2 Unit Trial

Qu1

(a) $\frac{12.35-8.66}{6.5} = 0.5676923$ … 0.2955288

$= 0.30$ (2 dec pl)

(b) $\frac{x-6}{3} = \frac{4x}{5}$

$5(x-6) = 12x$

$5x - 30 = 12x$

$\frac{-7x}{-7} = \frac{-30}{-7}$

$x = -4\frac{2}{7}$

(c) … $= 88$

(e) $\int x^3 + 4 \, dx$

$= \frac{x^4}{4} + 4x + C$

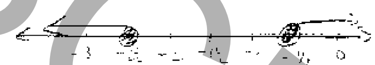
(f) $|2x+3| \geq 2$

$-2 \geq 2x+3 \geq 2$

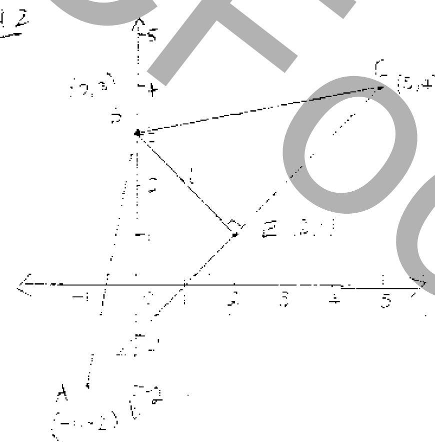
$-5 \geq 2x \geq -1$

$-\frac{5}{2} \geq x \geq -\frac{1}{2}$

$-2\frac{1}{2}$, $x \geq -\frac{1}{2}$



Qu2



(a) $E = \left(\frac{-1+5}{2}, \frac{-2+4}{2}\right)$

$= (2, 1)$

(b) m of $AC = \frac{4--2}{5--1}$

$= \frac{6}{6}$

$= 1$

(c) m_1 of $AC = 1$ (from (b))

thus m_2 of $L = -1$

equation of L:

$y - y_1 = m(x - x_1)$

$y - 3 = -1(x - 0)$

$y - 3 = -x$

$y = 3 - x$

(d) $E = (2, 1)$. If E lies on L then E satisfies the eqn

subst $E(2, 1)$ into $y = 3 - x$

$1 = 3 - 2$

$= 1$

TRUE, Thus E lies on L

(e) On diagram

Prove $\triangle BEC \equiv \triangle BEA$

BE is common S

$\angle BEC = \angle BEA = 90°$ A

$AE = EC$ (E is midpt) S

Thus $\triangle BEC \equiv \triangle BEA$ (SAS)

(f) … length of AE

$d = \sqrt{(-1-2)^2 + (-2-1)^2}$

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$

(g) $(x-h)^2 + (y-k)^2 = r^2$

where (h, k) is the centre

& r = radius

$h = 2$

$k = 1$

$r = \sqrt{18}$

$(x-2)^2 + (y-1)^2 = 18$

Qu3

(a) (i) $y = (3x^2 + 2)^3$

$\frac{dy}{dx} = 3(3x^2+2)^2 \cdot 6x$

$= 18x(3x^2+2)^2$

(ii) $y = 3x \cos 2x$

$\frac{dy}{dx} = 3x \cdot -\sin 2x \cdot 2 + \cos 2x \cdot 3$

$= -6x \sin 2x + 3\cos 2x$

$= 3\cos 2x - 6x \sin 2x$

(iii) $y = \frac{e^{3x}}{x}$

$\frac{dy}{dx} = \frac{x \cdot 3e^{3x} - e^{3x} \cdot 1}{x^2}$

$= \frac{3xe^{3x} - e^{3x}}{x^2}$

$= \frac{e^{3x}(3x - 1)}{x^2}$

(b) (i) $\int_0^{\frac{\pi}{4}} \sin 3x \, dx$

$= \left[-\frac{1}{3}\cos 3x\right]_0^{\frac{\pi}{4}}$

$= \left(-\frac{1}{3}\cos 3\left(\frac{\pi}{4}\right)\right) - \left(-\frac{1}{3}\cos 3(0)\right)$

