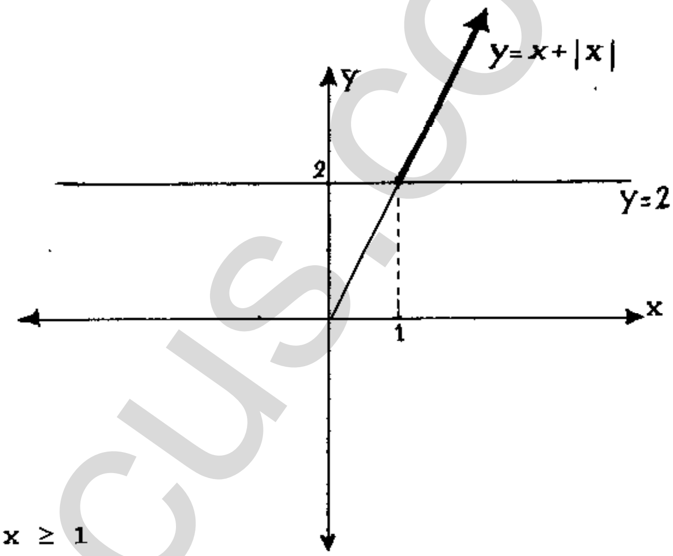
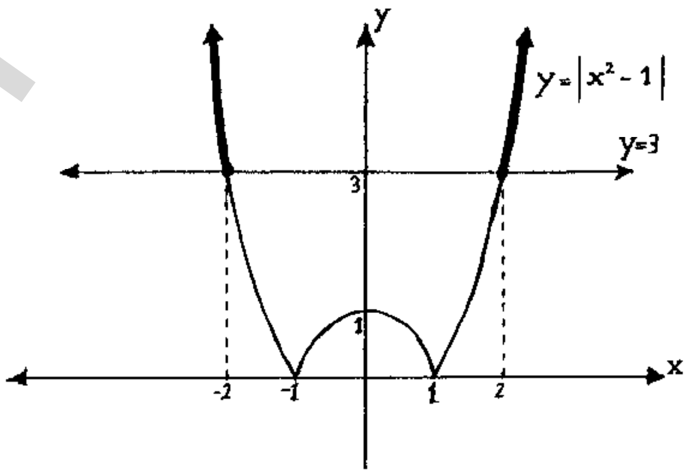
Exercises 30(c)

Q.1) $x + |x| \geq 2$
$x + |x| = 2$
When $x > 0$, $2x = 2$ $\therefore x = 1$
When $x < 0$ there are no solutions.



The general solution is $x \geq 1$

b) $|x^2 - 1| > 3$
$|x^2 - 1| = 3$
$\therefore x^2 - 1 = 3$ or $x^2 - 1 = -3$
$\therefore x = \pm 2$ has no real solutions.



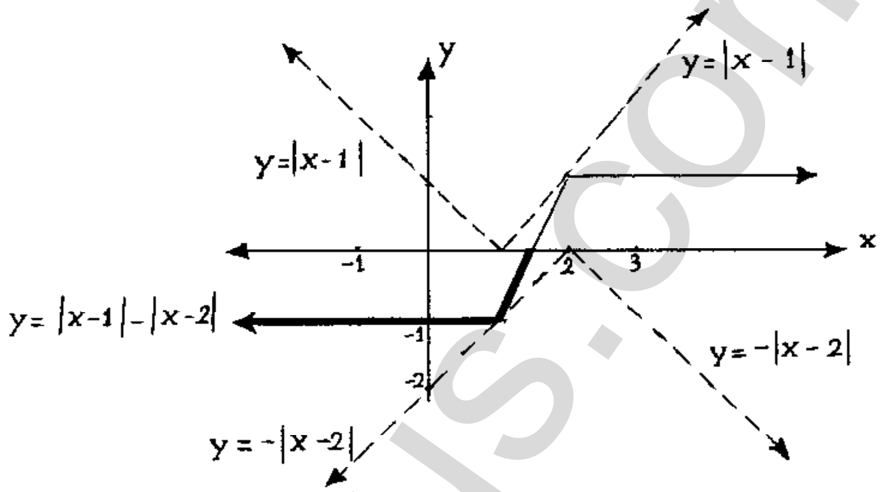
The general solution is $x < -2$ or $x > 2$

