Question 1 (12 marks) **Marks**

(a) Solve: $x(x^2-1)>0$ **2**

(b) Find the exact value of

(i) $\sin\dfrac{5\pi}{4}$ **1**

(ii) $\sin\left[2\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)\right]$ **2**

(c) Find the Cartesian equation of the curve whose parametric equations are $x=\sin\theta+\cos\theta$ and $y=\sin\theta-\cos\theta$. **1**

(d) Evaluate: $\lim\limits_{x\to 0}\dfrac{3x}{\sin 2x}$ **2**

(e) Sketch the curve: $y=4\cos^{-1}2x$ **2**

(f) Evaluate: $\displaystyle\int_0^3\sqrt{\left(9-x^2\right)}\,dx$ **2**

Question 2 (12 marks) **Use a separate page/booklet** **Marks**

(a) (i) Starting from the expansions for $\sin(A+B)$ and $\cos(A+B)$, prove that $\tan\left(A+B\right)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$ **1**

(ii) Solve: $\tan^{-1}x+\tan^{-1}2x=\dfrac{\pi}{4}$

Give the value of x (x >0) correct to 2 decimal places. **2**

(b) Find the coordinates of the points on the line $y=2x$ which are $\sqrt{2}$ units from the line $y = 4-x$. **2**

(c) Find the coefficient of a^5b^7 in $\left(2a-b\right)^{12}$ **2**

(d) Find the coordinates of point P which divides the join of $A(3,5)$ to $B(-4,2)$ externally in the ratio 2:3 **2**

(e) Use Simpson's Rule with 5 function values to find

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x\,dx$$ **3**

Question 3 (12 marks) **Use a separate page/booklet** **Marks**

(a) How many 11 letter words can be made using the letters of the word MATHEMATICS **2**

(b) A particle moves in a straight line and its position at a time t is given by $x = a\cos(4t+\alpha)$. The particle is initially at the origin moving with a velocity of $6m/s$ in the negative direction.

(i) Show that the particle is undergoing simple harmonic motion. **1**

(ii) Find the constants α and a. **2**

(iii) Find the position of the particle after 4 seconds **1**

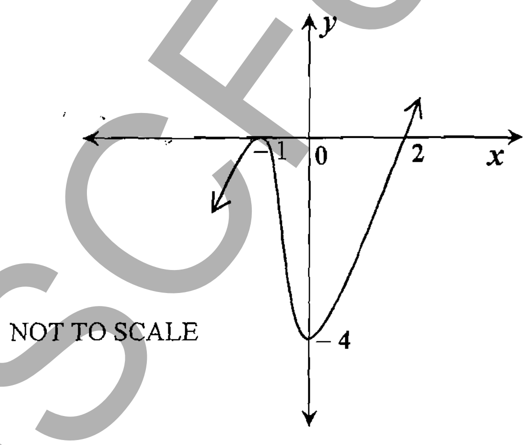
(c) Four blue and four white marbles are arranged in a circle

(i) How many different arrangements are possible? **1**

(ii) How many arrangements are possible if at least two of the blue marbles are together. **1**

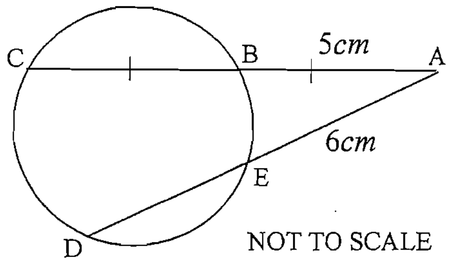
(iii) How many arrangements are possible if all the blue marbles are together? **2**

(d) A cubic curve touches the x-axis at -1 and intersects it at 2. It intersects the y-axis at -4 as shown below. Write the equation of the curve. **2**



Question 4 (12 marks) **Use a separate page/booklet** **Marks**

(a) In the circle below, the chords CB and DE are extended to intersect at A. CB=BA=5*cm* and EA=6*cm*. Find DE. **1**

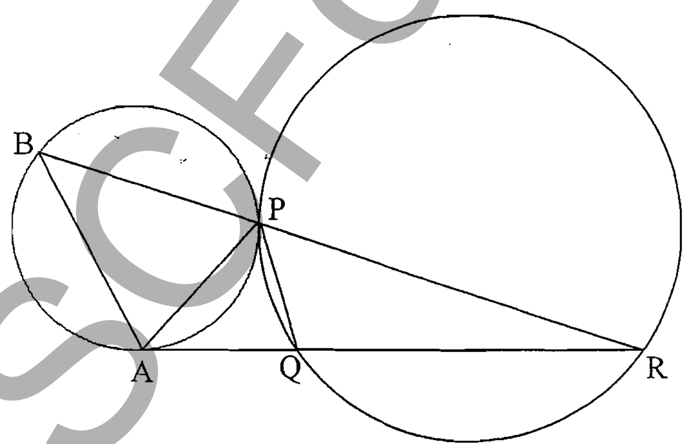


(b) **(i)** Show that one root of the equation $x+\sin x=\frac{\pi}{3}$ lies between $\frac{\pi}{6}$ and $\frac{\pi}{4}$ **1**

(ii) By letting $x=\frac{\pi}{6}$ find, using one application of Newton's method, a better approximation. **2**

(c) Two roots of $2x^3-x^2+kx+4=0$ are reciprocals of each other. Find the value of *k*. **2**

