsuccess) = (0.35) ${}^{10}_{9}C(0.65)^9(0.35)^1$ = 0.072 = 7%	1	

Question 12	**Trial HSC Examination - Mathematics Extension 1**	**2012**
Part	**Solution**	**Marks**	**Comment**
ii)	We need the greatest term in the expansion $\frac{n-r+1}{r}\times\frac{b}{a}\geq 1$ $\frac{10-r+1}{r}\times\frac{0.35}{0.65}\geq 1$ $r\leq 3.85$ $\therefore r=3$ ${}^{10}_{3}C(0.65)^7(0.35)^3$ $\therefore$ 7 will be the most likely number of people to sign up.	2	1 for finding the greatest term 1 for correct number of sign ups.
c)	$\int_0^{\frac{\pi}{4}}\sin x\cos^2 x\ \ dx$ $=\left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{4}}$ $=-\frac{1}{3}\left[\left(\cos\frac{\pi}{4}\right)^3-(\cos 0)^3\right]$ $=-\frac{1}{3}\left[\left(\frac{1}{\sqrt{2}}\right)^3-1\right]$ $=-\frac{1}{3}\left[\frac{1}{2\sqrt{2}}-1\right]$ $=-\frac{1}{6\sqrt{2}}+\frac{1}{3}$ $=-\frac{-1+2\sqrt{2}}{6\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ $=\frac{4-\sqrt{2}}{12}$	2	Can also be done by substitution 1 correct integration 1 for answer in any form

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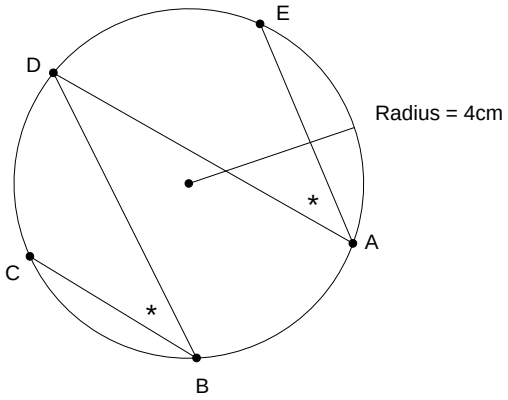
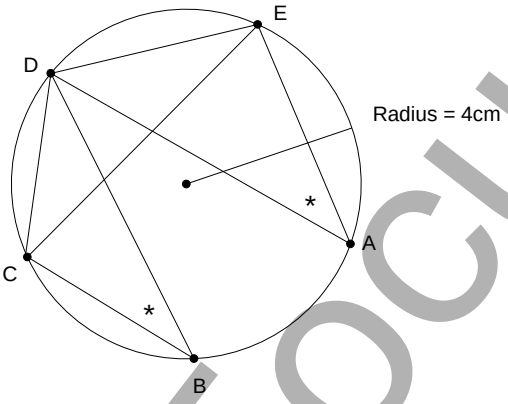
Part	**Solution**	**Marks**	**Comment**
d)i)		1	Must show all information, including radius
ii)	 $\angle DBC = \angle DAE (given)$ $Arc CD = Arc DE$ (converse of angles on the same arc) $\therefore CD = DE$ (equal arcs subtend equal chords) $\therefore \Delta CDE$ is isosceles as $CD = DE$	2 1 1	

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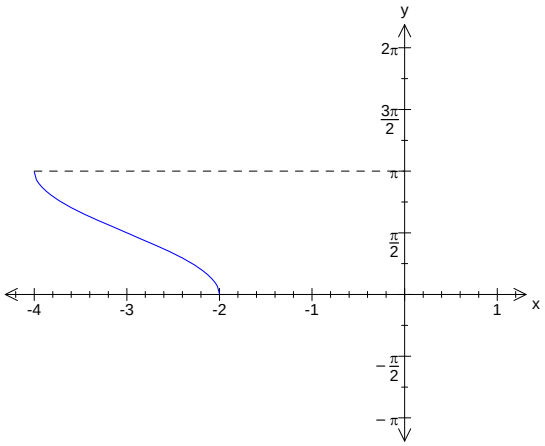
e)		2	
f)	$\left(3x-\frac{4}{5x^2}\right)^9$ $T_{k+1}={}^nC_k a^{n-k}x^k$ $T_6={}^9C_5(3x)^{9-5}(-\frac{4}{5x^2})^5$ $=126(81x^4)-\frac{1024}{3125x^{10}}$ $=-\frac{10450944}{3125x^6}$	2	1 1
		/15	