c) $|x - 1| - |x - 2| < 0$

$|x - 1| - |x - 2| = 0$ $\therefore$ $|x - 1| = |x - 2|$

$\therefore$ $x - 1 = x - 2$ or $x - 1 = -x + 2$

has no real solutions. $x = 1\frac{1}{2}$



$\therefore$ The general solution is $x < 1\frac{1}{2}$

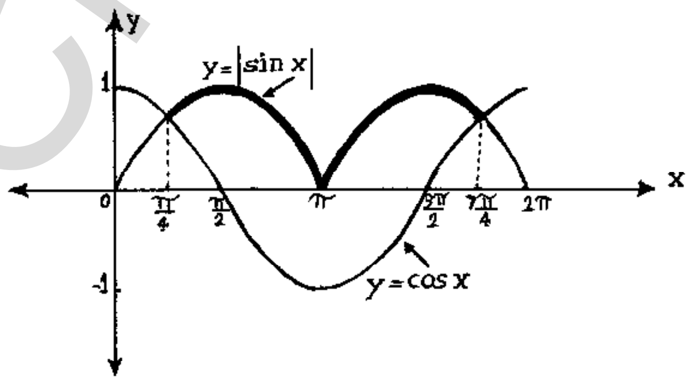
d) $|\sin x| > \cos x$ $\quad 0 < x < 2\pi$

$|\sin x| = \cos x$

$\therefore \sin x = \cos x$ or $\sin x = -\cos x$

$\therefore \tan x = 1$ $\qquad \therefore \tan x = -1$

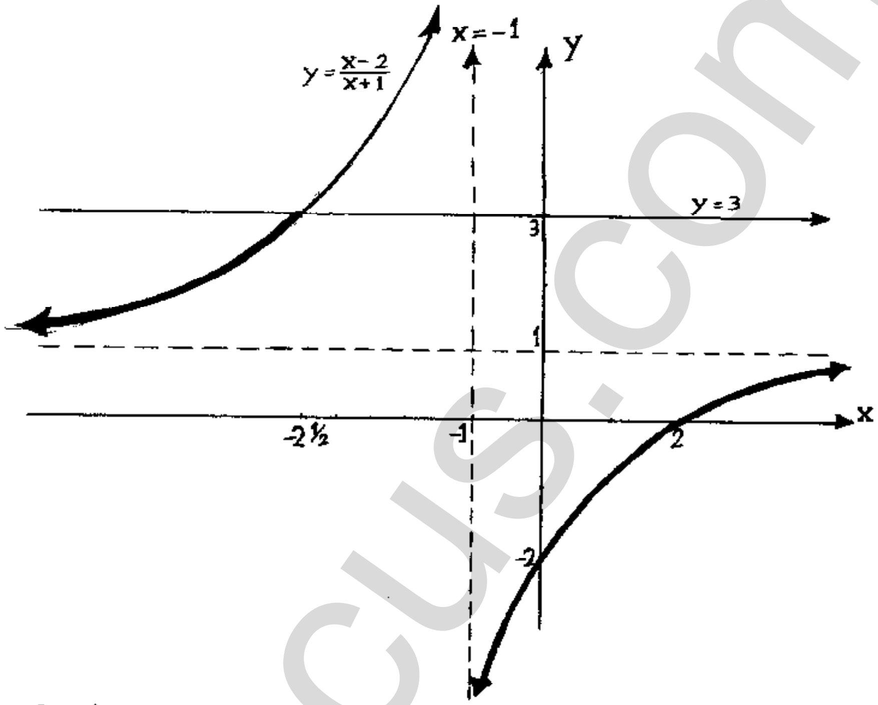
$\therefore x = \frac{\pi}{4}$ $\qquad \therefore x = \frac{7\pi}{4}$



