2006
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

General Mathematics

General Instructions

- o Reading Time- 5 minutes
- o Working Time - 2½ hours
- o Write using a blue or black pen
- o Calculators may be used
- o A Formulae Sheet is provided at the back of this paper.

Total marks - 100

Section I   pages   2 – 9

22 marks

- o Attempt Questions 1-22
- o Allow about 30 minutes for this section
- o Answer on the multiple choice answer sheet provided

Section II   pages   10 - 21

78 marks

- o Attempt questions 23 – 28
- o Allow about 2 hours for this section
- o Answer on writing paper.

Section I

Total marks (22)

Attempt Questions 1-22

Allow about 30 minutes for this section

Use the multiple choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample   **2 + 4 = ?**   **(A) 2 (B) 6 (C) 8 (D) 9**

A ⬤   B ⬤   C ◯   D ◯

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ⬤   B ⊗   C ◯   D ◯

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct

A ⬤   B ⊗   C ◯   D ◯

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1. Which is true for the set of scores?

1, 4, 4, 6, 9, 11, 13, 15, 15, 15

(A) The mode is 4 and the median is 9.
(B) The mode is 4 and the median is 10.
(C) The mode is 15 and the median is 9.
(D) The mode is 15 and the median is 10.

2. Solve the equation.

$$6x - 4 = 4x + 12$$

(A) $x = 16$
(B) $x = 8$
(C) $x = 1.6$
(D) $x = 0.8$

3. Marcus follows four teams in four different sports. Each team's recent record of wins and losses is shown below.

| Sport	Team	Record
Rugby League	Leopards	14 wins from 20 games
Rugby Union	Banksias	6 wins and 2 losses
Basketball	Emperors	A 65% success rate
Aussie Rules	Geese	Lost $\frac{1}{3}$ of their games

Based on recent records, which team has the greater probability of winning their next game?

    (A)   Leopards
    (B)   Banksias
    (C)   Emperors
    (D)   Geese

4. The photograph below shows a car which is 3.6 metres long.

The scale of the photograph is nearest to:

    (A)   1 : 3
    (B)   1: 12
    (C)   1 : 30
    (D)   1 : 36

5. Two ships sail from a port P.
    Ship A sails 20 km on a bearing 049
    Ship B sails 15 km on a bearing 139
    The distance AB between the ships is :
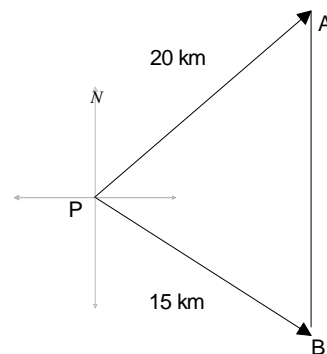
    (A)   12 km
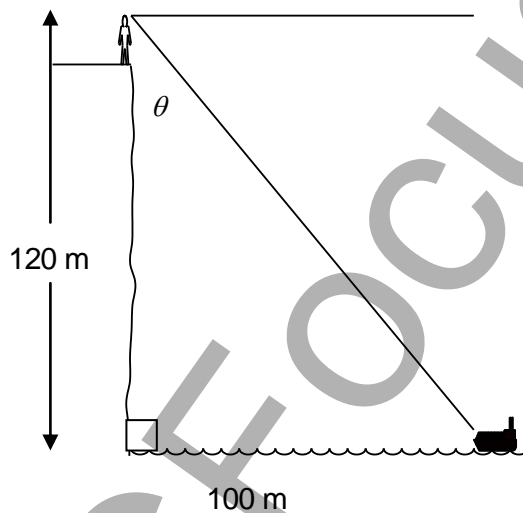   (B)   5.5 km
    (C)   35 km
    (D)   25 km

6. Rebecca is a salesperson and is paid a retainer of \$450.00 per week plus a commission of 12% of all sales over \$5 000.00. What is she paid for a week in which her sales were \$7 400.00?

(A) \$ 1 338
(B) \$ 942
(C) \$ 738
(D) \$ 1050

7. Expand and simplify $3x^2 - 10xy - 4x(2y - 3x)$
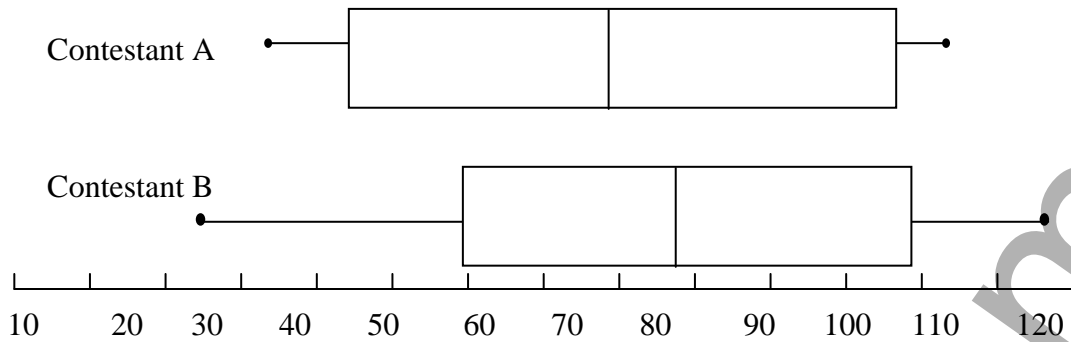
(A) $15x^2 - 18xy$
(B) $-9x^2 - 18xy$
(C) $15x^2 - 2xy$
(D) $-9x^2 - 2xy$

8. Find the size of the angle θ to the nearest minute.

120 m

100 m

`

(A) $56^o\ 26'$
(B) $39^o\ 48'$
(C) $33^o\ 56'$
(D) $50^o\ 12'$

9. The scores gained by two contestants A and B, in a competition over 12 rounds are shown in two box and whisker plots.

Which is **not** true ?

