Name: ____________________

Teacher: ____________________

YEAR 12 MATHEMATICS

EXTENSION 1

HALF YEARLY EXAMINATION 2005

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working must be shown in every question
- All questions are of equal value
- Start each question on a separate page

Question 1 **Marks**

(a) Find the size of the acute angle between the lines **3**

$$y = -x$$

$$\sqrt{3}y = x$$

(b) Solve the inequality $\frac{2x+1}{x-1} > 3$. **2**

(c) A sector of angle 135° at the centre, is cut from a circular piece of cardboard of radius 8cm. The cut edges are brought together to form a cone. Find the circumference of the base of the cone. **2**

(d) For the given function $f(x) = \frac{8}{4+x^2}$,

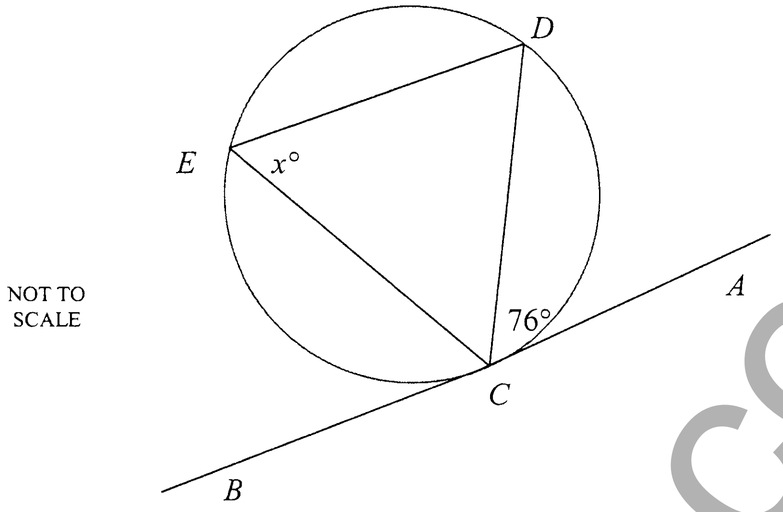
(i) show that $f(x)$ is an even function. **2**

(ii) evaluate $\lim_{x \to \infty} \frac{8}{4+x^2}$. **2**

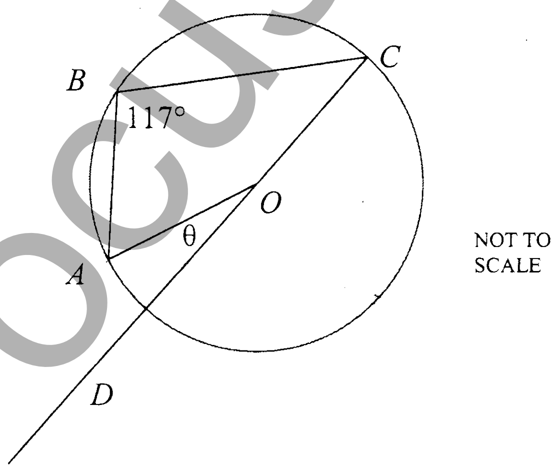
(iii) sketch the graph of $y = f(x)$. **1**

Question 2 *Start a new page* **Marks**

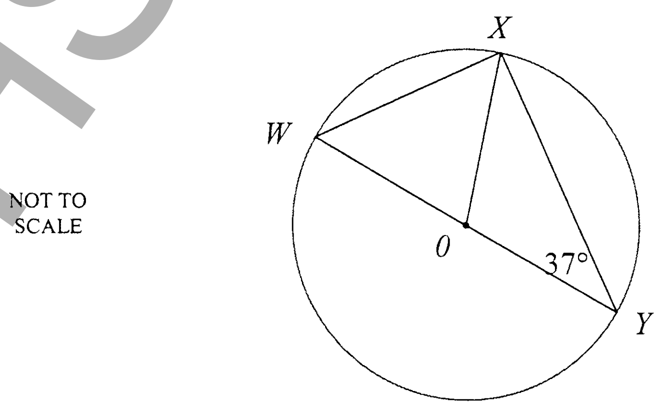
(a) In the diagram drawn below AB is a tangent to the circle and $\angle DCA = 76°$. Find the value of the pronumeral, giving reasons for your answer. **2**