$\therefore$ the general solution $\frac{\pi}{4} < x < \frac{7\pi}{4}$

e) $\dfrac{x - 2}{x + 1} < 3$

$\dfrac{x - 2}{x + 1} = 3 \quad \therefore x - 2 = 3x + 3 \quad \therefore x = -2\frac{1}{2}$

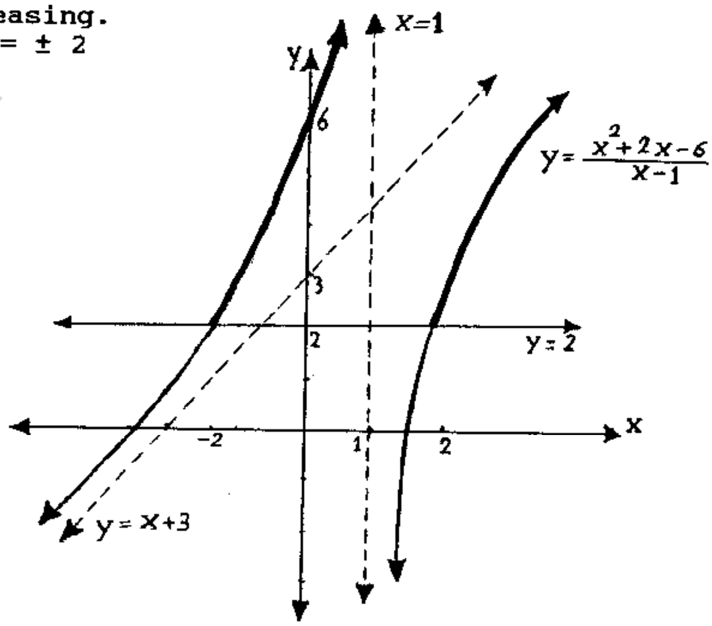


The general solution $x < -2\frac{1}{2}$ or $x > -1$.

f) $\dfrac{x^2 + 2x - 6}{x - 1} > 2 \quad \therefore x + 3 - \dfrac{3}{x - 1} > 2$

Note : the function $y = x + 3 - \dfrac{3}{x - 1}$ has no turning points and strictly increasing.

$\dfrac{x^2 + 2x - 6}{x - 1} = 2 \quad \therefore x = \pm 2$



The general solution is :
$-2 < x < 1$ or $x > 2$

g) $x + 2 + \dfrac{4}{x - 1} < -2$

$y = x + 2 + \dfrac{4}{x - 1}$

Domain:the function is defined for all x (real numbers) except for x = 1

Asymptotic behaviour:

When $x \to \pm \propto$, $y - (x + 2) \to 0$ $\therefore$ $y = x + 2$ is an asymptote.

$x \to 1$, $y \to \pm \propto$ $\therefore$ $x = 3$ is a vertical asymptote.

Turning points:

$\dfrac{dy}{dx} = 1 - \dfrac{4}{(x - 1)^2} = \dfrac{x^2 - 2x - 3}{(x - 1)^2}$

Let $\dfrac{dy}{dx} = 0$ To get the stationary turning points.

$\therefore$ $x^2 - 2x - 3 = 0$ $\therefore$ $x = 3$ or $x = -1$

x		*-1*		*1*		*3*	
$\frac{dy}{dx}$	+	*0*	–		–	*0*	+
y		max -1				min 7	

When $x < -1$, $\dfrac{dy}{dx} > 0$

$x > -1$, $\dfrac{dy}{dx} < 0$

$\therefore$ At $(-1 , -1)$ there is a maximum turning point.