(A) Contestant B had the highest interquartile range.
(B) Contestant B had the highest median.
(C) Contestant B had the highest range.
(D) Contestant B had the highest score.

10. Calculate the value, after two years, of an investment of $ 40 000 which is invested at 8% p.a. compounding quarterly.

(A) $ 46 866.38
(B) $ 46 656.00
(C) $ 41 616.00
(D) $ 46 794.34

11. A game uses two special dice which are both coloured red on one face, blue on two faces and green on three faces. To start the game a player must roll green with the first die and anything except red on the second die. The probability of starting on a given roll would be given by:

(A) $\frac{1}{2} \times \frac{1}{6}$

(B) $\frac{1}{2} + \frac{1}{6}$

(C) $\frac{1}{2} \times \frac{5}{6}$

(D) $\frac{1}{2} + \frac{5}{6}$

12. The height of a student is given as 155 cm (to the nearest cm.) The percentage error in this measurement is:

(A) ± 0.65%
(B) ± 0.%
(C) ± 0.5%
(D) ± 0.32%

13. Use the table below to find the tax payable by Kristy who has a gross income of $30 000 and allowable deductions of $2 400.00

| Taxable income	Tax on this income
$0 – $6 000	NIL
$6 001 – $22 000	16 cents for each $1 over $6 000
$22 001 – $45 000	$2 560 plus 25 cents for each $1 over $22 000
$45 001 – $60 000	$8 310 plus 40 cents for each $1 over $45 000
$60 001 and over	$14 310 plus 48 cents for each $1 over $60 000

(A) $6 900
(B) $3 960
(C) $9 460
(D) $4 560

14. The distance that an object falls varies directly with the square of the time taken. An object falls 40 metres in 2 seconds. How far would the object fall in 4 seconds?

(A) 80 metres
(B) 160 metres
(C) 320 metres
(D) 120 metres

15. Find the present value of an annuity invested at 8% p.a. compounding annually for 4 years if it's future value is $20 000.

(A) $90 122.24
(B) $88 323.38
(C) $14 700.60
(D) $13 611.66

16. A survey of males and females showed their ownership of mobile phones. The results are summarised in the table below.

	Males	Females
Own a mobile phone	450	520
Don't own a mobile phone	250	180

If a person were chosen at random from this group, what is the probability that they would be a male who owned a mobile phone?

(A) $\frac{9}{28}$

(B) $\frac{5}{28}$

(C) $\frac{9}{14}$

(D) $\frac{1}{2}$

17. Which of the following graphs could have an equation $y = 2 - x^2$?

(A)

(B)

(C)

(D)
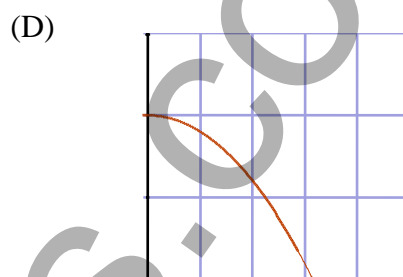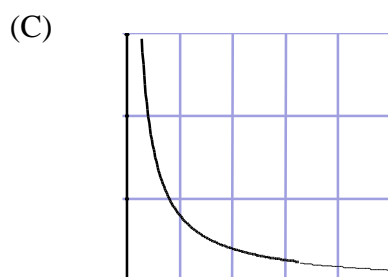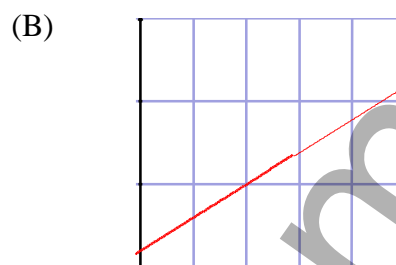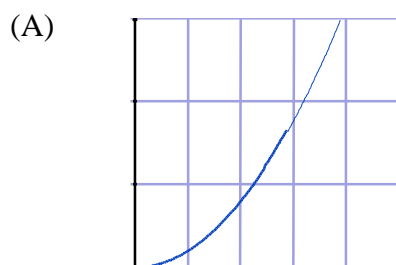
18. The course for a cross country race is shown below. The course is formed completely by straight lines and the two triangles are similar. What is the total length of the course?

300 m

1.2 km

600 m

800 m
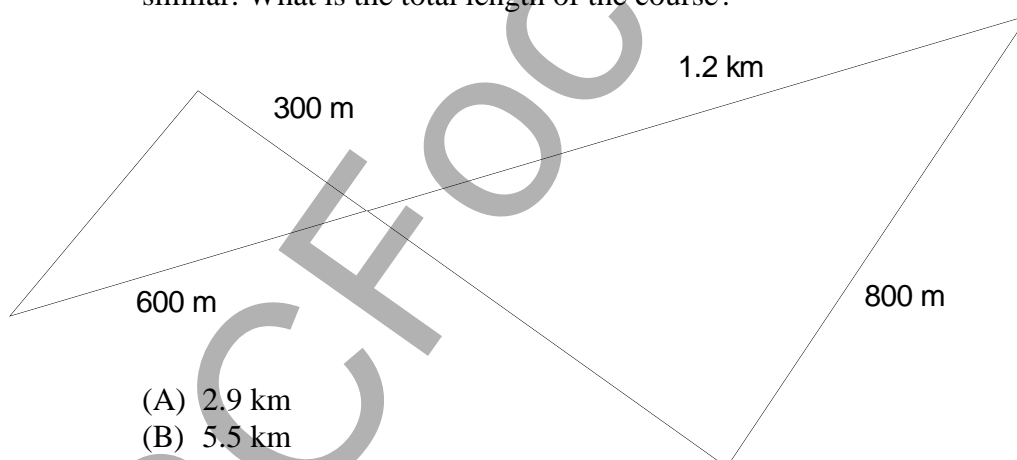
(A) 2.9 km
(B) 5.5 km
(C) 3.9 km
(D) 3.5 km