$= \left(-\frac{1}{3} \times -\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{3}\right)$

$= \frac{+1}{3\sqrt{2}} + \frac{1}{3}$

$= \frac{+\sqrt{2}}{6} + \frac{1}{3}$

$= \frac{+\sqrt{2}+2}{6}$

$= \frac{2+\sqrt{2}}{6} = \frac{1}{3}\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)$

(ii) $\int_0^3 e^{2x+3} \, dx$

$= \left[\frac{e^{2x+3}}{2}\right]_0^3$

$= \frac{e^{2(3)+3}}{2} - \frac{e^{2(0)+3}}{2}$

$= \frac{e^9}{2} - \frac{e^3}{2} = \frac{1}{2}\left(e^9 - e^3\right)$

(c) $\int \frac{x}{x^2+1} \, dx$

$= \frac{1}{2}\ln(x^2+1) + C$

Q4

(a) $u_3 = 7$
$u_{10} = 42$

(i) $u_3 = a + 2d = 7$ —— ①
$u_{10} = a + 9d = 42$ —— ②

② − ①

$7d = 35$
$d = 5$

Subst $d = 5$ into ①
$a + 2(5) = 7$
$a + 10 = 7$
$a = -3$

Thus first term is -3 & difference is 5

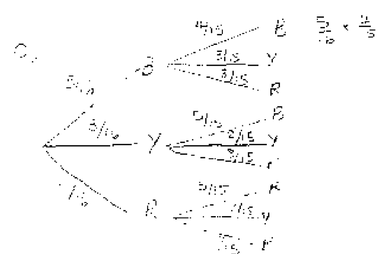
(ii) $S_n = \frac{n}{2}(a + l)$

$S_{10} = \frac{10}{2}(-3 + 42)$

$= 5(39)$

$= 195$

(b)



$\frac{5}{16} \times \frac{4}{15}$

$\therefore P(BB) = \frac{5}{16} \times \frac{4}{15} = \frac{1}{12}$

$P(\text{at most } 1\,B) = 1 - P(BB)$
$= 1 - \frac{1}{12}$
$= \frac{11}{12}$

(c) $y = 9 - x^2$
$\frac{dy}{dx} = -2x$

(i) When $x = 1$
$m = -2(1)$
$= -2$

$y - y_1 = m(x - x_1)$
$y - 8 = -2(x - 1)$
$y - 8 = -2x + 2$
$2x + y = 10$

(ii) Subst $(5, 0)$ into eqn

$2(5) + 0 = 10$
$10 + 0 = 10$
$10 = 10$
True. Thus tangent passes through $(5, 0)$

OR

tangent crosses the x axis when $y = 0$
$2x + 0 = 10$
$2x = 10$
$x = 5$
the pt is $(5, 0)$

(iii) Required A = A of Δ − A of curve
$= \left(\frac{1}{2} \times 4 \times 8\right) - \int_1^3 9 - x^2 \, dx$

$= 16 - \left[9x - \frac{x^3}{3}\right]_1^3$

$= 16 - \left[\left(9(3) - \frac{3^3}{3}\right) - \left(9(1) - \frac{1}{3}\right)\right]$

$= 16 - 9\frac{1}{3}$

$= 6\frac{2}{3}$ units^2

Q5

(a) (i) $\frac{BD}{\sin 41°29'} = \frac{9.8}{\sin 77°}$
$BD = 6.66933493$
$\doteqdot 6.7$ cm

(ii) $\angle ABC = 180 - (41°29' + 61°19')$
$= 77.2°$

A of Δ $= \frac{1}{2} \times 9.8 \times 7.4 \times \sin 77.2°$

$= 35.3589559$ cm^2
$\doteqdot 35$ cm^2

(b) $y = 4x^3 + 6x^2$

(i) $\frac{dy}{dx} = 12x^2 + 12x$

$\frac{d^2y}{dx^2} = 24x + 12$

Stat pts occur when $\frac{dy}{dx} = 0$,

$12x^2 + 12x = 0$
$12x(x+1) = 0$
$12x = 0$ or $x + 1 = 0$
$x = 0$, $x = -1$

when $x = 0$
$y = 4(0)^3 + 6(0)^2$
$= 0$

when $x = -1$
$y = 4(-1)^3 + 6(-1)^2$
$= -4 + 6$
$= 2$

The co-ordinates of stat pts are $(0,0)$ & $(-1, 2)$

Nature of stat. pts.

