Question 1

a. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• rearranges in terms of known trigonometric limit	1
• evaluates limit	1

Answer

$$\lim_{x\to 0}\frac{\sin 3x}{2x}=\tfrac{3}{2}\lim_{x\to 0}\frac{\sin 3x}{3x}=\tfrac{3}{2}\times 1=\tfrac{3}{2}$$

b. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• identifies a and r for the G.P	1
• applies formula for limiting sum	1

Answer

$\left(\frac{e}{e+1}\right)+\left(\frac{e}{e+1}\right)^2+\left(\frac{e}{e+1}\right)^3+\ldots$ is G.P. with $a=\frac{e}{e+1}$, and $r=\frac{e}{e+1}\Rightarrow 0<r<1$

$\therefore$ Limiting sum is $\frac{a}{1-r}=\frac{e}{e+1}\div\frac{1}{e+1}=e$

c. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
• expresses sum of reciprocals of roots in terms of sums of products	1
• evaluates using relationships between roots and coefficients	1

Answer

α, β and γ roots of $x^3+2x^2+3x+6=0$. $\quad\therefore \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\beta\gamma+\gamma\alpha+\alpha\beta}{\alpha\beta\gamma}=\frac{3}{-6}=-\frac{1}{2}$

d. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• substitutes values of gradients into formula for tangent of acute angle between the lines	1
• evaluates required angle	1

Answer

Acute angle θ between lines $y=2x$ and $x+y-3=0$ is given by $\tan\theta=\left|\frac{2-(-1)}{1+2.(-1)}\right|=3$

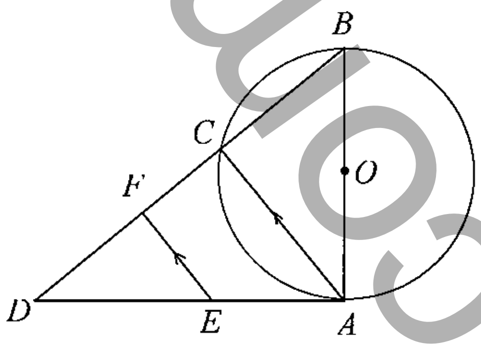
$\therefore\theta\approx 72°$ (to the nearest degree)

e. Outcomes assessed : PE2, PE3

Marking Guidelines

Criteria	Marks
i • quotes alternate segment theorem	1
ii • gives a sequence of deductions resulting in a test for a cyclic quadrilateral	1
• justifies these deductions by quoting geometric properties and tests	1
iii • explains why BE subtends a right angle at A or at F	1

Answer



Let O be the centre of the circle.

i. The angle between the tangent at A and the chord AC is equal to the angle subtended by that chord in the alternate segment, hence $\angle EAC = \angle ABC$.

ii. $\angle EAC = \angle DEF$ (*Corresp. ∠'s with parallel lines AC, EF are equal*)

$\therefore \angle DEF = \angle ABC$ (*Both equal to ∠EAC*)

$\therefore EABF$ is cyclic (*Exterior ∠ equal to interior opp. ∠*)

iii. $\angle BAE = 90°$ (*Tangent to circle ABC at A is perpendicular to radius OA drawn to point of contact*)

$\therefore BE$ is a diameter of circle $EABF$. (*subtends right ∠ at circumference*)

Question 2

a. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• finds primitive	1
• evaluates in surd form	1

Answer

$$\int_0^{\frac{\pi}{8}} \sec 2x \tan 2x \, dx = \tfrac{1}{2}\left[\sec 2x\right]_0^{\frac{\pi}{8}} = \tfrac{1}{2}\left(\sqrt{2}-1\right)$$

b. Outcomes assessed : PE3

Marking Guidelines

Criteria	Marks
• counts arrangements for one possible pattern of B's and G's	1
• adds number of arrangements for the second possible pattern of B's and G's	1

Answer

B G B G G B or B G G B G B $\therefore 2\times 3!\times 3! = 72$ ways

c. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• finds x coordinate of P	1
• finds y coordinate of P	1

Answer

$A(-2, 3) \quad B(6, -1)$

$3 : 2$

$$P\left(\frac{3\times 6+2\times(-2)}{3+2}, \frac{3\times(-1)+2\times 3}{3+2}\right)$$

