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IN OMNIBUS GLORIFICETUR DEUS

2005
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**84**)

- Attempt Questions 1-7
- All questions are of equal value

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Attempt Questions 1-7
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QUESTION 1 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Calculate the acute angle (to the nearest minute) between the lines : **2**

$$2x + y = 4$$
and
$$x - 3y = 6$$

b) Use the table of standard integrals to show that $\int_6^{15} \frac{dx}{\sqrt{x^2 + 64}} = \ln(2)$ **2**

c) Solve $\frac{2x-3}{x-1} \leq 4$ **3**

d) Evaluate $\sum_{n=2}^{6} \left(n^2 - n\right)$ **1**

e) Show that $2x - 1$ is a factor of $2x^3 + 5x^2 + x - 2$ **2**

f) Find $\int \sin x \cos x \, dx$ using the substitution $u = \sin x$ **2**

QUESTION 2 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Consider the parametric equations:

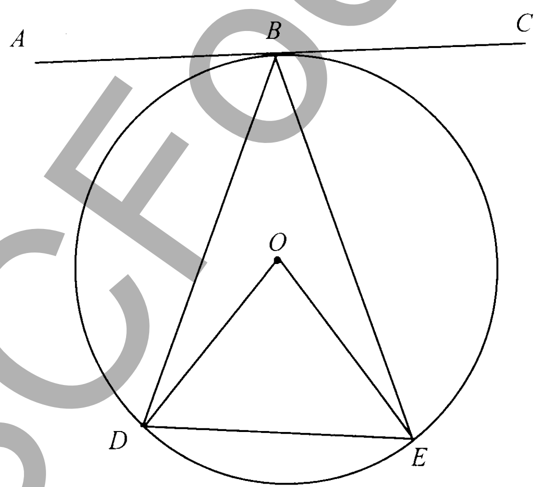
$$x = 2t - 1$$
$$y = t^2 + 2t$$

(i) Find the Cartesian equation of the curve represented by these parametric equations. **1**

(ii) Show that this Cartesian equation represents a parabola and state its vertex. **3**

b) Find the volume of the solid of revolution formed, when the section of the curve $y = \sqrt{\dfrac{1}{x^2 + 9}}$, between the lines $x = 0$ and $x = 3$, is rotated about the x axis. **3**

c) In the diagram below, O is the centre of the circle, AC is a tangent at B and D and E are points on the circumference. If $\angle ABD = 80^\circ$ and $\angle DBE = 40^\circ$, find the size of $\angle BEO$, giving reasons. **3**



d) Find $\int 4\sin^2 3x \, dx$. **2**

QUESTION 3 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) The polynomial $P(x) = 2x^5 + x^3 - 1$ has a root close to $x = 0 \cdot 85$. **3**

Use one application of Newton's method to find a second approximation for this root, giving your result correct to 2 decimal places.

b) (i) Show that $\sin x - \cos 2x = 2\sin^2 x + \sin x - 1$. **2**

(ii) Hence, or otherwise, solve: **3**

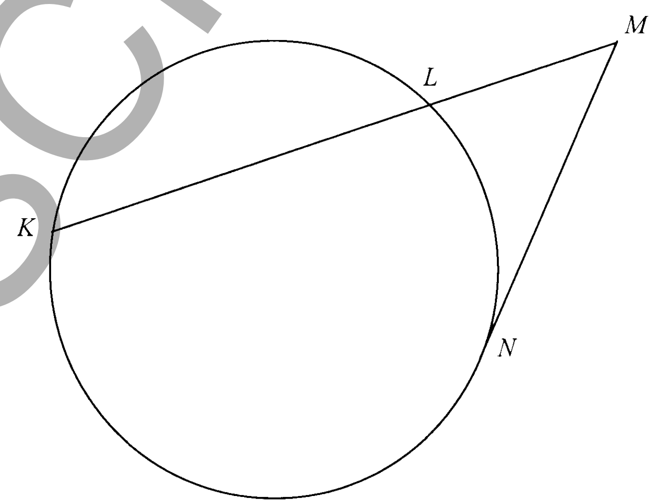
$$\sin x - \cos 2x = 0, \text{ for } 0 \le x \le 2\pi .$$

c) Evaluate: $\lim_{x \to 0} \frac{\sin 3x}{2x}$. **1**

d) In the diagram below the chord KL is produced to M so that $LM = \frac{1}{3}KL$. **3**

The tangent MN is then drawn.