When $x = 0$
$\frac{d^2y}{dx^2} = 24(0) + 12$
$= 12 > 0$ ∴ min

When $x = -1$
$\frac{d^2y}{dx^2} = 24(-1) + 12$
$= -12 < 0$ ∴ max

So $(0,0)$ is a min tn. pt
& $(-1,2)$ is a max tn. pt

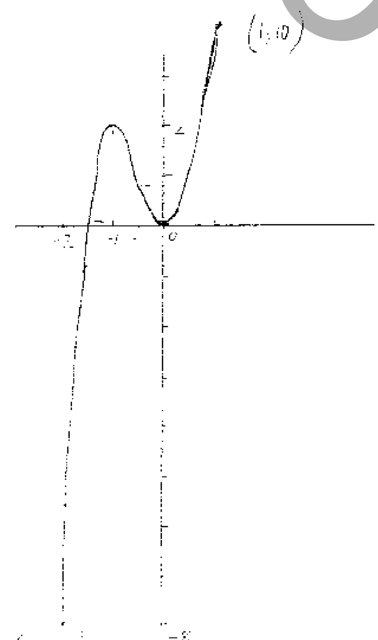
(ii) Pts of inflexion when $\frac{d^2y}{dx^2} = 0$;

∴ $24x + 12 = 0$
$12(2x + 1) = 0$
$2x + 1 = 0$
$2x = -1$
$x = -1/2$

x	-1	$-1/2$	0
$\frac{d^2y}{dx^2}$	-12	0	12

Concavity changes
Pt of inflexion at the pt where $x = -1/2$

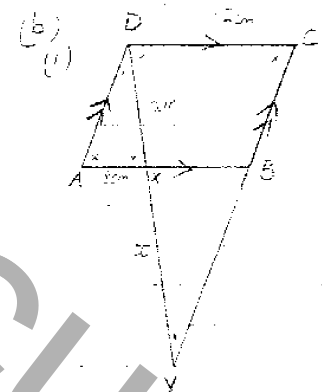
(iii)



(iv) The max value of $4x^3 + 6x^2$ in the domain $-2 \le x \le$ is 10

Qu6
(a) $\ln x = 2 \ln x$
∴ $x = x^2$
$x^2 - x = 0$
$x(x-1) = 0$
∴ $x = 0$ or 1
$x \ne 0$ thus $x = 1$ only

(b) (i)



(ii) $\angle ADX = \angle DYC$ (alt ∠s, AD∥CY)
$\angle DAX = \angle DCB$ (opp equal ∠s in parallelogram)
Thus $\angle AXD = \angle XDC$ (3rd ∠s equ in Δ)
So ΔADX ||| ΔDYC (equal ∠s)

(iii) $\frac{8}{12} = \frac{x}{10+x}$
$8(10+x) = 120$
$80 + 8x = 120$
$8x = 40$
$x = 5$