$\therefore$ P has coordinates $P\left(\frac{14}{5}, \frac{3}{5}\right)$

d. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• simplifies $1-\cos x$ in terms of t	1
• completes simplification of given expression in terms of t to establish required result	1

Answer

$t = \tan\frac{x}{2}$

$$1-\cos x = 1-\frac{1-t^2}{1+t^2} = \frac{2t^2}{1+t^2}$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\therefore \frac{\sin x}{1-\cos x} = \frac{2t}{1+t^2}\times\frac{1+t^2}{2t^2} = \frac{1}{t} = \cot\frac{x}{2}$$

e. Outcomes assessed : PE3, PE4

Marking Guidelines

Criteria	Marks
i • finds $\frac{dy}{dx}$ as a function of t	1
• finds equation of normal in required form	1
ii • finds coordinates of M	1
• finds equation of locus of M	1

Answer

i.

$$y = at^2 \Rightarrow \frac{dy}{dt} = 2at$$

$$x = 2at \Rightarrow \frac{dx}{dt} = 2a$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = t$$

$\therefore$ Normal at P has gradient $-\frac{1}{t}$ and equation

$$y - at^2 = -\frac{1}{t}(x-2at)$$

$$ty - at^3 = -x + 2at$$

$$x + ty = 2at + at^3$$

ii. $N(0, 2a+at^2) \quad \therefore M(at, a+at^2)$

$P(2at, at^2)$

Locus of M has equation $y = a + a\left(\frac{x}{a}\right)^2$

$$x^2 = a(y-a)$$

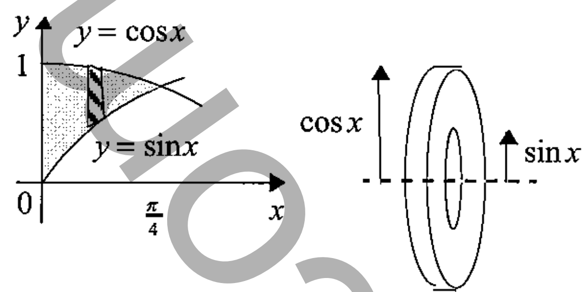
Question 3

a. Outcomes assessed : H5

Marking Guidelines

Criteria	Marks
• writes definite integral for the volume in terms of $\cos x$ and $\sin x$	1
• evaluates the integral.	1

Answer



$$V = \pi \int_0^{\frac{\pi}{4}} (\cos^2 x - \sin^2 x)\, dx$$
$$= \pi \int_0^{\frac{\pi}{4}} \cos 2x \, dx$$
$$= \tfrac{1}{2}\pi \left[\sin 2x \right]_0^{\frac{\pi}{4}}$$
$$= \tfrac{1}{2}\pi (1 - 0)$$

Volume is $\frac{\pi}{2}$ cubic units.

b. Outcomes assessed : HE2

Marking Guidelines

Criteria	Marks
• defines an appropriate sequence of statements $S(n)$ and shows the first member is true	1
• writes the LHS of $S(k+1)$ in terms of RHS of $S(k)$, conditional on truth of $S(k)$	1
• rearranges conditional expression for LHS of $S(k+1)$ to obtain RHS	1
• completes proof by Mathematical Induction	1

Answer

Let $S(n)$, $n = 2, 3, 4, \ldots$, be the sequence of statements defined by

$$S(n): \quad 2\times 1 + 3\times 2 + 4\times 3 + \ldots + n(n-1) = \frac{n(n^2-1)}{3}$$

Consider $S(2)$: $\quad LHS = 2\times 1 = 2$; $\quad RHS = \dfrac{2(2^2-1)}{3} = 2$. $\quad$ Hence $S(2)$ is true.