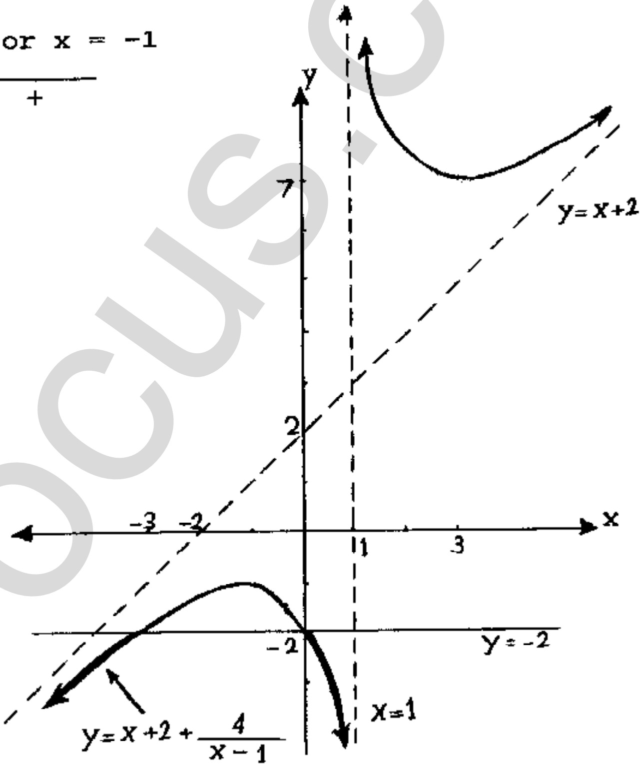
When $1 < x < 3$, $\dfrac{dy}{dx} < 0$

$x > 3$, $\dfrac{dy}{dx} > 0$

$\therefore$ At $(3 , 7)$ there is a minimum turning point.

Note: $x + 2 + \dfrac{4}{x - 1} = -2$

$\therefore$ $x = 0$ or -3



$\therefore$ **The general solution is : $x < -3$ or $0 < x < 1$**

h) $\dfrac{x^2}{x^2 - 4} \geq -1$

$y = \dfrac{x^2}{x^2 - 4}$

Domain:the function is defined for all x (real numbers) except for x = ± 2

Asymptotic behaviour:

When $x \to \pm \propto$, $y \to 1$ $\therefore$ $y = 1$ is a horizontal asymptote.

$x \to \pm 2$, $y \to \pm \propto$ $\therefore$ $x = \pm 2$ are vertical asymptotes.

Turning points:

$$\frac{dy}{dx} = \frac{-8x}{(x^2-4)^2} \quad \text{(gradient function)}$$

Let $\frac{dy}{dx} = 0$ To get the stationary turning points.

$\therefore x = 0$

x		-2		0		2	
$\frac{dy}{dx}$	$+$		$+$		$-$		$-$
y	$\nearrow$		$\nearrow$	max 0	$\searrow$		$\searrow$

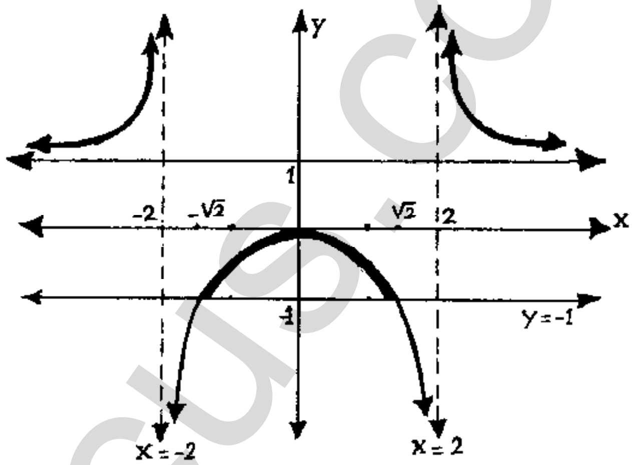
When $x < 0$, $\frac{dy}{dx} > 0$

$x > 0$, $\frac{dy}{dx} < 0$

$\therefore$ At $(0\ ,\ 0)$ there is a maximum turning point.