19. Town A is located at $(45°S, 45°W)$ and Town B at $(45°N, 45°W)$
Which is not correct?

(A) They are on the same large circle.
(B) They have the same local time.
(C) They are on the same small circle.
(D) They are on the same meridian of longitude.

20. Josie tosses three coins. What is the probability that at least two of the coins show a head?

(A) $\frac{5}{8}$

(B) $\frac{1}{2}$

(C) $\frac{3}{8}$

(D) $\frac{1}{8}$

21. Which scatter plot below shows a strong negative correlation.

(A)

(B)

(C)

(D)
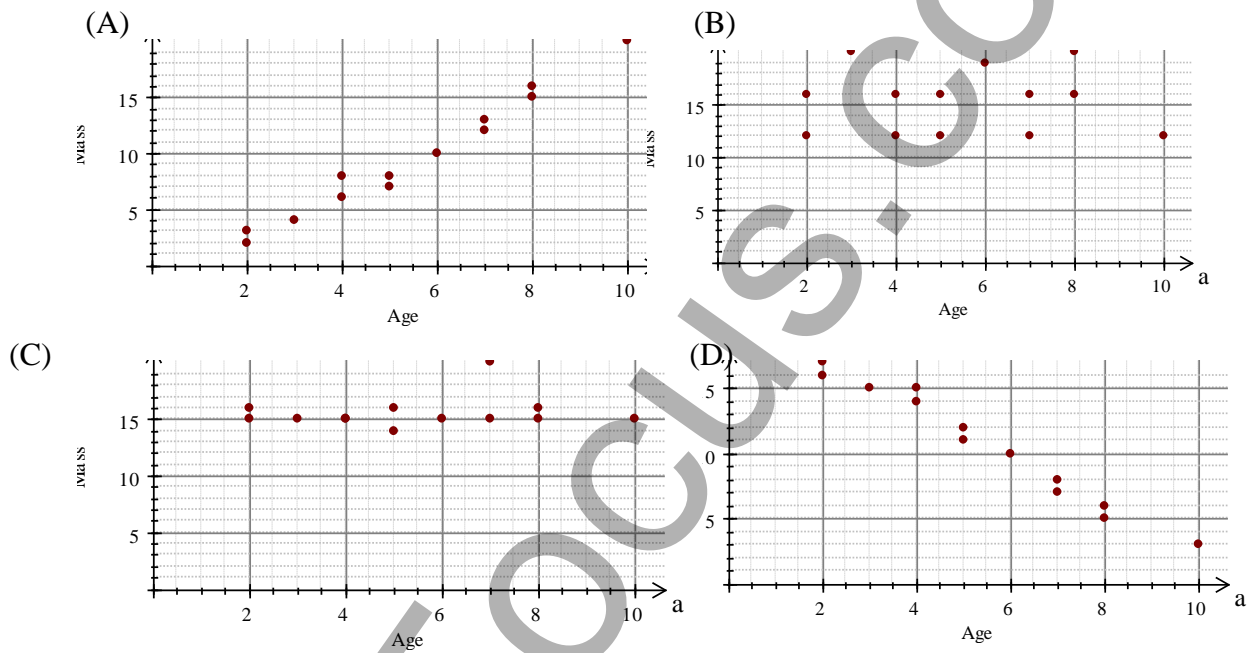
22. The radar chart shows the water level in two identically shaped tanks over a 24 hour period.

Water Level in Two Tanks
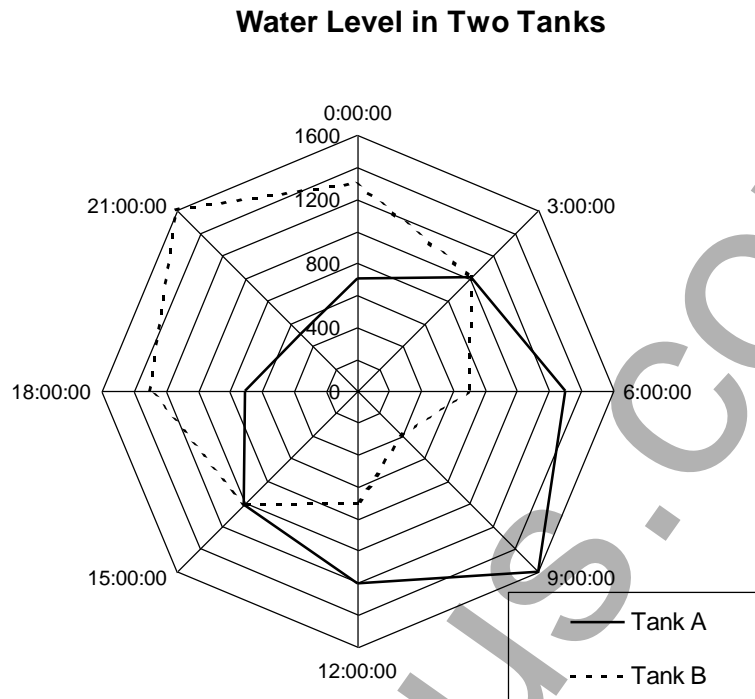
The greatest total volume of the two tanks taken together occurred at

(A)   21:00:00
(B)   18:00:00
(C)   3:00:00
(D)   15:00:00

78 marks
Attempt Questions 23 - 28
Allow about 2 hours for this section.

