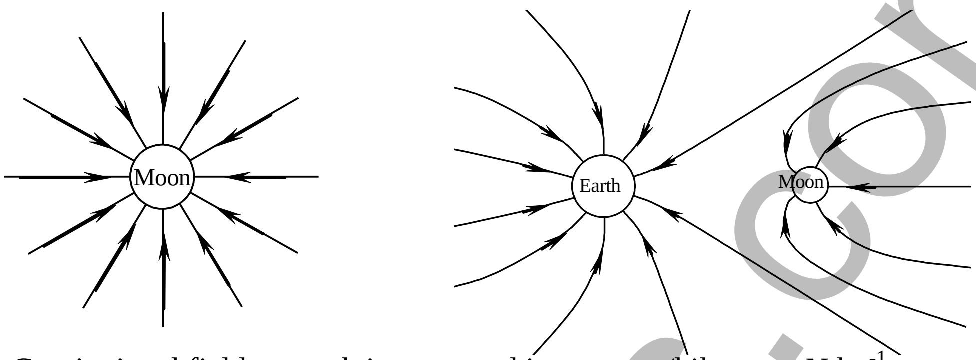
YEAR 12 PHYSICS ANSWERS TO ASSIGNMENT #3

Q1. a) A gravitational field is a region within which a unit mass would experience a force (its weight).

b) A gravitational field line of force represents the direction a unit mass would tend to move if placed at any point on the line (in the absence of any other forces).

c)



d) Gravitational field strength is measured in newtons/kilogram, N kg^{-1}.

e) Since the force acting on a 1-kg mass at the Earth's surface is 9.8 N (downwards), so the gravitational field strength on the surface is 9.8 N kg^{-1}.

f) The mass of the astronaut is 90 kg, and will be 90 kg anywhere in the Universe!

g) $F_W = m\,g \quad \therefore F_W = 120 \times 8.9 = 1\,068$ N

Q2. a) In order for any object to orbit the Earth it must possess sufficient orbital speed, in the direction perpendicular to the radius of its orbit. There is no way to launch an object from the Earth's surface to provide this orbital speed. But if the object was launched upwards at escape velocity or more, it would not return. [Compare it with Newton's mountain.]

b)
* The vertical ascent phase; the booster lifts the payload up through the first 100 km of atmosphere in a direct path, because almost all the air resistance is found there, so energy losses are limited. The roll manoeuvre, if required, occurs during this phase.
* The tip-over; the direction of the lift is slightly altered from the vertical. This is how the payload begins to accelerate around the Earth, rather than just away from it.
* The gravity turn; since the thrust force is no longer in the exact line of the weight force, the angle the rocket makes with the vertical continues to increase. This means that the tangential component of the thrust force becomes greater and greater.
* The vacuum phase; high enough so that there is effectively no drag force, the speed of the payload increases to reach orbital speed as the rocket exhausts its fuel.

c)

Factor	*Low-Earth orbit*	*Geosynchronous orbit*
orbital radius	250 – 500 km	41 900 km
orbital period	89 – 94 minutes	~ 24 hours [23 h 56 m 4 s]
orbital speed	~ 7.7 km s^{-1}	3.08 km s^{-1}

d) A stationary orbit is a geosynchronous circular orbit that is directly above the equator at all points, such that the satellite is always found at the same point in the sky. This is essential for communications satellites and satellite TV reception.

e) T (period of rotation of Earth) = 86 168 s.

$$r = \sqrt[3]{\frac{G\,m_E\,T^2}{4\pi^2}} = \sqrt[3]{\frac{(6.668\times10^{-11})(6.00\times10^{24})(86168)^2}{4\pi^2}} \qquad \therefore\ r = 4.22\times10^{7}\ \text{m}$$

f) $$v = \frac{2\pi\,r}{T} = \frac{2\pi\times(4.22\times10^{7})}{86168} = 3077\ \text{m s}^{-1}$$

Q3. (a) $$E_P = -\frac{G\,m_1\,m_2}{r} = \frac{-(6.668\times10^{-11})(6.0\times10^{24})\times10\,000}{(6380+370)\times10^{3}} = -5.93\times10^{11}\ \text{J}$$

