BAULKHAM HILLS HIGH SCHOOL
EXTENSION 1 TRIAL HSC 2011 SOLUTIONS

Solution	Marks	Comments
QUESTION 1		
1a) $-1 \le \frac{x}{3} \le 1$ $-3 \le x \le 3$	1	
1b) $\tan 55^\circ = \tan(45+10)$ $= \frac{\tan 45^\circ + \tan 10^\circ}{1 - \tan 45^\circ \tan 10^\circ}$ $= \frac{1+p}{1-p}$	1	
1 c) $(-5, 6)$ $(4, -3)$ $3 : 1$ $P = \left(\frac{-5\times1+4\times3}{3+1}, \frac{6\times1-3\times3}{3+1}\right)$ $= \left(\frac{7}{4}, -\frac{3}{4}\right)$	2	**1 mark** • Uses a correct method
1d) $\frac{2x}{x-2} \le 3$ $x - 2 \ne 0$ $x \ne 2$ $\frac{2x}{x-2} = 3$ $2x = 3x - 6$ $x = 6$  $x < 2$ *or* $x \ge 6$	3	**3 marks** • Correct graphical solution on number line or algebraic solution, with correct working **2 marks** • Bald answer • Identifies the two correct critical points via a correct method • Correct conclusion to their critical points obtained using a correct method **1 mark** • Uses a correct method • Acknowledges a problem with the denominator. **0 marks** • Solves like a normal equation, with no consideration of the denominator.
1e) $f(x) = e^x \cos^{-1} x$ $f'(x) = (e^x)\left(\frac{-1}{\sqrt{1-x^2}}\right) + (\cos^{-1} x)(e^x)$ $= -\frac{e^x}{\sqrt{1-x^2}} + e^x \cos^{-1} x$	2	**1 mark** • differentiates $\cos^{-1} x$ correctly.
1f) (v) $\int_{-1}^{7} \frac{x}{\sqrt{x+2}} dx$ $\quad u = \sqrt{x+2}$, $du = \frac{dx}{2\sqrt{x+2}}$ $\quad$ when $x = -1$, $u = 1$; when $x = 7$, $u = 3$ $= 2\int_1^3 (u^2 - 2)\, du$ $= 2\left[\frac{u^3}{3} - 2u\right]_1^3$ $= 2\left(\frac{27}{3} - 6 - \frac{1}{3} + 2\right) = 9\frac{1}{3}$ $\left(= \frac{28}{3}\right)$	3	**2 marks** • correct primitive in terms of u • correct substitution into their primitive **1 mark** • correct integrand in terms of u • substitiutes correct limits

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QUESTION 2		
2a) $2\sin x = \sqrt{3}$ $\sin x = \frac{\sqrt{3}}{2}$ $x = \pi k + (-1)^k \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$, where k is an integer $x = \pi k + \frac{(-1)^k \pi}{3}$	2	**2 marks** • equivalent correct expressions **1 mark** • correct answer neglecting the condition for k • leaving answer in terms of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
2b) $\binom{10}{4} \times 4^6 \times (3x)^4$ $= 210 \times 4096 \times 81x^4$ $= 69672960x^4$	2	**1 mark** • Correct use of binomial theorem
2c) $\int \sin^2 3x\,dx = \frac{1}{2}\int (1 - \cos 6x)dx$ $= \frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + c$ $= \frac{x}{2} - \frac{1}{12}\sin 6x + c$	2	**1 mark** • Correctly uses $\cos 2\theta$ identity • Integrates correctly from incorrect use of $\cos 2\theta$ identity
2d) (i) $f(1) = 1^3 - 4\times1^2 - 7\times1 + 10$ $= 1 - 4 - 7 + 10$ $= 0$ $\therefore (x-1)$ is a factor	1	
2 d) (ii) $x^3 - 4x^2 - 7x + 10 = 0$ $(x-1)(x^2 - 3x - 10) = 0$ $(x-1)(x-5)(x+2) = 0$ $\therefore x = 1,\ x = 5$ or $x = -2$	2	**1 mark** • Correctly finds quadratic factor • Identifies another linear factor (or root)
2 e)(i) $P(\text{socks match}) = \frac{{}^{10}C_2 + {}^{6}C_2 + {}^{4}C_2}{{}^{20}C_2}$ $= \frac{33}{95}$	2	**1 mark** • Progress towards answer, involving the use of combinations or similar logic
2 e)(ii) As there are three different colours, four socks would ensure a matching pair.	1	