Note : $\frac{x^2}{x^2-4} = -1$

$\therefore x = \pm\sqrt{2}$



$\therefore$ **The general solution is:** $x < -2 \ ,\ -\sqrt{2} \le x \le \sqrt{2}$ **or** $x > 2$

i) $\frac{1}{3x-2} - \frac{1-x}{x-6} < 0 \qquad \therefore \frac{x-6-(1-x)(3x-2)}{(3x-2)(x-6)} < 0$

$\therefore \frac{3x^2-4x-4}{(3x-2)(x-6)} < 0 \qquad \therefore \frac{(3x+2)(x-2)}{(3x-2)(x-6)} < 0$

Let $y = \frac{(3x+2)(x-2)}{(3x-2)(x-6)}$

***Domain:**the function is defined for all x except for x = 2/3, 6*

Asymptotic behaviour:

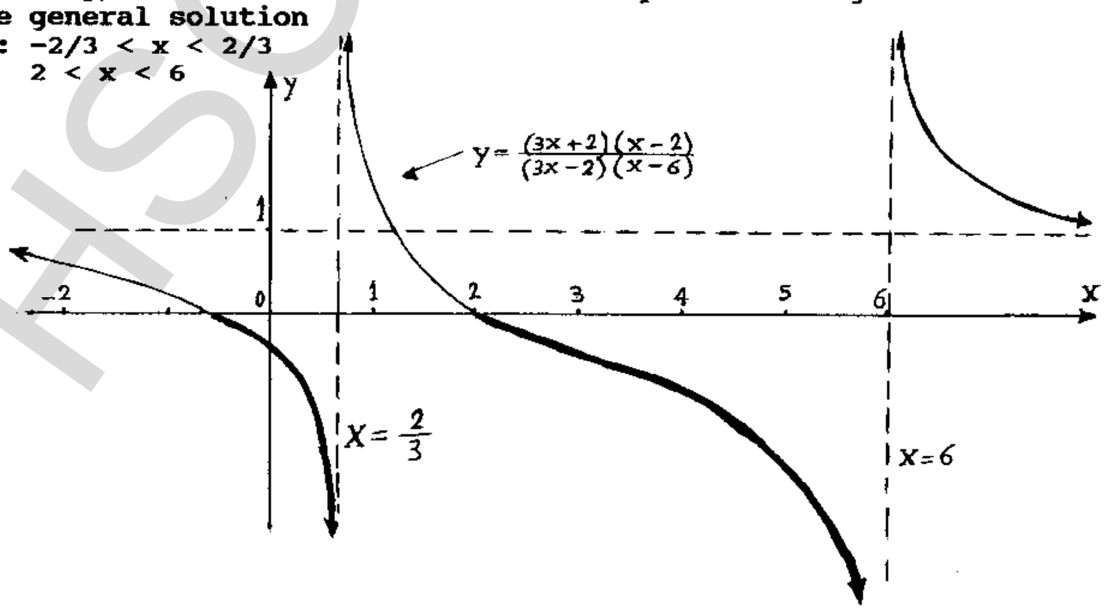
When $x \to \pm\infty$, $y \to 1$ $\therefore y = 1$ is a horizontal asymptote.

$x \to 2/3$, $y \to \pm\infty$ $\therefore x = 2/3$ is a vertical asymptote.

$x \to 6$, $y \to \pm\infty$ $\therefore x = 6$ is a vertical asymptote.

Since $dy/dx < 0$ $\therefore$ the function is always decreasing.

The general solution is: $-2/3 < x < 2/3$ **or** $2 < x < 6$



j) $\dfrac{1 - x^2}{1 + x^2} < \dfrac{1}{2}$

$y = \dfrac{1 - x^2}{1 + x^2}$

Domain: *the function is defined for all x.*

Asymptotic behaviour:

When $x \to \pm \infty$, $y \to -1$ $\therefore$ $y = -1$ is a horizontal asymptote.

Turning points:

$\dfrac{dy}{dx} = \dfrac{-2x(1 + x^2) - 2x(1 - x^2)}{(1 + x^2)^2} = \dfrac{-4x}{(1 + x^2)^2}$

Let $\dfrac{dy}{dx} = 0$ To get the stationary turning points.