(b) Given that O is the centre of the circle and $\angle ABC = 117°$, find the value of θ, giving reasons for your answer. **2**

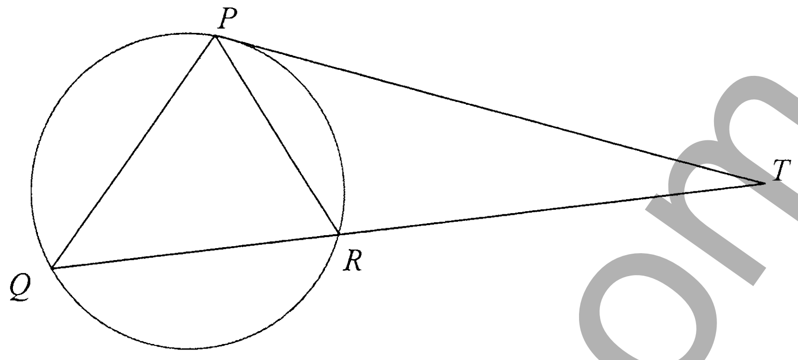


(c) In the diagram drawn below, WY is a diameter of a circle, centre O. If $\angle WYX = 37°$, find the size of $\angle WXO$. Give reasons for your answer. **2**



Question 2 (cont.) **Marks**

(d) PT is a tangent to the circle drawn below and QR is a secant, intersecting the circle at Q and at R. The line QR intersects PT at T.



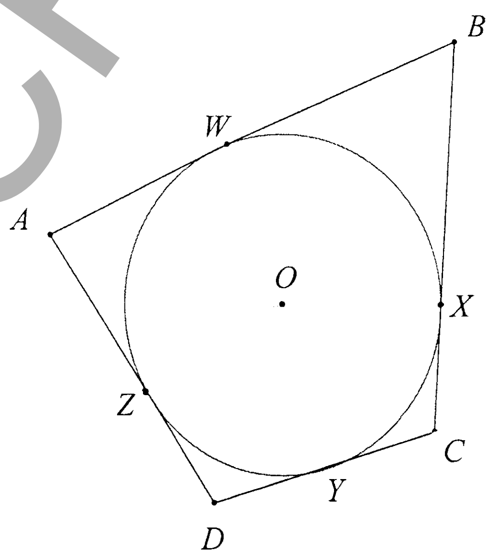
(i) Prove that the triangles PRT and QPT are similar. **2**

(ii) Hence prove $PT^2 = QT \times RT$. **1**

(e) A quadrilateral $ABCD$ is constructed so that AB, BC, CD and DA are tangents to a circle. W, X, Y and Z are the points of contact of the tangents AB, BC, CD and DA respectively.

(i) Copy the diagram neatly.

(ii) Prove that AB+DC=AD+BC. **3**



Question 3 *Start a new page* **Marks**

(a) Find the value of k for which $(x+2)$ is a factor of the polynomial **3**

$$2x^3 + kx^2 - 18x - 8.$$

Hence, express the polynomial as a product of its linear factors.

(b) Sketch the graph of the polynomial **2**

$$P(x) = x(x+1)^2(2x-1).$$

(c) If α, β, γ are the roots of the equation $x^3 + 2x^2 + 3x + 4 = 0$, **3**

find the value of (i) $\alpha + \beta + \gamma$

(ii) $\alpha\beta + \alpha\gamma + \beta\gamma$

(iii) $\alpha^2 + \beta^2 + \gamma^2$.

(d) The polynomial $x^3 - 6x^2 + 9x - k$ has a double root. **4**

Show that there are two possible values of k.
Find the roots for each value of k.