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)	$\int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$ $\sqrt{4-2x^2}$ $=\sqrt{2(2-x^2)}$ $=\sqrt{2}\sqrt{2-x^2}$ $\therefore \int_0^1 \frac{1}{\sqrt{4-2x^2}}\,dx$ $=\frac{1}{\sqrt{2}}\int_0^1 \frac{1}{\sqrt{2-x^2}}$ $=\frac{1}{\sqrt{2}}\left[\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_0^1$ $=\frac{1}{\sqrt{2}}\left(\left[\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)-\sin^{-1}\left(\frac{0}{\sqrt{2}}\right)\right]\right)$ $=\frac{1}{\sqrt{2}}\left[\frac{\pi}{4}-0\right]$ $=\frac{\pi}{4\sqrt{2}}$ $=\frac{\sqrt{2}\pi}{8}$	2	1 for getting in correct form and using standard integrals 1 answer
b)i)	$T=R+Ce^{-kt}$ $\frac{dT}{dt}=kCe^{-kt}$ and $T-R=Ce^{-kt}$ $\therefore \frac{dT}{dt}=-k(T-R)$ $\therefore T=R+Ce^{-kt}$ is a solution to the differential equation	1	

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	*When* $t=0$ $T=540°$ $R=20°$ $540=20+Ce^{0}$ $520=Ce^{0}$ $C=520$ $\therefore T=20+520e^{-kt}$ *When* $t=50$ $T=100°$ $R=20°$ $100=20+520e^{-50k}$ $\frac{80}{520}=-50k$ $k=\frac{\ln\left[\frac{8}{52}\right]}{-50}$ $k=0.037436043$ $\therefore T=20+520e^{-0.037436043t}$ *When* $T=40°$ $R=20°$ $40=20+520e^{-0.037436043\times t}$ $t=\frac{\ln\left[\frac{1}{26}\right]}{0.037436043}$ $t=87\,\text{min}\,utes$ Extra time $87-50=37$ So Storm must wait another 37 minutes for the Toffee to cool enough	3	1 for C 1 for k 1 for correct answer must have used full value of k
iii)	$40=25+520e^{-0.037436043\times t}$ $t=\frac{\ln\left[\frac{3}{104}\right]}{0.037436043}$ $t=95\text{minutes}$ $\therefore$ it would take longer for the toffee to cool in a room at 25°	2	1 for the calculation or other explanation 1 for a statement stating the result

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
c)	$y = \ln x - 1$ $y + 1 = \ln x$ $e^{y+1} = x$ $x^2 = \left(e^{y+1}\right)^2$ $= e^{2y+2}$ $V = \pi \int_a^b x^2 dy$ $= \pi \int_0^1 e^{2y+2} dy$ $= \pi \left(\frac{e^{2y+2}}{2} \right)_0^1$ $= \pi \left\{ \left[\frac{e^4}{2} \right] - \left[\frac{e^2}{2} \right] \right\}$ $= \frac{\pi}{2}\left(e^4 - e^2\right) units^3$	2	1 for finding x^2 1 for answer
d)i)	$P(x) = x^4 - 2x^3 + 5x^2 - 16x + 12$ $P(1) = 1^4 - 2 \times 1^3 + 5 \times 1^2 - 16 \times 1 + 12$ $P(1) = 1 - 2 + 5 - 16 + 12$ $P(1) = 0 \ \therefore (x-1)$ is a factor $P(2) = 2^4 - 2 \times 2^3 + 5 \times 2^2 - 16 \times 2 + 12$ $P(2) = 16 - 16 + 20 - 32 + 12$ $P(2) = 0 \ \therefore (x-2)$ is a factor $\therefore (x-1)(x-2)$ is a factor of $P(x) = x^4 - 2x^3 + 5x^2 - 16x + 12$	1	

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	$(x-1)(x-2) = x^2 - 3x + 2$ $x^2 - 3x + 2 \overline{)x^4 - 2x^3 + 5x^2 - 16x + 12}$ $\quad x^2 + x + 6$ $x^4 - 3x^3 + 2x^2$ $x^3 + 3x^2 - 16x$ $x^3 - 3x^2 + 2x$ $6x^2 - 18x + 12$ $6x^2 - 18x + 12$ 0 $\therefore$ The remaining factor is $(x^2 + x + 6)$ which cannot be reduced to linear factors.	2	1 1
e)i)	$A = \frac{1}{2}ab\sin C$ $A = \frac{1}{2} \times (x-2) \times (x-2) \times \sin 60°$ $A = \frac{1}{2}(x-2)^2 \times \frac{\sqrt{3}}{2}$ $A = \frac{\sqrt{3}(x-2)^2}{4}$	1	