Show that $MN = \frac{2}{3}KL$.



QUESTION 4 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) (i) Write down the $(k+1)^{th}$ term in the expansion of $\left(3x - \frac{2}{x^2}\right)^9$. **1**

(ii) Hence, determine the value of the term that is independent of x. **2**

b) For the cubic equation $2x^3 - 3x^2 + 5x - 2 = 0$ with roots, $x = \alpha$, $x = \beta$ and $x = \gamma$, find the value of :

(i) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(ii) $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\alpha^2\gamma^2}$ **2**

c) The population N, of Keysville first reached 25 000 on 1st January 2000. The population of Keysville is set to increase according to the equation:

$$\frac{dN}{dt} = k(N - 8000),$$

where t represents time in years after the population first reached 25 000. On 1st January 2005, the population of Keysville was 29 250.

(i) Verify that $N = 8000 + Ae^{kt}$, is a solution to the above equation, where A is a constant. **1**

(ii) Find the values of A and k. **2**

d) Find $\frac{d}{dx}\left(3x^2 \cos^{-1} x\right)$. **2**

QUESTION 5 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) For the function $f(x) = x^2 - 6x$:

(i) Find a domain of $f(x)$ for which there exists an inverse function, $f^{-1}(x)$. **1**

(ii) Find the equation of this inverse function $f^{-1}(x)$ and state its domain. **3**

b) Consider the expansion of $(a+b)^n$.

Show that:

(i) $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+....+\binom{n}{n}=2^n$ **1**

(ii) $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}+....+(-1)^n\binom{n}{n}=0$ **1**

(iii) Hence show that $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+....=2^{n-1}$ **2**

c) A particle moves in a straight line with an acceleration given by

$$\ddot{x} = 9(x-2)$$

where x is the displacement in metres from the origin O after t seconds. Initially, the particle is 4 metres to the right of the origin, moving with a velocity of $-6m/s$.

(i) Show that $v^2 = 9(x-2)^2$. **2**

(ii) Find an expression for v and hence find x as a function of t. **2**

QUESTION 6 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Using the fact that the inverse trigonometric function

$y = sin^{-1} x \quad \{-1 \le x \le 1\}$ is equivalent to the function

$x = sin\, y \quad \left\{-\frac{\pi}{2} \le y \le \frac{\pi}{2}\right\}$

(i) Show that $\frac{d}{dx}\left(sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$ **2**

(ii) Hence find the value of the derivative of $sin^{-1}\left(\sqrt{x}\right)$ when $x = \frac{1}{2}$. **2**

b) A projectile is fired with an initial velocity of $40m/s$, at an angle of projection of 30^0, from a point O on horizontal ground. Air resistance is to be neglected and acceleration due to gravity is $10m/s^2$.

(i) Derive the equations for both the horizontal and the vertical displacement. **4**

(ii) Find the maximum height reached by the projectile. **2**

(iii) Calculate the range of the flight. **2**

QUESTION 7 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) The chord joining P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ on the parabola $x^2 = 4ay$ subtends a right angle at the vertex of the parabola.

(i) Show that $pq = -4$ **1**

(ii) Show that the locus of the point M, the midpoint of PQ, is also a parabola. **2**

b) A sphere is expanding so that its surface area is increasing at the rate $24cm^2/s$. When the radius of the sphere is $12cm$, find the rate of increase of the:

(i) radius **3**

(ii) volume. **3**

c) Given the series expansion for e^h:

$$e^h = 1 + \frac{h}{1!} + \frac{h^2}{2!} + \frac{h^3}{3!} + \dots ,$$

(i) show that $\frac{e^h - 1}{h} = 1 + \frac{h}{2!} + \frac{h^2}{3!} + \dots$ **1**

(ii) Hence, use the definition $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, to find the derivative of $f(x) = e^x$. **2**

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0 \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a}\sin ax, a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax, a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax, a \neq 0$$

$$\int \sec ax \tan ax dx = -\frac{1}{a}\sec ax, a \neq 0$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a}, a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}dx = \ln(x + \sqrt{x^2 - a^2}), x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}dx = \ln(x + \sqrt{x^2 + a^2})$$

Note: $\ln x = \log_e x, x > 0$