Solution	Marks	Comments
QUESTION 3		
3a) $\lim_{x\to 0}\frac{3x}{\sin 4x} = \lim_{x\to 0}\frac{3}{4}\times\frac{4x}{\sin 4x}$ $=\frac{3}{4}\times 1$ $=\frac{3}{4}$	1	**0 marks** • $\frac{3}{4}\times\frac{x}{\sin x}$ or equivalent
3b) (i) $m_{OP}\times m_{OQ} = -1$ $\frac{p^2-0}{2p-0}\times\frac{q^2-0}{2q-0} = -1$ $p^2q^2 = -4pq$ $pq = -4$	2	**1 mark** • Correctly finds the two slopes • Identifies the condition for perpendicular lines
3 b)(ii) $M=\left(\frac{2p+2q}{2},\frac{p^2+q^2}{2}\right)$ $=\left(p+q,\frac{p^2+q^2}{2}\right)$ $x=p+q$ $y=\frac{p^2+q^2}{2}$ $=\frac{(p+q)^2-2pq}{2}$ $=\frac{x^2-2\times -4}{2}$ $=\frac{x^2+8}{2}$ ∴ locus of M is $y=\frac{1}{2}x^2+4$	2	**1 mark** • Correctly finds M • Attempts to eliminate parameters from their answer for M
3 c) (i) $x=3+\sqrt{y}$ $x-3=\sqrt{y}$ $y=(x-3)^2,\ x\ge 3$	2	**1 mark** • Correctly swaps the x and y pronumerals • Does not consider domain (or range) of the solution
3 c)(ii) Point of intersection must lie on the line $y = x$ $x=(x-3)^2$ $x=x^2-6x+9$ $x^2-7x+9=0$ $x=\frac{7\pm\sqrt{13}}{2}$ But $x\ge 3$ ∴ intersect at $\left(\frac{7+\sqrt{13}}{2},\frac{7+\sqrt{13}}{2}\right)$	2	**1 mark** • Recognises the significance of $y = x$
3 d) $\frac{d^2x}{dt^2}=-e^{-2x}$ $\frac{d}{dx}\left(\frac{1}{2}v^2\right)=-e^{-2x}$ $\frac{1}{2}v^2=\frac{1}{2}e^{-2x}+c$ when $x=0$, $v=1$ $\frac{1}{2}=\frac{1}{2}+c$ $c=0$ ∴ $v^2=e^{-2x}$ When $x=4$, $v^2=e^{-8}$ $v=\pm e^{-4}$ However ∵ $v\ne 0$, i.e. particle does not stop and thus cannot change direction ∴ when $x=4$, $v=e^{-4}$ m/s	3	**2 marks** • $v=\pm e^{-4}$ • No justification for why v must be positive **1 mark** • Correctly integrates using $\frac{d}{dx}\left(\frac{1}{2}v^2\right)$