If $S(k)$ is true : $\quad 2\times 1 + 3\times 2 + 4\times 3 + \ldots + k(k-1) = \dfrac{k(k^2-1)}{3}$ $\quad$ *

Consider $S(k+1)$: $\quad LHS = \left\{2\times 1 + 3\times 2 + 4\times 3 + \ldots + k(k-1)\right\} + (k+1)k$

$$= \frac{k(k^2-1)}{3} + (k+1)k \qquad \text{if } S(k) \text{ is true, using *.}$$
$$= \frac{k(k+1)\left\{(k-1)+3\right\}}{3}$$
$$= \frac{(k+1)\left\{k^2+2k\right\}}{3}$$
$$= \frac{(k+1)\left\{(k+1)^2-1\right\}}{3}$$
$$= RHS$$

Hence if $S(k)$ is true then $S(k+1)$ is true. But $S(2)$ is true, and hence $S(3)$ is true and so on. Hence by Mathematical Induction, $S(n)$ is true for all positive integers $n \ge 2$.

c. Outcomes assessed : HE4

Marking Guidelines

Criteria	Marks
i • rearranges and interchanges x and y to obtain equation of inverse function	1
ii • sketches graph of $y = f(x)$ showing endpoints and intercepts	1
• sketches inverse function by reflection in $y = x$	1
• shows endpoints and intercepts for inverse function	1
iii • writes equation for x	1
• solves for x in simplest exact form	1

Answer

i. $f(x) = (x+2)^2 - 9,\ \ -2 \le x \le 2.$

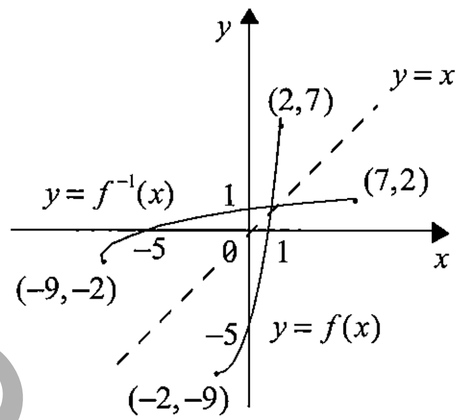
$(x+2)^2 = y + 9$ and $0 \le x + 2 \le 4$

$x + 2 = +\sqrt{y+9}$

$\therefore\ \ x = -2 + \sqrt{y+9}\ ,\ \ -9 \le y \le 7$

$\therefore x \leftrightarrow y\ \Rightarrow\ f^{-1}(x) = -2 + \sqrt{x+9}\ ,\ -9 \le x \le 7$

ii. Graphs of inverse functions are reflections of each other in $y = x$



iii. Graphs intersect on the line $y = x$.

Hence $(x+2)^2 - 9 = x$

$x^2 + 3x - 5 = 0$

$\therefore x > 0 \Rightarrow x = \dfrac{-3+\sqrt{29}}{2}$

Question 4

a. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
• writes expression for probability in terms of binomial coefficients	1
• evaluates required probability	1

Answer

$P(\textit{none in common}) = \dfrac{^{34}C_6}{^{40}C_6} \approx 0 \cdot 35$ (to 2 decimal places)

b. Outcomes assessed : HE6

Marking Guidelines

Criteria	Marks
• writes du in terms of dx and converts limits for x into limits for u	1
• finds equivalent definite integral in terms of u	1
• finds primitive and substitutes limits	1
• simplifies exact answer	1

Answer

$u = \sin^2 x$

$du = 2\sin x \cos x \, dx$

$du = \sin 2x \, dx$

$x = \frac{\pi}{4} \Rightarrow u = \frac{1}{2}$

$x = \frac{\pi}{3} \Rightarrow u = \frac{3}{4}$

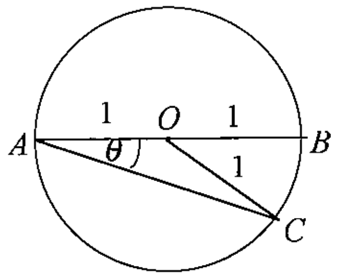
$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin 2x}{1+\sin^2 x}\, dx = \int_{\frac{1}{2}}^{\frac{3}{4}} \frac{1}{1+u}\, du$$
$$= \left[\ln(1+u)\right]_{\frac{1}{2}}^{\frac{3}{4}}$$
$$= \ln\tfrac{7}{4} - \ln\tfrac{3}{2}$$
$$= \ln\tfrac{7}{6}$$

c. Outcomes assessed : H5, PE3

Marking Guidelines

Criteria	Marks
i • finds area of ΔAOC in terms of $\sin 2\theta$	1
• uses area information to complete equation for θ	1
ii • shows that $f(0\cdot 4)$, $f(0\cdot 5)$ have opposite signs	1
• notes that f is continuous, and deduces equation has one root θ, $0\cdot 4<\theta<0\cdot 5$	1
iii • applies Newton's rule to write numerical expression for next approximation	1
• evaluates this approximation	1