(d) Two circles touch externally at P. The tangent drawn at a point A on one of these circles meets the other circle in Q and R. RP meets the other circle in B. Prove $\angle BPA=\angle QPA$. **4**



(e) Solve for: $\frac{2x-3}{x}\leq 1, \qquad x\neq 0$ **2**

Question 5 (12 marks) **Use a separate page/booklet** **Marks**

(a) Find : $\int \sin^2 3x \, dx$ **2**

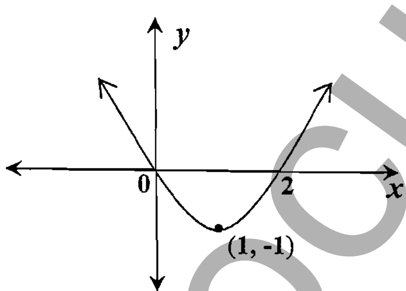
(b) The figure below shows a sketch of $f(x) = (x-1)^2 - 1$

(i) Explain why $f(x)$ does not have an inverse function. **1**

(ii) Sketch the graph of the inverse function, $g^{-1}(x)$, of $g(x)$, where $g(x) = (x-1)^2 - 1, \ x \geq 1$. **1**

(iii) State the domain of $g^{-1}(x)$. **1**

(iv) Find the expression for $y = g^{-1}(x)$ in terms of x. **1**



(c) Some bacteria are introduced into a solution. After 't' hours there are y grams of bacteria present, where $y = \dfrac{Pe^{qt}}{1+Pe^{qt}}$ $(t \geq 0)$ and P and q are constants.

(i) Show that $0 < y < 1$ for all values of t. **1**

(ii) Show that $\dfrac{dy}{dt} = q\,y(1-y)$ **2**

(iii) The maximum value of $\dfrac{dy}{dt}$ occurs when $y = 0{\cdot}5$, if $P = 0{\cdot}01$ and q= $0{\cdot}7$ find, to the nearest hour, when the bacteria will be increasing at the fastest rate. **3**

Question 6 (12 marks) **Use a separate page/booklet** **Marks**

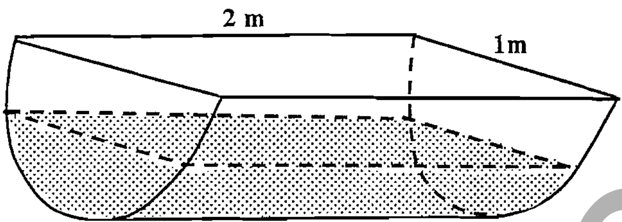
(a) The acceleration of a particle P is given by the equation $\frac{d^2x}{dt^2} = 8x\left(x^2+1\right)$ where the displacement in *cm* of P from a fixed point O, after 't' seconds, is given by x

Initially, P is projected from O with velocity $2 cm/s$ in the negative direction.

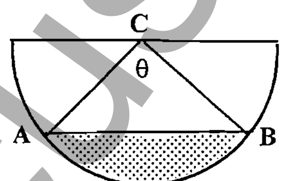
(i) Show that the speed in any position x is $2(x^2+1)\ cm/s$ and hence find an expression for x in terms of t. **3**

(ii) Determine the displacement of P after $\frac{\pi}{8}$ seconds. **1**

(b) An aquarium tank is 2m long and has a semi-circular cross-section of diameter 1m as shown in the diagram.



Tank



Cross-section

NOT TO SCALE

C is at the centre of the top edge. AB represents the water level and $\angle ACB = \theta$ where θ is measured in radians.

(i) Show that the volume of the water in the tank is given by

$$V = \frac{1}{4}(\theta - \sin\theta)$$ **2**

(ii) Water is poured into the tank at the rate of $0\cdot1\ m^3/\text{min}$. Find the rate at which the water level is rising when the depth of water is 0·2m. Give answer correct 3 sig. fig. **4**

(c) Find the values of x for which $f(x) = xe^{-2x}$ decreases. **2**

Question 7 (12 marks) **Use a separate page/booklet** **Marks**

(a) A tennis club consists of $2n$ members, n being female and n male.

(i) How many possible committees are there consisting of 2 females and 1male ? **1**

(ii) How many possible committees are there consisting of 3 females? **1**

(iii) A committee consists of three people. Using the above results, or otherwise, prove that

$$n\left({}^{n}c_2\right) + {}^{n}c_3 = \frac{1}{2}\left({}^{2n}c_3\right)$$ **1**

(b) Find $\int x\sqrt{x^2+9}\,dx$ using $u = x^2 + 9$ **2**

(c) A particle is projected with a velocity **V** ***m/s*** from a point at a height ***3h*** metres above the ground, the direction of projection making an angle α with the horizontal. The greatest height above the point of projection is ***h*** metres.

(i) Show that $V\sin\alpha = \sqrt{2gh}$ **2**

(ii) Show that the horizontal distance travelled by the particle before hitting the ground is $6h\cot\alpha$. **3**

(d) By expanding $[x + (1-x)]^n$, for all real numbers x and all positive integers n, show that

(i) ${}^{n}c_0x^n + {}^{n}c_1x^{n-1}(1-x) + {}^{n}c_2x^{n-2}(1-x)^2 + \cdots + {}^{n}c_n(1-x)^n = 1$, when n is even **1**

(ii) Deduce that ${}^{n}c_0 + {}^{n}c_1 + {}^{n}c_2 + \cdots + {}^{n}c_n = 2^n$ **1**

End of paper