Answer each question on a new sheet of paper with your name and the question number at the top of the page. Do not write on the back of sheets.

All necessary working should be shown in every question.

_____

Question 23 (13 marks) Start a new sheet of paper.

Marks

(a) "Wendys Ice-Cream Shop" has 12 different flavours available and sells, single double or triple-decker cones or tubs with one, two or three flavours.
Suzie and Alex go to "Wendys Ice-Cream Shop" and each order an ice-cream.
Suzie orders a triple-decker cone and Alex orders a tub with three scoops.

    i)   The order that the triple-decker is arranged is important to Suzie as she will eat the top flavour first then the second and third. How many different arrangements of flavours are there from which she can choose if she wishes to have three different flavours?   **1**

    ii)   How many different arrangements of flavours are there from which Suzie can choose if she is prepared to have the same flavour repeated twice or even three times?   **1**

    iii)   How many different combinations of flavours can Alex choose in his tub if he wants three different flavours? The order doesn't matter as he can eat them in any succession.   **2**

(b) Homer and Marjorie and their family are leaving on a holiday. The day before leaving, Marjorie bought some magazines for the children to read on the way which cost $12.50 in total and Homer filled up the petrol tank with fuel costing $1.20/litre. After leaving on the holiday they discover after travelling 25 km to Brownville, that they have left the magazines at home.

    i.   If the car has a consumption of 15 litres/100 km, how much fuel would be used to return home and collect the magazines and return to Brownville?   **1**

    ii. How much would it cost in fuel to return to get the magazines?   **1**

    iii. How many litres of fuel could be bought for the $12.50 cost of the magazines? (Answer correct to 2 decimal places)   **1**

    iv. After what distance would it be cheaper to buy new magazines, rather than go back and get the original ones? (Nearest km)   **1**

Question 23 (continued)

(c) In the TV quiz Program "Who Wants to be a Millionaire" a contestant, Mike, has no idea of the answer to a question for which he is given four possible answers A, B, C and D.

He is offered three "Lifelines".

Fifty - Fifty: He can choose to have two of the incorrect answers removed allowing him to guess from the two remaining answers.

Phone a Friend : He can phone a friend who has a 70% chance of answering the question correctly.

Ask the Audience : He can ask the audience who have a 60% chance of answering the question correctly.

He decides to use the strategy of choosing one of the "Lifelines" at random. If the choice is Phone a Friend or Ask the Audience he will take the advice given, or he will guess from the remaining two answers if it is Fifty-Fifty.

i   Copy and complete the tree diagram for this strategy by writing the probabilities on the branches.

1
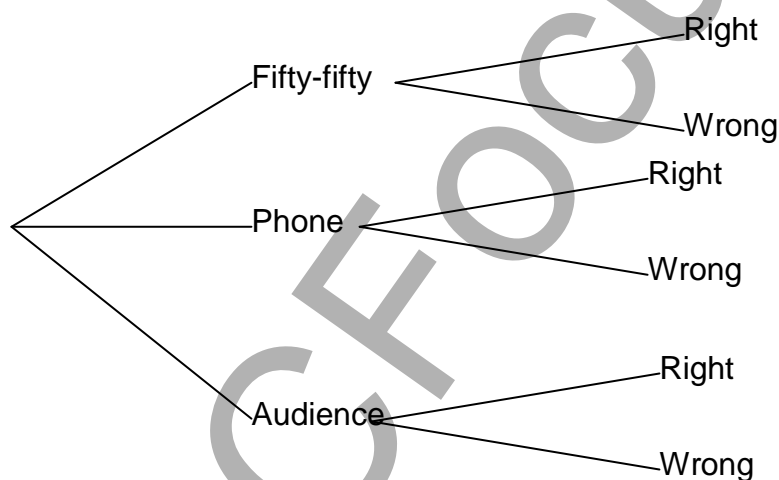
ii   What is the probability that he will answer the question correctly using this strategy?

2

iii   If he didn't have the Fifty-Fifty option but used the same strategy with the other two "Lifelines" what is the probability he will get the question right?

2

Question 24 (13 marks) Start a new sheet of paper.

(a) The graphs below compare the blood pressure of two groups of volunteers in a medical study. The trial group undertook a structured program of exercise and diet and the control group maintained their previous habits.
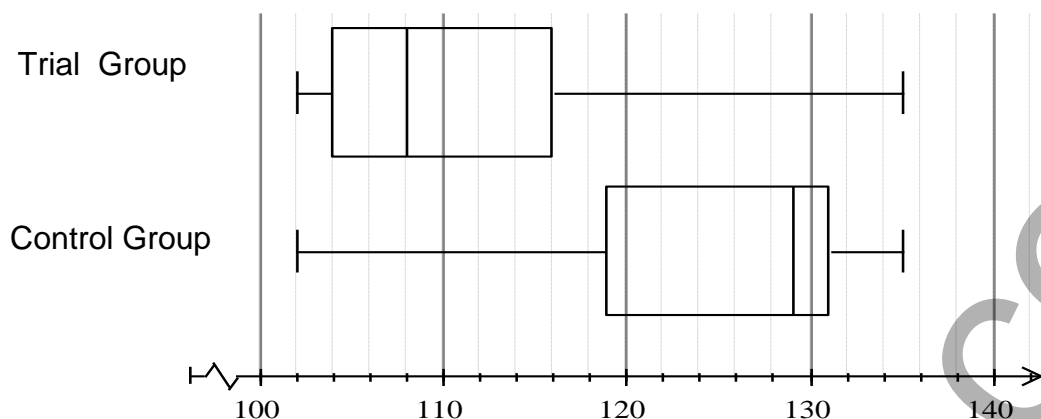