Answer



i. $\angle OCA = \theta$ ($\angle$'s opp. equal sides are equal in ΔAOC)

$\angle AOC = \pi - 2\theta$ ($\angle$ sum of Δ is π)

$\angle BOC = 2\theta$ (adj. supp. $\angle$'s add to π)

Area sector BOC + Area $\Delta AOC = \frac{1}{4}$ *Area circle*

$$\therefore \tfrac{1}{2}\times 1^2 \times 2\theta + \tfrac{1}{2}\times 1^2 \times \sin(\pi - 2\theta) = \tfrac{1}{4}\times\pi\times 1^2$$
$$\theta + \tfrac{1}{2}\sin 2\theta - \tfrac{\pi}{4} = 0$$

ii. Let $f(\theta) = \theta + \frac{1}{2}\sin 2\theta - \frac{\pi}{4}$

$f(0\cdot 4) \approx -0\cdot 03 < 0$

$f(0\cdot 5) \approx 0\cdot 14 > 0$ and f is continuous

Also $f'(\theta) = 1 + \cos 2\theta > 0 \Rightarrow f$ monotonic increasing

$\therefore f(\theta) = 0$ for exactly one value of θ, $0\cdot 4 < \theta < 0\cdot 5$

iii. Since $f'(\theta) = 1 + \cos 2\theta$,

$$\theta \approx 0\cdot 4 - \frac{f(0\cdot 4)}{f'(0\cdot 4)}$$
$$\approx 0\cdot 4 - \frac{-0\cdot 0267}{1\cdot 6967}$$
$$\approx 0\cdot 42 \text{ (to 2 dec. pl.)}$$

Question 5

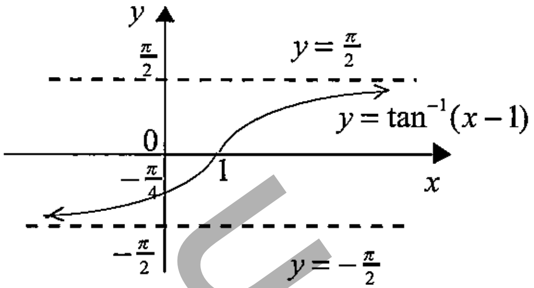
a. Outcomes assessed : HE4

Marking Guidelines

Criteria	Marks
i • shows correct shape and asymptotes	1
• shows intercepts on coordinate axes	1
ii • finds $\frac{dy}{dx}$ and evaluates for $x = 1$	1
• finds equation of tangent	1

Answer

i.



ii. $y=\tan^{-1}(x-1)$

$\frac{dy}{dx}=\frac{1}{1+(x-1)^2}$

$\therefore \frac{dy}{dx}=1$ when $x=1$

$\therefore$ Tangent at $(1,0)$ has gradient 1 and equation $y=x-1$

b. Outcomes assessed : HE5

Marking Guidelines

Criteria	Marks
i • shows by differentiation that a is constant	1
ii • integrates to find a primitive function for t in terms of x	1
• evaluates constant of integration using initial conditions then writes x as a function of t	1
iii • evaluates x at $t=2$ and $t=3$ to find distance travelled in third second.	1

Answer

i. $v=\sqrt{x} \Rightarrow \frac{1}{2}v^2=\frac{1}{2}x$

$\therefore a=\frac{d}{dx}(\frac{1}{2}v^2)=\frac{1}{2}$ for all x

Hence a is independent of x.

ii. $\frac{dx}{dt}=x^{\frac{1}{2}}$

$\frac{dt}{dx}=x^{-\frac{1}{2}}$

$t=2x^{\frac{1}{2}}+c$

$\left.\begin{matrix}t=0\\x=1\end{matrix}\right\} \Rightarrow c=-2$

$\therefore t=2\sqrt{x}-2$

$x=\frac{1}{4}(t+2)^2$

iii. Between $t=2$ and $t=3$, particle moves right from $x=4$ to $x=\frac{25}{4}$