$\therefore$ $-4x = 0$ $\quad\therefore$ $x = 0$, $y = 1$

x		0	
$\dfrac{dy}{dx}$	$+$		$-$
y		max 1	

When $x < 0$, $\dfrac{dy}{dx} > 0$

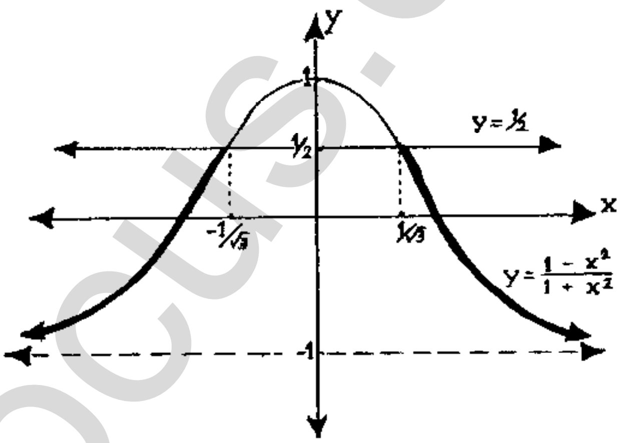
$x > 0$, $\dfrac{dy}{dx} < 0$

$\therefore$ *At (0 , 1) there is a maximum turning point.*

NOTE: $\dfrac{1 - x^2}{1 + x^2} = \dfrac{1}{2}$

$\therefore$ $x = \pm \dfrac{1}{\sqrt{3}}$

$\therefore$ **The general solution is:** $x < -1/\sqrt{3}$ or $x > 1/\sqrt{3}$



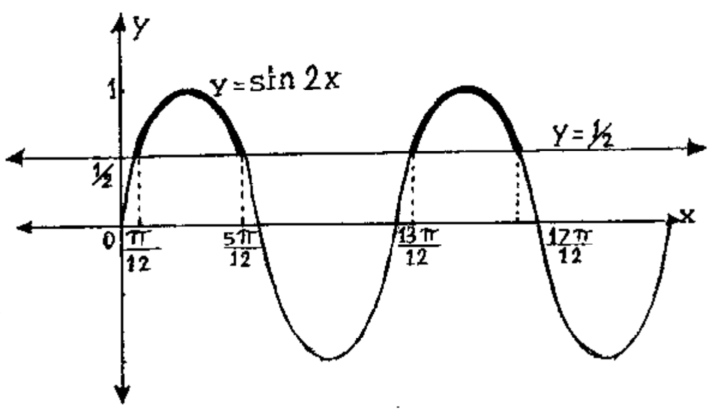
Q.2) a) $\sin 2x \geqslant \dfrac{1}{2}$, $0 \le x \le 2\pi$

Note: $\sin 2x = 1/2$

$\therefore$ $2x = \pi/6 + 2k\pi$ or $2x = 5\pi/6 + 2k\pi$

$\therefore$ $x = \pi/12 + k\pi$ or $x = 5\pi/12 + k\pi$

$\therefore$ $x = \pi/12,\ 13\pi/12$ or $x = 5\pi/12$, $17\pi/12$



$\therefore$ **The general solution is:** $\pi/12 \leqslant x \leqslant 5\pi/12$ or $13\pi/12 \leqslant x \leqslant 17\pi/12$

b) $\cos^2 x > \frac{1}{4}$, $0 \le x \le 2\pi$

Note: $\cos^2 x = 1/4$ $\therefore \cos x = \pm 1/2$

$\cos x = 1/2$ or $\cos x = -1/2$

$\therefore x = \pi/3$ or $x = 5\pi/3$ $\therefore x = 2\pi/3$ or $x = 4\pi/3$

$y = \cos^2 x$

Domain: the function is defined for $0 \le x \le 2\pi$.

Turning points:

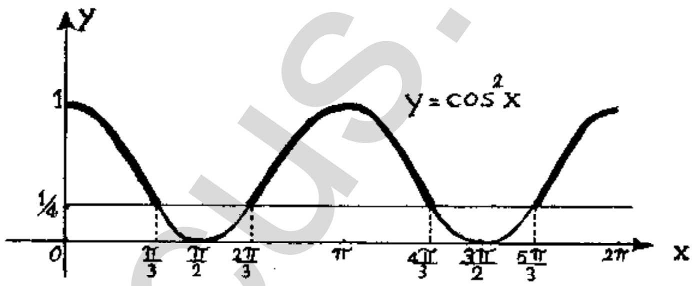
$\frac{dy}{dx} = -2\sin x.\cos x$ (gradient function)

Let $\frac{dy}{dx} = 0$ To get the stationary turning points.