    i) What is the median of each group? **1**

    ii) Explain why the box and whisker plot gives a better picture of the results of the study than either the *range* or *interquartile range* on their own. **2**

    iii) Comment on the skewness of the results for the two groups. **1**

(b) The capacity of a spherical water tank can be approximated by the formula
$C = 500D^3$

- C is the capacity of the tank in litres
- D is the diameter of the tank in metres.

    i) The spherical pressure tank on top of a bore has diameter 0.4m. What is its capacity? **1**

    ii) A larger spherical storage tank has a capacity of 4000 litres. What is its diameter? **1**

    iii) How many of the bore pressure tanks could be filled from the larger storage tank? **1**

    iv) Re-write the formula so that D is the subject. **1**

    v) Re-write the formula so that R is the subject, where R is the radius of the tank. **1**

(c)   The graphs below show what percentage of all money was spent on advertising in different media in 2004, and how it increased for 2005.

Money Spent on Advertising in Various Media - 2004
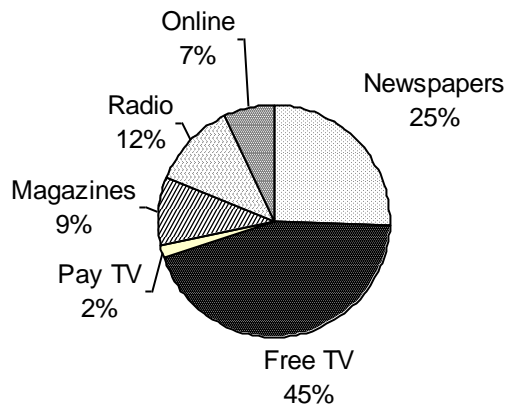
Percentage Increase in Money spent in 2005 compared to 2004 in Each Media
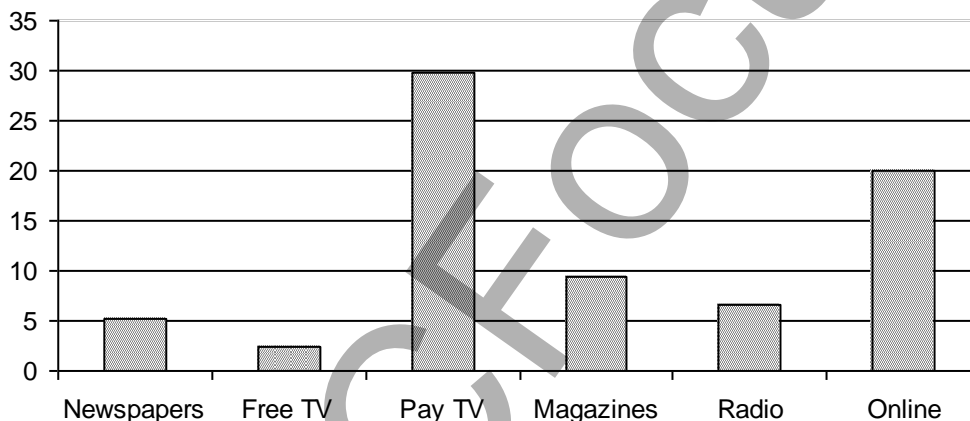
(i)   If $497 million was spent on Online advertising in 2004, how much was spent on Free TV in 2004?   **1**

(ii)   How much was spent on Online advertising in 2005 ? (to nearest $ million)   **1**

(iii)   If $112 million was spent on Pay TV and $840 million on radio in 2004, which of Radio or Pay TV had the biggest actual money increase between 2004 and 2005?   **2**

Question 25 (13 marks) Start a new sheet of paper.

(a)   The table below gives the future values of an annuity of $1 invested
       annually for different rates of interest over periods up to six years.

|	Interest Rate per annum.			
Years	5%	10%	15%	20%
1	1.00000	1.00000	1.00000	1.00000
2	2.05000	2.10000	2.15000	2.20000
3	3.15250	3.31000	3.47250	3.64000
4	4.31013	4.64100	4.99338	5.36800
5	5.52563	6.10510	6.74238	7.44160
6	6.80191	7.71561	8.75374	9.92992

i)   Use the above table to calculate the future value of an amount of
       $6000 invested annually in an annuity paying 10% pa for 4 years.
       (Answer to the nearest dollar.)   **1**

ii)   Denise wants to save up $40 000 for a house deposit in 5 years.
       Use the above table to find how much she would need to invest   **2**
       annually in an annuity paying 15% per annum to achieve this.

iii)   Joan has made annual payments of $4 000 into an annuity for 3
       years and its value is now $13 890. What was the annual interest   **2**
       rate on the annuity?

(b)   Pietro and Dominica are out walking and they come to a rectangular park
       which measures 200 metres from A to C with $\angle BAC = 35^\circ$. Pietro takes
       the short cut along the path that runs diagonally through the park from A to
       B, while Dominica walks around the perimeter path that goes from A to C
       and then to B.