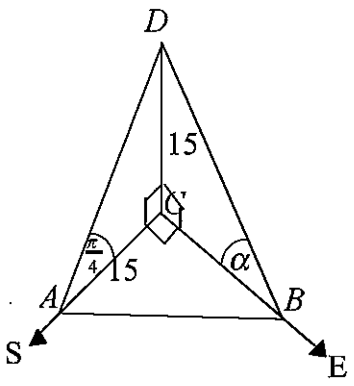
Distance travelled in third second is $2\cdot25$ m.

c. Outcomes assessed : H5, HE5, HE7

Marking Guidelines

Criteria	Marks
i • finds AC and finds BC in terms of $\cot\alpha$	1
• uses Pythagoras' theorem and an appropriate trig. identity to find AB in terms of $\operatorname{cosec}\alpha$	1
ii • differentiates AB with respect to t using chain rule or implicit differentiation	1
• substitutes given values and interprets result	1

Answer



i. In ΔACD,

$\angle DAC=\angle ADC=\frac{\pi}{4}$

$\therefore AC=15$.

In ΔBCD, $BC=15\cot\alpha$.

$\therefore$ In ΔABC,

$AB^2=15^2+15^2\cot^2\alpha$

$=15^2(1+\cot^2\alpha)$

$=15^2\operatorname{cosec}^2\alpha$

$\therefore AB=15\operatorname{cosec}\alpha$

ii. When $\alpha=\frac{\pi}{3}$,

$\frac{dAB}{dt}=-15\operatorname{cosec}\alpha\cot\alpha\frac{d\alpha}{dt}$

$=-15\times\frac{2}{\sqrt{3}}\times\frac{1}{\sqrt{3}}\times0\cdot01$

$=-0\cdot1$

$\therefore AB$ is decreasing at a rate of $0\cdot1\ \text{ms}^{-1}$

Question 6

a. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • integrates v with respect to t to find expression for x	1
• uses initial conditions to evaluate the constant of integration, giving x as a function of t	1
• differentiates v with respect to t to get $\ddot{x}$ then expresses $\ddot{x}$ in terms of x	1
ii • states period	1
• states extremities	1
iii • solves trig. equation to find time to first return	1

Answer

i. $v = -12\sin(2t + \frac{\pi}{3})$

$x = 6\cos(2t + \frac{\pi}{3}) + c$ $\quad\ddot{x} = -24\cos(2t + \frac{\pi}{3})$

$t = 0,\ x = 5 \Rightarrow c = 2$

$\therefore x = 2 + 6\cos(2t + \frac{\pi}{3})$ $\quad\therefore \ddot{x} = -4(x - 2)$

ii. Period is π seconds. $\quad -4 \le x \le 8$

iii. $x = 5 \Rightarrow \cos(2t + \frac{\pi}{3}) = \frac{1}{2}$

$2t + \frac{\pi}{3} = \frac{\pi}{3},\ 2\pi - \frac{\pi}{3}, \ldots$

$t = 0,\ \frac{2\pi}{3},\ \ldots$

First return after $\frac{2\pi}{3}$ seconds.

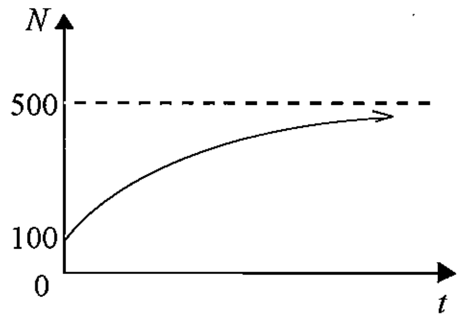
b. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • sketches graph of correct shape with correct vertical intercept	1
• shows asymptote for limiting population size	1
ii • differentiates with respect to t	1
iii • writes and solves equation for N	1

Answer

i.



ii.