Question 13	Trial HSC Examination - Mathematics Extension 1		2012
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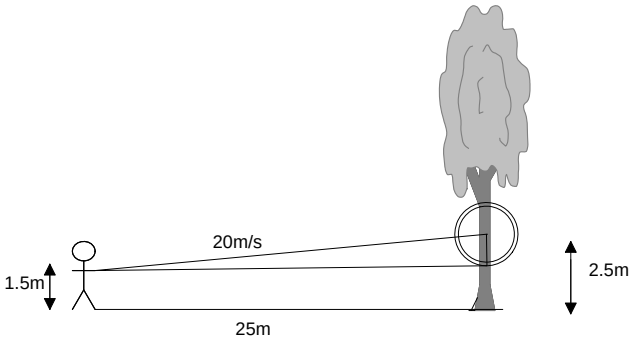
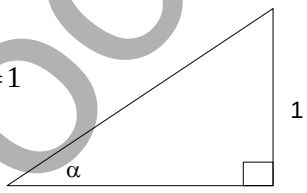
ii)	*Let* $s = x-2$ (*side length*) $\frac{ds}{dx}=1$ and $\frac{ds}{dt}=5mm/s=0.5cm/s$ $\frac{ds}{dt}=\frac{ds}{dx}\times\frac{dx}{dt}$ $0.5=1\times\frac{dx}{dt}$ $\frac{dx}{dt}=0.5$ $A=\frac{\sqrt{3}(x-2)^2}{4}$ $\frac{dA}{dx}=\frac{\sqrt{3}(x-2)}{2}$ $\frac{dA}{dt}=\frac{dA}{dx}\times\frac{dx}{dt}$ $\frac{dA}{dt}=\frac{\sqrt{3}(x-2)}{2}\times 0.5$ *when* s=10, $x=12$ $\frac{dA}{dt}=\frac{\sqrt{3}(12-2)}{2}\times 0.5$ $=\frac{5\sqrt{3}}{2}cm^2/s$ So the area of the triangle is increasing at $4.33cm^2/s$	2 1 Both exact or decimal answer OK 1	
		/15	

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
a)		2	1 for each
i)	Vertical Motion $\ddot{y} = -g = -9.8ms^{-2}$ $\therefore \dot{x} = \int -9.8dt$ $= -9.8t + C_2$ *When* $t = 0 \quad \dot{y} = V\sin\theta$ *So* $C = V\sin\theta$ $\dot{y} = V\sin\theta$ $20\sin\alpha = -9.8t + C_2$ $C_2 = 20\sin\alpha$ $\dot{y} = -9.8t + 20\sin\alpha$ $y = \int -9.8t + 20\sin\alpha dt$ $= -4.9t^2 + 20t\sin\alpha + C_3$ *When* $t = 0 \quad y = 1.5$ $0 = 0 + C_1$ $\therefore C_3 = 1.5$ $\therefore y = -4.9t^2 + 20t\sin\alpha + 1.5$		1 for using correct integrations 1 for final result.

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
ii)	$x = 20t\cos\alpha$.....(1) $y = -4.9t^2 + 20t\sin\alpha + 1.5$.....(2) *From*(1) $t = \dfrac{x}{20\cos\alpha}$.....(3) *sub* (3) into (2) $y = -4.9\left[\dfrac{x}{20\cos\alpha}\right]^2 + 20\left[\dfrac{x}{20\cos\alpha}\right]\sin\alpha + 1.5$ $= \dfrac{-4.9x^2}{400\cos^2\alpha} + \dfrac{x\sin\alpha}{\cos\alpha} + 1.5$ $= \dfrac{-4.9}{400}x^2\sec^2\alpha + x\tan\alpha + 1.5$ $= \dfrac{-4.9}{400}x^2(1+\tan^2\alpha) + x\tan\alpha + 1.5$ For the ball to hit the target $x = 25$ *and* $y = 2.5$ $2.5 = \dfrac{-4.9}{400}25^2(1+\tan^2\alpha) + 25\tan\alpha + 1.5$ $2.5 = -7.65625(1+\tan^2\alpha) + 25\tan\alpha + 1.5$ $0 = -7.65625\tan^2\alpha - 25\tan\alpha + 8.65625$	3	1
ii)	Let $x = \tan\alpha$ Solve using quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{25 \pm \sqrt{25^2 - (4\times 7.65625\times 8.65625}}{2\times 7.65625}$ $x = \dfrac{25 \pm 18.97}{15.3125}$ $x = 2.87$ or 0.39 $\tan\alpha = 2.87$ *or* $\tan\alpha = 0.39$ $\alpha = 70°47'$ *or* $21°29'$ However to avoid the air resistance Zanthie must shoot at an angle of 21°		1 1