(c) $V = \int \pi x^2 \, dy$
$y^2 = x - 5$
$x = y^2 + 5$
$x = 5y + 5$

∴ $\pi \int_0^5 (5y+5)^2 - (y^2+5)^2$

$= \pi \int_0^5 25y^2 + 50y + 25 - y^4 - 10y^2 - 25$

$= \pi \int_0^5 15y^2 - y^4 + 50y \; dy$

$= \pi \left[\frac{15y^3}{3} - \frac{y^5}{5} + \frac{50y^2}{2} \right]_0^5$

$= \pi \left[5y^3 - \frac{y^5}{5} + 25y^2 \right]_0^5$

$= \pi \left(5(5)^3 - \frac{(5)^5}{5} + 25(5)^2 \right) - (0)$

$= (625 - 625 + 625)\pi$

∴ Volume required is 625π units

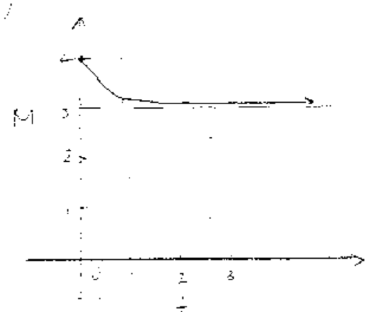
Qu7
(a) $M = e^{-2t} + 3$
(i) Initially $t = 0$;
$M = e^{-2(0)} + 3$
$= 1 + 3$
$M = 4$ kg

(ii) $\frac{dM}{dt} = e^{-2t} \cdot -2$
$= -2e^{-2t}$

when $t = 3$,
$\frac{dM}{dt} = -2e^{-6}$

(iii) As $t \to \infty$, $e^{-2t} \to 0$, $M \to 3$
Thus there will always be more than 3 mL of vaccine present in the cat's blood

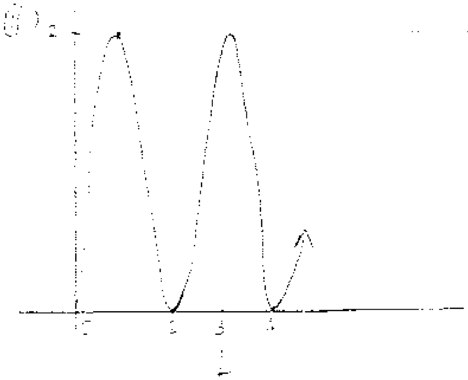
(iv)



(c) $x = 1 - \cos \pi t$

(i) Initially, $t = 0$,
$x = 1 - \cos \pi(0)$
$x = 0$

(ii)



(iii) $v = \frac{dx}{dt} = \pi \sin \pi t$

(iv) When $t = \frac{1}{6}$

$v = \pi \sin \pi\left(\frac{1}{6}\right)$

$= \pi \times \frac{1}{2}$

$= \frac{\pi}{2}$

(v) Max speed when
$a = 0$
$a = \frac{dv}{dx} = \pi \times \cos \pi t \times \pi$

$\pi^2 \cos \pi t = 0$
$\cos \pi t = 0$
$\pi t = \frac{\pi}{2}, \frac{3\pi}{2}$

$t = \frac{1}{2}, \frac{3}{2}$

first to reach max speed

Sol 8
(a) (i) Interest rate $= 18\%$ p.a
$= 1.5\%$ p.m
$= 0.015$

Int charged at end of 1st month $= \$(0.015 \times 15000)$

Total owing after making 1st instalment is

$= 15000 + 0.015 \times 15000 - M$
$= 15000(1 + 0.015) - M$
$A_1 = 15000(1.015) - M$

(ii) $A_2 = 0.015 \times A_1 + A_1 - M$
$= A_1(0.015 + 1) - M$
$= A_1(1.015) - M$
$= (15000(1.015) - M)(1.015) - M$
$= 15000(1.015)^2 - M(1.015) - M$
$= 15000(1.015)^2 - M(1.015 + 1)$

$A_3 = 0.015 \times A_2 + A_2 - M$
$= A_2(0.015 + 1) - M$
$= A_2(1.015) - M$
$= [15000(1.015)^2 - M(1.015+1)](1.015) - M$
$= 15000(1.015)^3 - M(1.015)(1.015+1) - M$
$= 15000(1.015)^3 - (1.015)^2 M - 1.015M - M$
$\therefore A_3 = 15000(1.015)^3 - M(1.015^2 + 1.015 + 1)$

(iii) $A_{60} = 15000(1.015)^{60} - M(1 + 1.015 + 1.015^2 + \ldots 1.015^{59})$
GP ; $a = 1$, $r = 1.015$, $n = 60$