$N = 500 - 400e^{-0\cdot 1t}$

$\frac{dN}{dt} = 0\cdot 1 \times 400e^{-0\cdot 1t}$

$= 0\cdot 1(500 - N)$

iii. Initial rate of growth is

$0\cdot 1(500 - 100) = 0\cdot 1 \times 400$

$\therefore$ want N such that

$0\cdot 1(500 - N) = 0\cdot 1 \times 200$

$500 - N = 200$

$N = 300$

c. Outcomes assessed : H5, HE4

Marking Guidelines

Criteria	Marks
• uses inverse trig. identity to simplify equation	1
• uses trig. expansion to evaluate x in terms of k	1

Answer

$\cos^{-1}x - \sin^{-1}x = k, \quad -\frac{\pi}{2} \le k \le \frac{3\pi}{2}$

$\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$

$\therefore 2\cos^{-1}x = k + \frac{\pi}{2}$

$\cos^{-1}x = \frac{k}{2} + \frac{\pi}{4}$

$x = \cos(\frac{k}{2} + \frac{\pi}{4})$

$\therefore x = \cos\frac{k}{2}\cos\frac{\pi}{4} - \sin\frac{k}{2}\sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}(\cos\frac{k}{2} - \sin\frac{k}{2})$

Question 7

a. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • uses integration to find expressions for $\dot{x}$ and x	1
• uses integration to find expressions for $\dot{y}$ and y	1
ii • writes simultaneous equations for V and θ	1
• finds the value of V	1
• finds the value of θ	1
iii • finds the values of $\dot{x}$ and $\dot{y}$ just before impact	1
• uses Pythagoras' theorem to find the magnitude of v	1
• uses trigonometry to find the direction of v as an angle relative to the horizontal	1

Answer

i.

$\ddot{x} = 0$

$\dot{x} = c_1$

$\therefore \dot{x} = V\cos\theta$

$x = Vt\cos\theta + c_2$

$\left.\begin{matrix} t=0 \\ x=0 \end{matrix}\right\} \Rightarrow c_2 = 0$

$\therefore x = Vt\cos\theta$

$\ddot{y} = -10$

$\dot{y} = -10t + c_3$

$\left.\begin{matrix} t=0 \\ \dot{y}=V\sin\theta \end{matrix}\right\} \Rightarrow c_3 = V\sin\theta$

$\therefore \dot{y} = -10t + V\sin\theta$

$y = -5t^2 + Vt\sin\theta + c_4$

$\left.\begin{matrix} t=0 \\ y=0 \end{matrix}\right\} \Rightarrow c_4 = 0$

$\therefore y = Vt\sin\theta - 5t^2$

ii. When $t = 8$

$\left.\begin{matrix} x=288 \\ y=64 \end{matrix}\right\} \Rightarrow \begin{matrix} 8V\cos\theta = 288 \\ 8V\sin\theta = 384 \end{matrix}$

$\therefore V^2(\cos^2\theta + \sin^2\theta) = 36^2 + 48^2$

$V = 60$

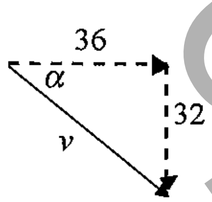
$\tan\theta = \frac{48}{36} = \frac{4}{3}$

$\theta = \tan^{-1}\frac{4}{3}$

iii. When $t = 8$

$\dot{x} = 60 \times \frac{3}{5} = 36$

$\dot{y} = -80 + 60 \times \frac{4}{5} = -32$



$v^2 = 36^2 + 32^2$

$v = 4\sqrt{145}$

$\tan\alpha = \frac{8}{9}$

$\alpha \approx 41 \cdot 6°$

Velocity of rocket just before impact is approximately 48 ms^{-1} inclined at 42° below the horizontal.

b. Outcomes assessed : HE3

Marking Guidelines

Criteria	Marks
i • writes a typical term in x^r in the expansion of the RHS of the identity	1
• collects like terms to find coefficient of x^r, then equates to coefficient of x^r on LHS	1
ii • writes single binomial coefficient for sum on LHS	1
• writes single binomial coefficient for sum on RHS then deduces result	1

Answer

i. $(1+x)^{m+n} = (1+x)^m(1+x)^n$

For $0 \le r \le m$ and $0 \le r \le n$,

terms in x^r in expansion of the RHS have the form ${}^mC_k x^k \times {}^nC_{r-k} x^{r-k}$, $k = 0, 1, 2, ..., r$.