(b) $$v^2 = \frac{G\,m}{r} = \frac{(6.668\times10^{-11})(6.0\times10^{24})}{(6380+370)\times10^{3}} \qquad \therefore\ v = 7.70\times10^{3}\ \text{ms}^{-1}$$

(c) $E_K = ½\,m\,v^2 \quad \therefore\ E_K = 0.5\times 10\,000\times(7.70\times10^3)^2 = 2.96\times10^{11}$ J

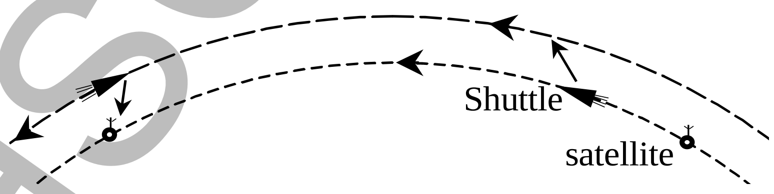
(d) $$E_P = -\frac{G\,m_1\,m_2}{r} = \frac{-(6.668\times10^{-11})(6.0\times10^{24})\times10\,000}{(6380+35\,800)\times10^{3}} = -9.485\times10^{10}\ \text{J}$$

(e) $$v^2 = \frac{G\,m}{r} = \frac{(6.668\times10^{-11})(6.0\times10^{24})}{(6380+35800)\times10^{3}} \qquad \therefore\ v = 3.08\times10^{3}\ \text{ms}^{-1}$$

$E_K = ½\,m\,v^2 \quad \therefore\ E_K = 0.5\times 10\,000\times(3.08\times10^3)^2 = 4.74\times10^{10}$ J

(f) $E_{TOTAL} = E_P + E_K = -2.96\times10^{11}$ J for the satellite in low-Earth orbit, and $E_P + E_K = -4.74\times10^{10}$ J for the satellite in a geosynchronous orbit. Note that in each case this is the same value as that of the E_K, but with the sign of the E_P.

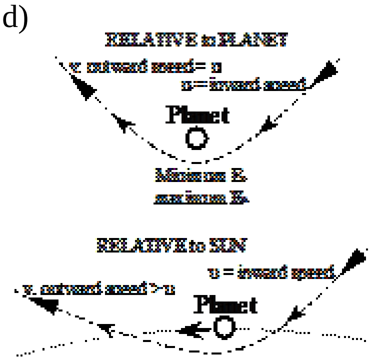
Q4. a) The pursuing Space Shuttle must fire its booster rocket! This increases the total energy of the shuttle E_{TOTAL} Since $E_{TOTAL} = E_P + E_K$ and E_P and E_K have opposite signs, the E_P is increased, while the E_K actually decreases! The Shuttle lifts to a higher orbit (E_P is ↑) and it moves slower within this orbit until the satellite catches it. It then fires its retro-rockets, reversing this process, and dropping to the initial orbit as its speed increases.



b) The firing of the rocket must be in the forward direction of the spacecraft as it orbits around Earth, but also must be in the direction of the Earth as it orbits around the Sun. In this way the spacecraft will have a speed greater than the 30 km s^{-1} orbital speed of the Earth, so its E_{TOTAL} will be greater, resulting in an increased E_P as it moves outward. Once it reaches Mars, it must lose a lot of kinetic energy to drop into a low-Mars orbit, so it will have to fire its retro rockets at that stage.

c) Since the spacecraft still has to escape from Earth, it can't fire its retro rockets! Instead it fires the rockets in the *forward* direction of the spacecraft as it orbits around Earth, but this must be in the direction *opposite to* that of the Earth's orbit around the Sun. In this way the spacecraft will have a speed *less than* the Earth's 30 km s^{-1} orbital speed, so its E_{TOTAL} will be less, resulting in a decreased E_P. Hence it moves inwards.

d)



As a spacecraft approaches a planet, since it has come from "infinity" it travels in a hyperbolic path. It is moving very quickly as it bypasses the planet, but relative to the planet its outward speed equals its inward speed. However, the planet has a velocity relative to the Sun, and therefore, if the angle of approach of the spacecraft is planned correctly, the "elastic collision" between planet and craft transfers a fraction of the planet's momentum to the craft, increasing its speed greatly.