

- (a) 10      (b)  $\frac{1}{10}$       (c) 100

- 3 (ii) The curves  $y = \cos x$ ,  $y = \sin x$  intersect at  $P\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$ . Find the gradients of these curves at P. If  $\theta$  is the acute angle between their tangents at P, show that  $\tan \theta = 2\sqrt{2}$ .
- (iii) The area under the curve  $y = \sec x$ ,  $0 \leq x \leq \frac{\pi}{4}$ , is rotated about the  $x$ -axis. What is the volume of the solid of revolution thus generated?

**QUESTION 4 (10 Marks)**

- (i) A parabola  $y = 5 + 2x - x^2$  is given.
- (a) Write down the coordinates of its vertex.
- (b) What is the equation of its axis?
- (c) Find the area under the curve between  $x = -1$  and  $x = 1$ .
- (ii) A parabola  $y = ax^2 + bx + c$  is drawn through the three points A(-1, 4), B(0, 7), C(1, 8).
- (a) Determine  $a$ ,  $b$  and  $c$ .
- (b) Show that D(3, 4) is also on the parabola.

**QUESTION 5 (10 Marks)**

- (i) The first three terms of an arithmetic progression are 26, 23, 20.
- (a) Write down the sum of the first  $n$  terms of the series.
- (b) What is the least  $n$  such that the sum of the first  $n$  terms of the series is negative.
- (ii) The sequence 5, 11, 29 ... has as its  $n$ -th term  $3^n + 2$ . Write down
- (a) the fourth term;
- (b) the sum of the first  $n$  terms.

**QUESTION 6 (10 Marks)**

- (i) The position of a particle moving along the  $x$ -axis is given by  $x = 1 - \cos 2(t-1)$ .
- (a) Show that the particle is at rest when  $t = 1$ .
- (b) When and where is the particle next at rest?
- (ii) For each of the following equations, state whether it has solutions and if so find all solutions in the interval  $0 \leq x \leq 2\pi$ . (a)  $2 \sin x + 1 = 0$  (b)  $\sin x + 2 = 0$  (c)  $\sin x - 2 = 0$

**QUESTION 7 (10 Marks)**

- (i) Sketch the curve  $y = x^3 - 4x^2 + 3x$ .
- (ii) The origin is a common point of the line  $y = m\pi$  and the curve  $y = x^3 - 4x^2 + 3x$ . Show that if  $m < -1$

N.S.W. DEPARTMENT OF EDUCATION  
HIGHER SCHOOL CERTIFICATE EXAMINATION 1972  
MATHEMATICS PAPER C (2S) (EQUIVALENT TO 2 UNIT)

Instructions: Time 3 hours. All questions may be attempted. In every question, all necessary working should be shown. Marks will be deducted for careless or badly arranged work.

**QUESTION 1 (12 Marks)**

- (i) Indicate on a diagram the region in which  $0 \leq x + y \leq 1$ .
- (ii) Find the equation of the tangent at the point (1, 5) on the parabola  $y = x^2 + 2x + 2$ .
- (iii) Differentiate (a)  $\frac{1}{1+x^2}$  (b)  $(\cos x - \sin x)^2$
- (iv) An integer  $n$ ,  $1 \leq n \leq 144$ , is picked at random. What is the probability that  $n$  is the square of an integer?

**QUESTION 2 (9 Marks)**

- (i) Find the second derivative of  $xe^{-x}$ .
- (ii) Write down a primitive (indefinite integral) of  $\frac{\sqrt{x}}{x}$ .
- (iii) The velocity  $\frac{dx}{dt}$  of a particle is given by  $\frac{dx}{dt} = 4t^3$ . If  $x = 5$  at  $t = 0$ , calculate  $x$  at  $t = 2$ .

**QUESTION 3 (9 Marks)**

- (i) Given that  $e^{2.3026} = 10$  (to five-figure accuracy) write down the natural logarithms of

they have no other point in common.

(iii) Discuss the case  $m = -1$  giving a geometrical interpretation.

**QUESTION 8 (10 Marks)**

(i) Sketch the curve  $y = x^2$  and the line  $y = x$ , then find the area enclosed between the curve and the line in the positive quadrant.

(ii)  $P_1, P_2$  are points on the curve  $y = \frac{1}{x}$  with x-coordinates  $\frac{1}{1+h}$  and  $1+h$ , respectively. Show that the equation of the line through  $P_1, P_2$  is  $x+y = 1+h + \frac{1}{1+h}$ .

**QUESTION 9 (10 Marks)**

(i) Show that the triangle whose sides satisfy  $2x, y = 0, x + 2y = 5, x - 3y = 20$  is isosceles and right-angled.

(ii) Simplify each of the following expressions. No negative or zero exponents should appear in your final answer and it is given that neither  $x$  nor  $y$  is zero.

(a)  $\frac{(8x)^{-1}}{2^{-6}}$

(b)  $\frac{xy^{-1} + yx^{-1}}{x^2 + y^2}$

**QUESTION 10 (10 Marks)**

(i) Calculate the following definite integrals:

(a)  $\int_0^1 (x^{1/5} - x^{1/3}) dx$

(b)  $\int_{\pi/2}^{\pi} \sin x dx$

(c)  $\int_0^3 \frac{x dx}{1+x^2}$

(ii) Show that the second derivative of  $\log_e (1 + \sin x)$  is  $\frac{-1}{1 + \sin x}$ .