$\therefore \sin x = 0$ or $\cos x = 0$

$x = 0 , \pi , 2\pi$ $x = \pi/2 , 3\pi/2$

$y = 0 , 0 , 0$ $y = 1 , 1$



$\therefore$ **The general solution is:** $0 \le x < \pi/3$ **or** $2\pi/3 < x < 4\pi/3$ **or** $5\pi/3 < x \le 2\pi$

c) $\sin^2 x \le \frac{1}{2}$, $0 \le x \le 2\pi$

Note: $\sin^2 x = 1/2$ $\therefore \sin x = \pm 1/\sqrt{2}$

$\sin x = 1/\sqrt{2}$ or $\sin x = -1/\sqrt{2}$

$\therefore x = \pi/4$ or $x = 3\pi/4$ $\therefore x = 5\pi/4$ or $x = 7\pi/4$

$y = \sin^2 x$

***Domain:** the function is defined for $0 \le x \le 2\pi$.*

Turning points:

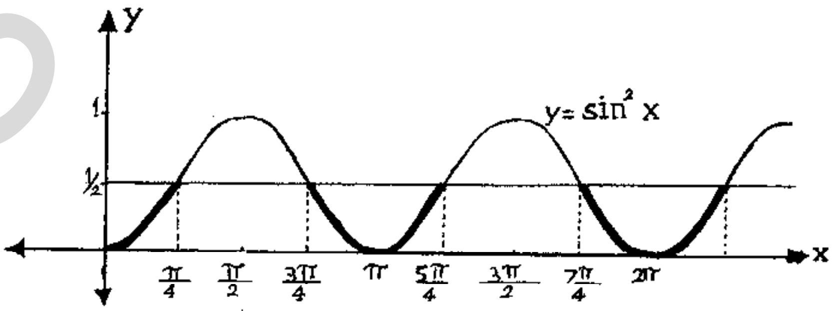
$dy/dx = 2\sin x.\cos x$ (gradient function)

Let $dy/dx = 0$ To get the stationary turning points.

$\therefore \sin x = 0$ or $\cos x = 0$

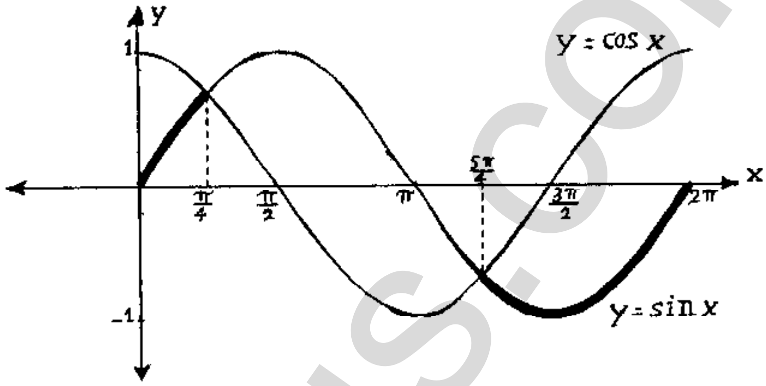
$x = 0 , \pi , 2\pi$ $x = \pi/2 , 3\pi/2$