Question 4 *Start a new page* **Marks**

(a) Sketch the function $y = 2\sin 3x$ for $0 \le x \le 2\pi$. **2**

(b) Differentiate $\cos^4 x$. **2**

(c) Solve $7\sin\theta + \cos\theta = 5$ for $0^\circ \le \theta \le 360^\circ$. **3**

Write your solution(s) to the nearest minute.

(d) Use the table of standard integrals to show that: **2**

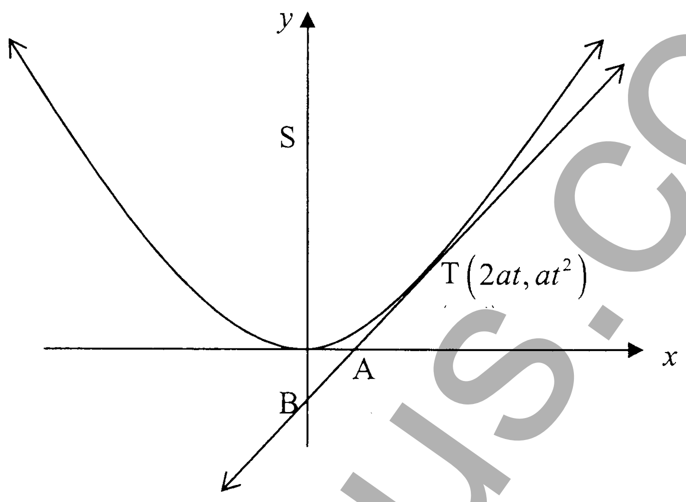
$$\int_0^{\frac{\pi}{9}} \sec 3x \tan 3x \, dx = \frac{1}{3}$$

(e) Prove that $\dfrac{1}{\tan A + \cot B} + \dfrac{1}{\cot A + \tan B} = \dfrac{\sin(A+B)}{\cos(A-B)}$. **3**

Question 5 *Start a new page* **Marks**

(a) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$, are two points on the parabola $x^2 = 4ay$. If PQ is a focal chord, prove that $pq = -1$. **3**

(b) $T(2at, at^2)$ is a variable point on the parabola $x^2 = 4ay$.



(i) Show that the equation of the tangent, to the parabola at the point T, is $y - tx + at^2 = 0$. **1**

(ii) If the tangent at T cuts the x-axis at A, and the y-axis at B, find the co-ordinates of A and B. **2**

(iii) Show that the tangent at T makes equal angles with the y-axis and the line TS, where S is the focus of the parabola. **3**

(iv) In what ratio does the point T, divide the interval AB? **3**

Question 6 *Start a new page* **Marks**

(a) Solve $\log_3(9x-2)-2\log_3 x=2$. **3**

(b) Evaluate $\int_0^1 \frac{3x^2}{1+x^3}dx$. **2**

(c) Calculate the volume of the solid of revolution, **2**

formed by rotating the curve $y=e^x+e^{-x}$, about the x-axis,

between $x=-1$ and $x=1$.

(d) Consider the function $y=xe^{-x}$.

(i) Determine the nature of any stationary point(s). **2**

(ii) Find any point(s) of inflexion. **2**

(iii) Sketch the function. **1**

Question 7 *Start a new page* **Marks**

(a) Evaluate $\int_0^4 x\sqrt{16-x^2}\,dx$, **3**

using the substitution $u = 16 - x^2$, or otherwise.

(b) A solid of revolution is formed by rotating the area under the curve $y = \tan x$ between $x = 0$ and $x = \frac{\pi}{3}$, around the $x-$axis. **3**

Find the exact volume of the solid.

(c) Show that $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$. **3**

Hence, find the exact value of $\int_0^{\frac{\pi}{2}} \sin^2 \frac{\theta}{2}\,d\theta$.

(d) For a certain function, $f''(x) = -18\cos 3x$. **3**

Determine the equation of this function,

given that there is a stationary point at the point $\left(\frac{2\pi}{3}, 1\right)$.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$