A

35°

C   B
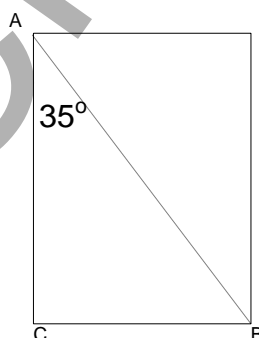
i)   How far does Dominica walk to get from A to B (nearest metre).   **1**

ii)   How much shorter, to the nearest metre, is the distance that Pietro   **2**
       walks from A to B? (to nearest metre)

Question 25 (continued.)

(c)   Steve plays the following game against a dealer. On each round of the game, the dealer draws a single card from a normal pack of 52 cards. Each round of the game costs Steve $2.00. The card is returned to the pack and shuffled before the next round.

If the card is an Ace, Steve receives $4 back.
If it is a picture card (K, Q or J), Steve receives $3 back.
If it is a 7 Steve receives his money back.
If it is any other card he loses his $2 bet.

   i)   If Steve plays 40 rounds of the game, how many times would you   **1**
          expect an Ace to be drawn?

   ii)   What is the financial expectation of the game?   **2**

   iii)   How could the payment for an Ace be changed to make the game   **2**
          fair?

Question 26 (13 marks) Start a new sheet of paper.

(a) Two printing machines, worth $150 000 each, are bought in December 2005. In December of each year, beginning in 2006, the value of each printing machine is depreciated.

> Machine A is depreciated by 10% using the declining balance method of depreciation.
> Machine B is depreciated using the straight line method of depreciation, so that its value in 2012 is $60 000.

The depreciation for Machine A has been calculated and plotted on the graph on page 25 at the end of the examination paper. Detach this page and hand it in with your answer to Question 26.

    i) Plot the line showing the value of Machine B on the graph on page 25.   **2**

    ii) In which year after 2006 was the value of the Machines approximately the same?   **1**

    iii) Use the declining balance method to find the value of machine A in 2015.   **1**

(b) An extract from a credit card statement is shown below   **2**

Interest rate = 18% p.a.   Daily rate = 0.049%

Date	Credit	Debit	Balance
01/03/06			$750.00
12/03/06		$200 purchase	$950.00
18/03/06	$300 repayment		$650.00

If interest is calculated daily on the maximum daily balance, calculate the interest that will be due for the month of March.

(c) A group of employees of Super Sales Pty Ltd are measured for their pulse rate. The data is found to be normally distributed with a mean of 55 and a standard deviation of 6.

    (i) Joanne has a pulse rate of 43. Convert this to a z - score.   **1**

    (ii) Adam has a z – score of 1. What was his pulse rate?   **1**

    (iii) What percentage of the employees had pulse rates between those of Joanne and Adam.   **2**

Question 26 (continued)

(d) Yasmin receives a lump sum of $200 000 when she retires. She uses it to purchase an annuity which pays 9% p.a. compounded monthly, to give her a monthly payment each month for the next 15 years.

(i) To find the amount of her monthly payment M, she substitutes into the Present value formula below. Write down the values of N, r and n which will be substituted into the formula. **1**

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

(ii) Use the Present value formula to find the amount of the monthly payment M (answer to the nearest dollar). **2**

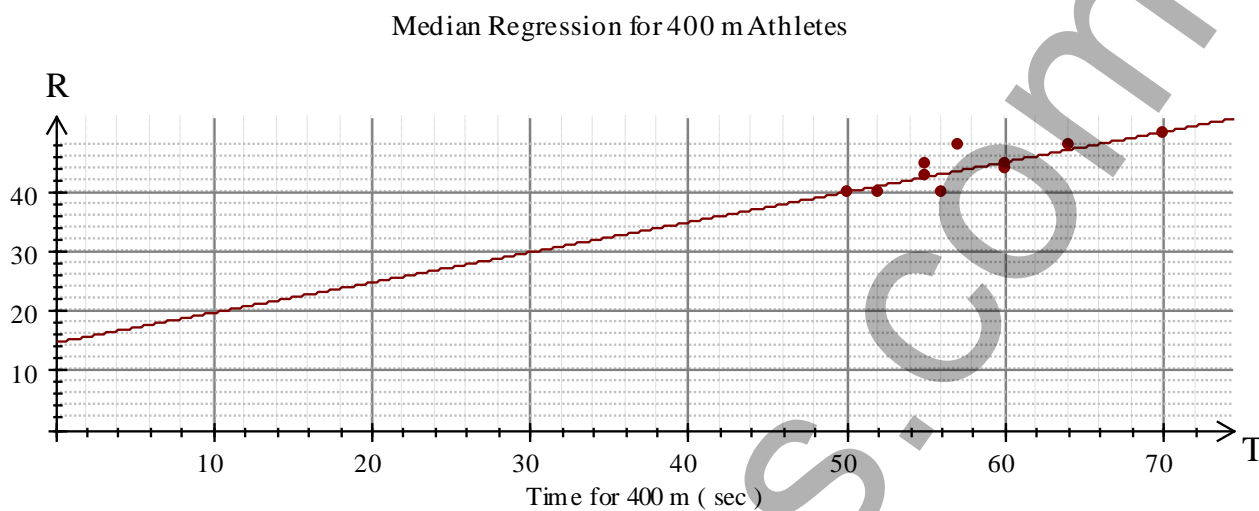**Question 27** (13 marks) Start a new sheet of paper.

(a)   The points scored by the Newtown Jets in their Home and Away rugby league
games are shown on the stem and leaf plot below.