$y = 0 , 0 , 0$ $y = 1 , 1$



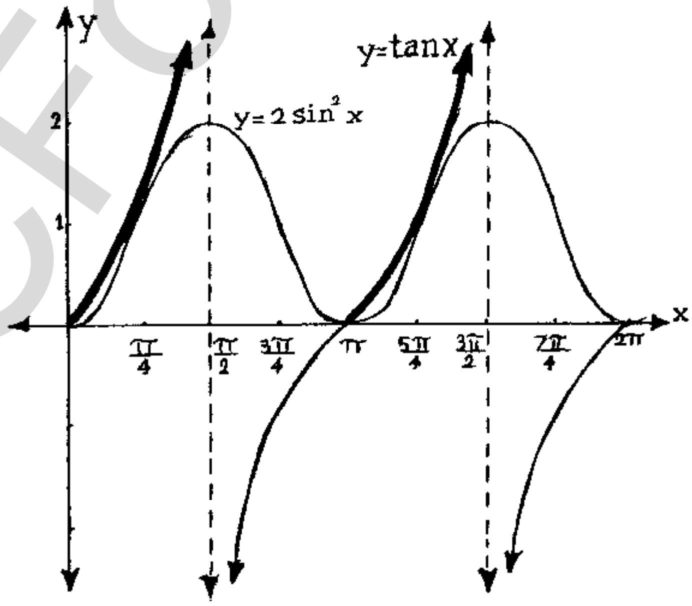
$\therefore$ **The general solution is:** $0 \le x \le \pi/4$ **or** $3\pi/4 \le x \le 5\pi/4$ **or** $7\pi/4 \le x \le 2\pi$

d) $\sin x < \cos x \quad , \quad 0 \le x \le 2\pi$
Note: $\sin x = \cos x \quad \therefore \tan x = 1$
$\therefore x = \pi/4 \quad \text{or} \quad x = 5\pi/4$



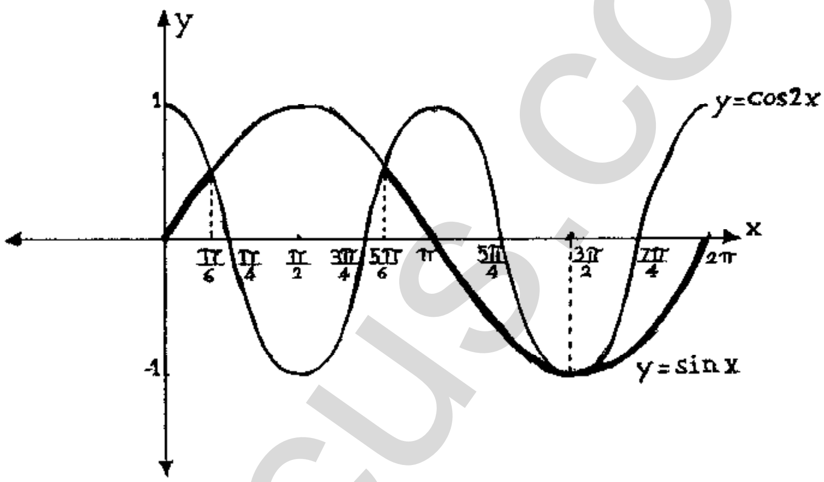
$\therefore$ **The general solution is: $0 \le x < \pi/4$ or $5\pi/4 < x \le 2\pi$**

e) $2\sin^2 x < \tan x \quad , \quad 0 \le x \le 2\pi$
Note: $2\sin^2 x = \tan x \quad \therefore \sin x(2\sin x\cos x - 1) = 0$
$\therefore \sin x = 0$ or $\sin 2x = 1$
$\therefore x = 0 , \pi$
$\therefore 2x = \pi/2 + 2k\pi$
$\therefore x = \pi/4 , 5\pi/4$



$\therefore$ **The general solution is: $0 < x < \pi/2$, except $x = \pi/4$ or $\pi < x < 3\pi/2$, except $x = 5\pi/4$.**

e)* $\sin x < \cos 2x \quad , \quad 0 \le x \le 2\pi$
Note: $\sin x = \cos 2x$ but $\cos 2x = 1 - 2\sin^2 x$
$\therefore 2\sin^2 x + \sin x - 1 = 0$
$\therefore \sin x = -1$ or $\sin x = 1/2$
$\therefore x = 3\pi/2$ $\quad\quad \therefore x = \pi/6 \ , \ 5\pi/6$



$\therefore$ The general solution is: $0 < x < \pi/6$ or $5\pi/6 < x < 2\pi$, except $x = 3\pi/2$.