$0 = 15000(1.015)^{60} - M \times \frac{1(1.015^{60} - 1)}{1.015 - 1}$

$\frac{M(1.015^{60} - 1)}{0.015} = 15000(1.015)^{60}$

$M = 15000(1.015)^{60} \times \frac{0.015}{1.015^{60} - 1}$

$= \$380.90$

(b) $A = \int_a^b f(x)\,dx$ $\quad h = \frac{b-a}{n}$, $h = \frac{2-0}{4}$

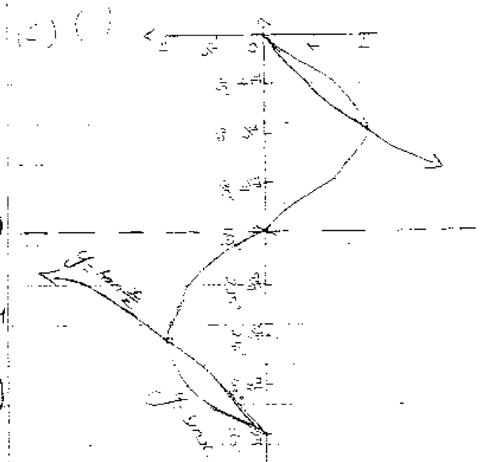
$= \frac{h}{3}\{(y_0 + y_4) + 4(y_1 + y_3) + 2y_2\}$

$= \frac{2}{3}\{(0.9 + 1.7) + 4(1.4 + 2.1) + 2(1.8)\}$

$= \frac{2}{3}\{2.6 + 14 + 3.6\}$

$= 13.47$ (to 2 dec pl)

(c) (i)



(ii) $0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$

Q9 (a)
(i) $A = \frac{1}{2} r^2 \theta$

$90\pi = \frac{1}{2} \times 15^2 \times \theta$

$\frac{180\pi}{15^2} = \frac{15^2\theta}{15^2}$

$\theta = \frac{4\pi}{5}$

(ii) Length of Arc ABC $= r\theta$

$= 15 \times \frac{4\pi}{5}$

$= 12\pi$

Length AC: $15^2 + 15^2 - 2 \times 15 \times 15 \times \cos\frac{4\pi}{5}$

$= 28.53169549$

Thus total perimeter is

$12\pi + 28.5 = 66.23$

(c) (i) $S = \frac{D}{T}$ $\quad D = \sqrt{6^2 + x^2}$ $\quad S = 6$ km/h

$6 = \frac{\sqrt{6^2+x^2}}{T}$

$T = \frac{\sqrt{6^2+x^2}}{6}$

(ii) $BC = 10 - x$

Jogging speed from B to C is 10 km/h

$S = \frac{D}{T}$

$10 = \frac{10 - x}{T}$

$T = \frac{10 - x}{10}$

Thus total time taken from lighthouse to [illegible] = $\frac{\sqrt{6^2+x^2}}{6} + \frac{10-x}{10}$ hours

(iii) $T = \frac{\sqrt{6^2+x^2}}{6} + \frac{10-x}{10}$

$T = \frac{(36+x^2)^{1/2}}{6} + \frac{10}{10} - \frac{x}{10}$

$\frac{dT}{dx} = 2x \cdot \frac{1}{2}\frac{(36+x^2)^{-1/2}}{6} - \frac{1}{10}$

$= \frac{x}{6\sqrt{36+x^2}} - \frac{1}{10}$

A minimum occurs when $\frac{dT}{dx} = 0$

ie $\frac{x}{6\sqrt{36+x^2}} - \frac{1}{10} = 0$

$\frac{x}{6\sqrt{36+x^2}} = \frac{1}{10}$

$10x = 6\sqrt{36+x^2}$

$100x^2 = 36(36+x^2)$

$100x^2 = 36^2 + 36x^2$

$100x^2 - 36x^2 = 36^2$

$\frac{64x^2}{64} = \frac{36^2}{64}$

$x = \sqrt{\frac{36^2}{64}}$

$= \frac{36}{8}$

$= 4\frac{1}{2}$

(iv) Quickest time when $T = \frac{\sqrt{36+x^2}}{6} + \frac{10-x}{10}$

When $x = 4\frac{1}{2}$

$T = \frac{\sqrt{36 + \left(\frac{9}{2}\right)^2}}{6} + \frac{10 - \frac{9}{2}}{10}$