Leaf (Home Games)	Stem	Leaf (Away Games)
9 8 5	0	1 4 5
5	1	1 8 9
7 3 2	2	3 6 6 9
3 0 0	3	

   i)   Find the mean and standard deviation of the points scored in the Home
games and in the Away games (correct to one decimal place).   **2**

   ii)   What do the means and standard deviations in part i) indicate about the
team's performance in Home games compared to in Away games?   **2**

(b)   Two ships are sailing across the Pacific Ocean. They cross the equator
simultaneously, at different longitudes. Ship A is at 160° E and Ship B is at
155° W.
(Radius of Earth is 6400 km. 1.852 km = 1 nautical mile)

   i)   What is the shortest distance between the ships, to the nearest nautical
mile?   **2**

   ii)   If the local time is 6 a.m. on Saturday 29th March on Ship A, what is the
local time on ship B.   **2**

(c)   A farm is surveyed using a radial survey. One triangular section AOB of the
surveyed land is shown below. Find the area of the section, correct to 3 significant
figures.   **2**

Question 27 (continued)
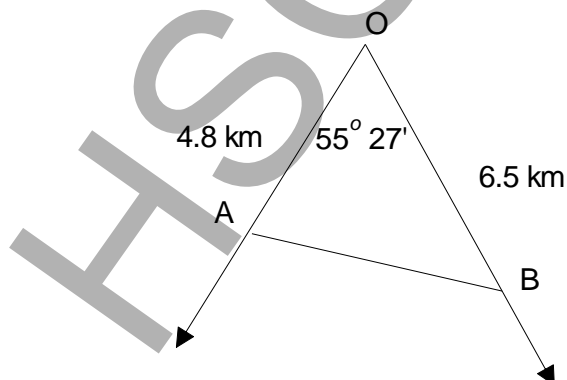
(d)   The resting heart rate for a group of ten 400 m Athletes was compared with their most recent times for the race. The result is shown on the scatter graph below with the Median Regression line drawn.

Median Regression for 400 m Athletes


Time for 400 m (sec)

i)   Give the equation of the median regression line in the form
            R = mT + b.

2

ii)   If an athlete had a 400 m time of 80 seconds, estimate his resting heart rate using this equation.

1

Question 28 (13 marks) Start a new sheet of paper.

(a)   An observatory is in the shape of a cylinder with a hemisphere (half of a sphere) on top as shown in the diagram below. There is a slot in the roof for the telescope to view the night sky. The outside diameter of the base is 30 metres and the height of the vertical walls is 18 metres.


18 metres

30 metres
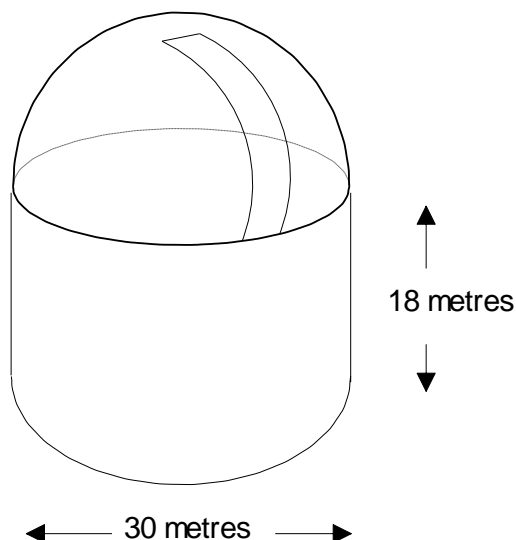
i)   The walls are to be constructed of solid concrete, one metre thick. Find the volume of the concrete in the walls, to the nearest cubic metre.

1

ii)   The hemispherical roof is to be tiled in ceramic tiles. If the slot removes 5% of the original surface area of the roof, find the area to be tiled, to the nearest square metre.

2

iii)   The inside surface of the roof is also hemispherical and fits exactly on top of the inside edge of the walls. Find the internal volume of the observatory building, to the nearest cubic metre.

2

iv)   Internal walls, floors and equipment take up 3500 cubic metres, with the remaining volume being air. An air conditioning unit circulates 120 000 litres of air per minute. How long would it take to theoretically circulate all the air in the observatory (to the nearest ten minutes)?
(One m^3 contains 1000 litres of air.)

Question 28 (continued)

(b)   The graph below shows the height in metres of a rocket and a plane which take off
at the same time, for the first 6 seconds of their flight.
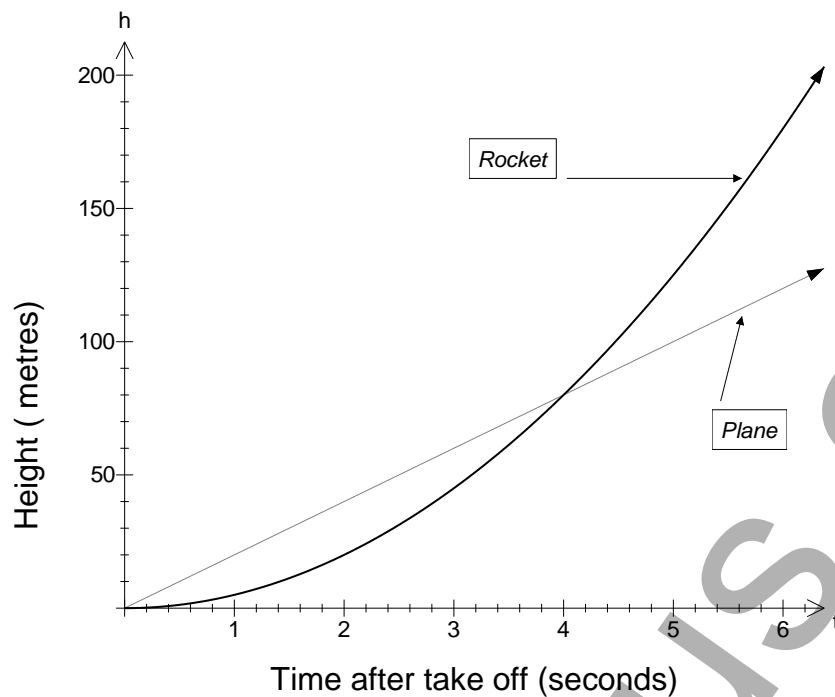
i)   After how many seconds are the plane and the rocket at the
same height, and what is this height?