Collecting such like terms gives the coefficient of x^r as $\sum_{k=0}^{r} {}^mC_k\, {}^nC_{r-k}$.

The coefficient of x^r in the expansion of the LHS is ${}^{m+n}C_r$.

Hence equating coefficients of x^r on both sides of the identity gives ${}^{m+n}C_r = \sum_{k=0}^{r} {}^mC_k\, {}^nC_{r-k}$.

ii. Using i., for $m \ge 2$ and $n \ge 2$,

${}^{m+1}C_0\, {}^nC_2 + {}^{m+1}C_1\, {}^nC_1 + {}^{m+1}C_2\, {}^nC_0 = {}^{(m+1)+n}C_2$ and ${}^mC_0\, {}^{n+1}C_2 + {}^mC_1\, {}^{n+1}C_1 + {}^mC_2\, {}^{n+1}C_0 = {}^{m+(n+1)}C_2$

$\therefore\ {}^{m+1}C_0\, {}^nC_2 + {}^{m+1}C_1\, {}^nC_1 + {}^{m+1}C_2\, {}^nC_0 = {}^mC_0\, {}^{n+1}C_2 + {}^mC_1\, {}^{n+1}C_1 + {}^mC_2\, {}^{n+1}C_0 = {}^{m+n+1}C_2$

Independent Trial Examination 2009 Mathematics Extension 1 Mapping Grid

Question	Marks	Content	Syllabus Outcomes	Targeted Performance Bands
1 a	2	Trigonometric functions	H5	E2-E3
b	2	Series and applications	H5	E2-E3
c	2	Polynomials	PE3	E2-E3
d	2	Angle between two lines	H5	E2-E3
e i	1	Circle geometry	PE3	E2-E3
ii	2	Circle geometry	PE2, PE3	E2-E3
iii	1	Circle geometry	PE3	E2-E3
2 a	2	Trigonometric functions	H5	E2-E3
b	2	Permutations and combinations	PE3	E2-E3
c	2	Division of an interval	H5	E2-E3
d	2	Trigonometric functions	H5	E2-E3
e i	2	Parametric representation	PE3, PE4	E2-E3
ii	2	Parametric representation	PE3, PE4	E2-E3
3 a	2	Trigonometric functions	H5	E2-E3
b	4	Induction	HE2	E3-E4
c i	1	Inverse functions	HE4	E2-E3
ii	3	Inverse functions	HE4	E2-E3
iii	2	Inverse functions	HE4	E2-E3
4 a	2	Further probability	HE3	E2-E3
b	4	Methods of integration	HE6	E2-E3
c i	2	Trigonometric functions	H5	E2-E3
ii	2	Polynomials	PE3	E2-E3
iii	2	Polynomials	PE3	E2-E3
5 a i	2	Inverse functions	HE4	E2-E3
ii	2	Inverse functions	HE4	E2-E3
b i	1	Velocity and acceleration as functions of displacement	HE5	E2-E3
ii	2	Velocity and acceleration as functions of displacement	HE5	E2-E3
iii	1	Velocity and acceleration as functions of displacement	HE5	E2-E3
c i	2	Trigonometric functions	H5	E3-E4
ii	2	Rates of change	HE5, HE7	E3-E4
6 a i	3	Simple harmonic motion	HE3	E3-E4
ii	2	Simple harmonic motion	HE3	E3-E4
iii	1	Simple harmonic motion	HE3	E3-E4
b i	2	Exponential growth and decay	HE3	E2-E3
ii	1	Exponential growth and decay	HE3	E2-E3
iii	1	Exponential growth and decay	HE3	E2-E3
c	2	Trigonometric functions, inverse functions	H5, HE4	E2-E3
7 a i	2	Projectile motion	HE3	E3-E4
ii	3	Projectile motion	HE3	E3-E4
iii	3	Projectile motion	HE3	E3-E4
b i	2	Binomial theorem	HE3	E3-E4
ii	2	Binomial theorem	HE3	E3-E4