$= \frac{\sqrt{36 + \frac{81}{4}}}{6} + \frac{11}{20}$

$= \frac{15}{2} \div 6 + \frac{11}{20}$

$= \frac{9}{5}$ of 60 mins

$= 108$ min

$= 1$ hr $+ 48$ mins

SHD

(a) (i) $AO = OP$ (equal radii)

Thus $\triangle APO$ is isosceles

$\therefore \angle OAP = \angle OPA$ equal [illegible]

$\angle OAP + \angle OPA = \angle POB = 2x$

Thus $\angle OAP = \angle OPA = x$

(ii) $\sin 2x = \frac{[illegible]}{[illegible]}$

$= \frac{PN}{OP}$ $\quad OP = \frac{1}{2}AB$

$= \frac{PN}{\frac{1}{2}AB}$

$= \frac{2PN}{AB}$

(iii) In $\triangle PAB$, $\sin x = \frac{PB}{AB}$

$\cos x = \frac{AP}{AB}$

In $\triangle APN$, $\sin x = \frac{PN}{AP}$

$\cos x = \frac{AN}{AP}$

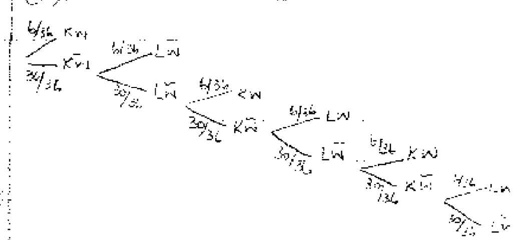
$2\sin x \cos x$

$= 2 \times \frac{PN}{AP} \times \frac{AP}{AB}$

$= \frac{2PN}{AB}$

$= \sin 2x$

(b)



(i) $P(LW) = P(K\bar{W}) \times P(LW)$

$= \frac{30}{36} \times \frac{6}{36}$

$= \frac{5}{36}$

(ii) $P(\text{Lois wins on 2nd}) = \{P(K\bar{W}) \times P(L\bar{W})\} + \{P(K\bar{W}) \times P(L\bar{W}) \times P(K\bar{W})\}$

$= \frac{5}{36} + \left\{\frac{30}{36} \times \frac{30}{36} \times \frac{30}{36} \times \frac{6}{36}\right\}$

$= \frac{5}{36} + \left[\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right]$

$= \frac{5}{36} + \frac{5^3}{6^4}$

(iii) $P(LW) = P(K\bar{W}) \times P(LW)$ or $P(K\bar{W}) \times P(L\bar{W}) \times$
$P(K\bar{W}) \times P(LW)$ or $P(K\bar{W}) \times P(L\bar{W}) \times P(K\bar{W})$
$P(L\bar{W}) \times P(K\bar{W}) \times P(LW) \times$

$= \left(\frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}\right)$

$+ \ldots$

$= \left(\frac{5}{6} \times \frac{1}{6}\right) + \left(\frac{5^3}{6^3} \times \frac{1}{6}\right) + \left(\frac{5^5}{6^5} \times \frac{1}{6}\right) + \left(\frac{5^7}{6^7} \times \frac{1}{6}\right) + \ldots$

$S_\infty = \frac{a}{1-r}$ $\qquad a = \frac{5}{6} \times \frac{1}{6}$

$= \frac{\frac{5}{6} \times \frac{1}{6}}{1 - \frac{5^2}{6^2}}$ $\qquad r = \frac{5^2}{6^2}$

$= \frac{5}{36} \div \frac{11}{36}$

$= \frac{5}{11}$