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
b)	$(1+x)^{n+4} = (1+x)^n (1+x)^4$ $\binom{n+4}{r} = \binom{n}{r} + 4\binom{n}{r-1} + 6\binom{n}{r-2} + 4\binom{n}{r-3} + \binom{n}{r-4}$ *LHS* $(1+x)^{n+4} = 1 + \binom{n+4}{r}x + ... + \binom{n+4}{r}x^r + ...$ coefficient of $x^r = \binom{n+4}{r}$ *RHS* $(1+x)^n (1+x)^4 = \left[1 + \binom{n}{1}x + ... + \binom{n}{r}x^r + ...\right] \times$ $\left[1 + \binom{4}{1}x + \binom{4}{2}x^2 + \binom{4}{3}x^3 + \binom{4}{4}x^4\right]$ coefficient of $x^r = \binom{n}{r} + \binom{n}{r-1}\binom{4}{1} + \binom{n}{r-2}\binom{4}{2} +$ $\binom{n}{r-3}\binom{4}{3} + \binom{n}{r-4}\binom{4}{4}$ $= \binom{n}{r} + 4\binom{n}{r-1} + 6\binom{n}{r-2} + 4\binom{n}{r-3} + \binom{n}{r-4}$ Because coefficients of x^r in both expansions are the same it follows $\binom{n+4}{r} = \binom{n}{r} + 4\binom{n}{r-1} + 6\binom{n}{r-2} + 4\binom{n}{r-3} + \binom{n}{r-4}$	2 1	

Question 14	Trial HSC Examination - Mathematics Extension 1		2012
Part	**Solution**	**Marks**	**Comment**
c)	Equation of the chord of contact $xx_1 = 2a(y + y_1)$ This passes through $B(0 , 2a)$ $0x_1 = 2a(2a + y_1)$ $0 = 4a^2 + 2ay_1$ $\frac{2ay_1}{2a} = \frac{-4a^2}{2a}$ $y_1 = -2a$ Locus of the Midpoint AB $A(x_1 , -2a)$ & $B(0 , 2a)$ $y = \frac{-2a + 2a}{2}$ $\quad x = \frac{x_1}{2}$ $y = 0$ $\therefore y = 0$ is the equation of the locus of the midoint AB	2	1 for finding y_1 1 for the equation of the locus
d)i)	$R\sin(4t - \alpha) = R\sin 4t\cos\alpha - R\cos 4t\sin\alpha$ *If* $R\sin(4t - \alpha) = \frac{1}{\sqrt{3}}\sin 4t - \cos 4t$ *then* $r\cos\alpha = \frac{1}{\sqrt{3}}$ and $r\sin\alpha = 1$  continued over $\frac{1}{\sqrt{3}}$	2	

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Part	**Solution**	**Marks**	**Comment**
	$R = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}$ $= \sqrt{\frac{1}{3} + 1}$ $= \sqrt{\frac{4}{3}}$ $= \frac{2}{\sqrt{3}}$ $\tan\alpha = \frac{R\sin\alpha}{R\cos\alpha}$ $= \frac{1}{\frac{1}{\sqrt{3}}}$ $= \sqrt{3}$ $\therefore \tan\alpha = \sqrt{3}$ $\alpha = \frac{\pi}{3}$ $\therefore \frac{\sin 4t}{\sqrt{3}} - \cos 4t = R\sin(4t - \alpha)$ $\frac{\sin 4t}{\sqrt{3}} - \cos 4t = \frac{2}{\sqrt{3}}\sin(4t - \frac{\pi}{3})$		1 for deriving R 1 for deriving α
ii)	$x = 4 + \frac{\sin 4t}{\sqrt{3}} - \cos 4t$ *from part (i)* $x = 4 + \frac{2}{\sqrt{3}}\sin(4t - \frac{\pi}{3})$ $\dot{x} = \frac{8}{\sqrt{3}}\cos(4t - \frac{\pi}{3})$ $\ddot{x} = -\frac{32}{\sqrt{3}}\sin(4t - \frac{\pi}{3})$ $\ddot{x} = -16\left[\frac{2}{\sqrt{3}}\sin(4t - \frac{\pi}{3})\right]$ and from (1) $x - 4 = \frac{2}{\sqrt{3}}\sin(4t - \frac{\pi}{3})$ $\ddot{x} = -16(x - 4)$	2	1 1

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Part	**Solution**	**Marks**	**Comment**
iii)	The maximum speed occurs when the particle is at the centre of motion. When $x = 4$ $4 = 4 + \frac{2}{\sqrt{3}}\sin(4t - \frac{\pi}{3})$ $\frac{2}{\sqrt{3}}\sin(4t - \frac{\pi}{3}) = 0$ $\sin(4t - \frac{\pi}{3}) = 0$ $4t - \frac{\pi}{3} = 0, \pi, 2\pi, ...$ $4t = \frac{\pi}{3}, \frac{4\pi}{3}, ...$ $t = \frac{\pi}{12}, \frac{\pi}{3}, ...$ The particle first reaches maximum speed when $t = \frac{\pi}{12}$ seconds	2 1 1	
		/15	