Exercise 3

Extracts From

Taylors College COMPLEX NUMBERS Study Guide

+ ANSWERS

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Geometrical Interpretation of kz

$|kz| =$ _______
$\arg kz =$ _______
The point corresponding to kz liues on the same _______ through z and the origin, i.e., kz, z and O are _______ points.

Geometrical interpretation of $\mathbf{i^n z}$

$|iz| =$ _______, $\arg iz =$ _______
$|i^n z| =$ _______, $\arg i^n z =$ _______
z and $i^n z$ have the same _______ and lie on a _______
The effect of multiplying a complex number by i is an anticlockwise rotation of _______

Geometrical Representation of a Complex Number as a Vector.

Let the point Z represents z and O represents 0.
Consider the complex number $z = 1+i\sqrt{3}$. Then $|z| =$ _______ and $\arg z =$ _______.
Thus we can represent z by the vector $\overrightarrow{OZ}$ length _______ units which makes an angle of _______ with the positive x-axis.

The Triangle Inequality

1. T or F? $\arg(z_1 + z_2) = \arg z_1 + \arg z_2$
What is the geometrical significance of $|z_1 + z_2|$?
T or F? $|z_1 + z_2| = |z_1| + |z_2|$
2. O, Z_1, Z_2 collinear with Z_1 between O and Z_2.
T or F? $\arg(z_1 + z_2) = \arg z_1 + \arg z_2$
T or F? $|z_1 + z_2| = |z_1| + |z_2|$
Complete:

$$\boxed{|z_1 + z_2| \_\_\_\_ |z_1| + |z_2|}$$

HW (i) If $z_1 = 3 + 4i$ and $z_2 = 13$, find the maximum value of $|z_1 + z_2|$.
(ii) If $|z_1 + z_2|$ has its greatest value, and also $0 < \arg z_2 < \frac{\pi}{2}$, express z_2 in the form $x + iy$ where x and y are real. (86 Catholic Trial)

Subtraction of Vectors

T or F? $\arg(z_1 - z_2) = \arg(z_2 - z_1)$

Multiplication of Vectors

Given $z_1 = 2 + i$, $z_2 = -2 + 3i$, use both evaluation and vectors to find
(i) $z_3 = z_1 + z_2$ **(ii)** $z_4 = z_1 - z_2$ **(iii)** $z_5 = z_1 z_2$ **(iv)** $z_6 = iz_1$

Locus in the Complex Plane Q1-2

1. Let $z = x + iy$ then $|z| =$ _______ $= r$ i.e., _______ $= r^2$.
The graph of $|z| = r$ is a _______.
2. $|z - z_1| = r$ is a _______.

ANSWERS

Geometrical Interpretation of kz

$|kz| = |k||z|$, $\arg kz = \arg k + \arg z = 0 + \arg z$ or $\pi + \arg z$.
The point corresponding to kz lies on the same line through z and the origin, i.e., kz, z and 0 are collinear points.

Geometrical Interpretation of $\mathbf{i^n z}$

$|iz| = |i^n z| = |z|$, $\arg iz = \frac{\pi}{2} + \arg z$, $\arg i^n z = \frac{\pi}{2}(n \mod 4) + \arg z$. z and $i^n z$ have the same modulus and lie on the same circle. The effect of multiplying a complex number by i is an anticlockwise rotation of $\frac{\pi}{2}$.

Geometrical representation of a complex number as a vector

$z = 1 + i\sqrt{3} \Rightarrow |z| = 2$ and $\arg z = \frac{\pi}{3}$. Thus we can represent Z by the vector $\overrightarrow{OZ}$ length 2 units which makes an angle of $\frac{\pi}{3}$ with the positive x-axis.

Triangle inequality

1. F. $|z_1 + z_2|$ is the length of the diagonal of a parallelogram formed from vector addition of z_1 and z_2. F
2. F. T.

$$\boxed{|z_1+z_2| \le |z_1|+|z_2|}$$

HW (i) 18 **(ii)** $\frac{52}{5}+\frac{39}{5}i$

Subtraction of vectors

F

Multiplication of vectors

(i) $4i$ **(ii)** $4-2i$ **(iii)** $-7+4i$ **(iv)** $-1+2i$

Locus in the Complex Plane Q1-2

1. $|z|=\sqrt{x^2+y^2}$, $x^2+y^2=r^2$. The graph of $|z|=r$ is a circle of radius r, centre 0.
2. $|z-z_1|=r$ is a circle, centre z_1, radius r.