2

ii)   Which has climbed the greater distance in the first 2 seconds
and in the first 6 seconds? What does this tell you about the
speed of the rocket?

2

iii)   After 6 seconds the relative positions of the two are shown on
the diagram below. Use the cosine rule to find the distance RP
between the plane and the rocket to the nearest metre.

2
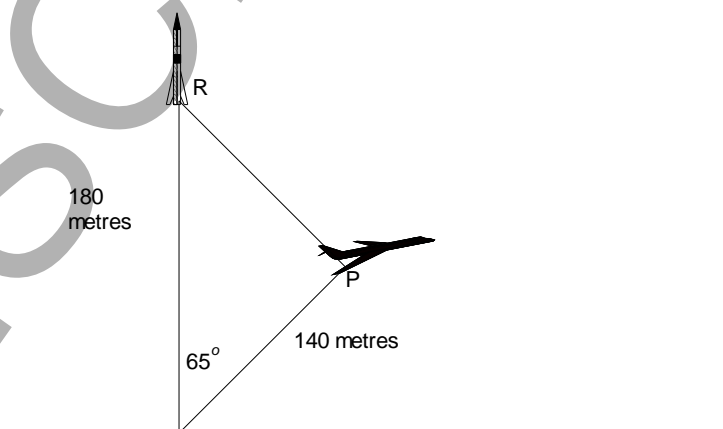
End of Exam

FORMULAE SHEET

Area of an annulus

$$A = \pi\left(R^2 - r^2\right)$$

R = radius of outer circle

r = radius of inner circle

Area of an ellipse

$$A = \pi ab$$

a = length of semi-major axis

b = length of semi-minor axis

Area of a sector

$$A = \frac{\theta}{360}\pi r^2$$

θ = number of degrees in central angle

Arc length of a circle

$$l = \frac{\theta}{360}2\pi r$$

θ = number of degrees in central angle

Simpson's rule for area approximation

$$A \approx \frac{h}{3}\left(d_f + 4d_m + d_l\right)$$

h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Surface area

Sphere $\qquad A = 4\pi r^2$

Closed cylinder $\qquad A = 2\pi rh + 2\pi r^2$

r = radius

h = perpendicular height

Volume

Cone $\qquad V = \frac{1}{3}\pi r^2 h$

Cylinder $\qquad V = \pi r^2 h$

Pyramid $\qquad V = \frac{1}{3}Ah$

Sphere $\qquad V = \frac{4}{3}\pi r^3$

r = radius

h = perpendicular height

A = area of base

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle

$$A = \frac{1}{2}ab\sin C$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

FORMULAE SHEET

Simple interest

$$I = Prn$$

P = initial quantity

r = percentage interest rate per period, expressed as a decimal

n = number of periods

Compound interest

$$A = P(1+r)^n$$

A = final balance

P = initial quantity

n = number of compounding periods

r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$$S = V_0 - Dn$$

S = salvage value of asset after n periods

V_0 = purchase price of the asset

D = amount of depreciation apportioned per period

n = number of periods

Declining balance formula for depreciation

$$S = V_0(1-r)^n$$

S = salvage value of asset after n periods

r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x}$ = mean

x = individual score

n = number of scores

f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form of a straight line

$$y = mx + b$$

m = gradient

b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Multiple Choice Answer Sheet

Name _____

Completely fill the response oval representing the most correct answer.

1.   A ◯   B ◯   C ◯   D ◯

2.   A ◯   B ◯   C ◯   D ◯

3.   A ◯   B ◯   C ◯   D ◯

4.   A ◯   B ◯   C ◯   D ◯

5.   A ◯   B ◯   C ◯   D ◯

6.   A ◯   B ◯   C ◯   D ◯

7.   A ◯   B ◯   C ◯   D ◯

8.   A ◯   B ◯   C ◯   D ◯

9.   A ◯   B ◯   C ◯   D ◯

10. A ◯   B ◯   C ◯   D ◯

11. A ◯   B ◯   C ◯   D ◯

12. A ◯   B ◯   C ◯   D ◯

13. A ◯   B ◯   C ◯   D ◯

14. A ◯   B ◯   C ◯   D ◯

15. A ◯   B ◯   C ◯   D ◯

16. A ◯   B ◯   C ◯   D ◯

17. A ◯   B ◯   C ◯   D ◯

18. A ◯   B ◯   C ◯   D ◯

19. A ◯   B ◯   C ◯   D ◯

20. A ◯   B ◯   C ◯   D ◯

21. A ◯   B ◯   C ◯   D ◯

22. A ◯   B ◯   C ◯   D ◯

Comparision of the Depreciation of Two Printing Machines

Machine A   Machine B