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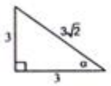
QUESTION 4		
4a)  $\alpha=\tan^{-1}\left(\frac{1}{1}\right)$ $=\tan^{-1}1$ $=\frac{\pi}{4}$ $3\cos x+3\sin x$ $=3\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$	2	**1 mark** • Correctly finds R and/c α
4 b)(i) $3\cos x+3\sin x\ne 0$ $3\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)\ne 0$ $x-\frac{\pi}{4}\ne\frac{\pi}{2}$ $x\ne\frac{3\pi}{4}$	1	**1 mark** • Any valid value of x
4 b)(ii) $\frac{1}{3\cos x+3\sin x}=\frac{\sqrt{6}}{9}$ $3\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=\frac{9}{\sqrt{6}}$ $\cos\left(x-\frac{\pi}{4}\right)=\frac{3}{\sqrt{12}}$ $=\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{\sqrt{3}}$ $=\frac{\sqrt{3}}{2}$ $x-\frac{\pi}{4}=-\frac{\pi}{6},\frac{\pi}{6}$ $x=\frac{\pi}{12},\frac{5\pi}{12}$ ∴ first positive value of x is $\frac{\pi}{12}$	2	**1 mark** • $\cos\left(x-\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}$
4c)(i) As problem starts at lowest amplitude of graph, it is most effectively modelled with a cosine curve which will obey SHM i.e. $x=-a\cos nt$ amplitude = 1.5 period = 12 i.e. $a=\frac{3}{2}$ i.e. $\frac{2\pi}{n}=12$ $n=\frac{\pi}{6}$ ∴ $x=-\frac{3}{2}\cos\left(\frac{\pi}{6}t\right)$	2	**1 mark** • Correctly evaluating n • Explanation of why th situation is modelled b a negative cosine curv
4 c)(ii) $\dot{x}=\frac{\pi}{4}\sin\left(\frac{\pi}{6}t\right)$ when $t=4$, $\dot{x}=\frac{\pi}{4}\times\sin\frac{2\pi}{3}$ $=\frac{\pi\sqrt{3}}{8}$ ∴ water level is rising at $\frac{\pi\sqrt{3}}{8}$ m/hour	2	**1 mark** • Correctly finds an expression for $\frac{dx}{dt}$
4 d) Let the roots be $\frac{\alpha}{r},\ \alpha,\ \alpha r$ $\alpha\beta\gamma=-c$ $\alpha^3=-c$ $\alpha=\sqrt[3]{-c}$ However α is a root of the equation ∴ $\alpha^3+a\alpha^2+b\alpha+c=0$ $-c+a\left(\sqrt[3]{-c}\right)^2+b\sqrt[3]{-c}+c=0$ $a\sqrt[3]{(-c)^2}=-b\sqrt[3]{-c}$ $a^3c^2=-b\times -c$ $a^3c=b$	3	**2 marks** • Significant progress towards a correct solution **1 mark** • solution correctly identifying the relationship between roots and coefficients

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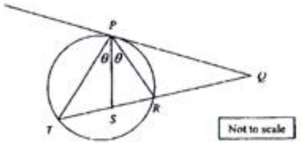
QUESTION 5		
5a)(i) $e^x = x + 2$ Let $f(x) = e^x - x - 2$ $\therefore$ looking for solutions to $f(x) = 0$ $f(1) = e^1 - 1 - 2$ $= -0.2817181715\ldots < 0$ $f(2) = e^2 - 2 - 2$ $= 3.389056099\ldots > 0$ As $f(x)$ is continuous and $f(1)$ and $f(2)$, a solution must lie in the interval $1 < x < 2$	2	**1 mark** • Shows the difference in signs without talking about continuity
5 a)(ii) $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ $= x_0 - \frac{e^x - x - 2}{e^x - 1}$ $= 1.5 - \frac{e^{1.5} - 1.5 - 2}{e^{1.5} - 1}$ $= 1.218042292\ldots$ $= 1.22$ (correct to 3 decimal places)	2	**1 mark** • Correctly uses Newton's Method
5 b)  $\angle RSP = \angle SPT + \angle PTS$ (exterior $\angle$ ΔSPT) $\angle RSP = \theta + \angle PTS$ $\angle SPQ = \angle SPR + \angle QPR$ (common $\angle$) $\angle QPR = \angle PTS$ (alternate segment theorem) $\therefore \angle SPQ = \theta + \angle QPR = \angle RSP$ ΔSPQ is isosceles (two $=$ $\angle$s) $\therefore$ $PQ = SQ$ ($=$ sides in isosceles ΔSPQ)	3	**2 marks** • Correct solution with poor reasoning • Significant progress towards solution with good reasoning. **1 mark** • Significant progress towards solution with poor reasoning. • Progress towards solution with good reasoning.
5 c)(i) $T = 35 + Ae^{-kt}$ $\frac{dT}{dt} = -kAe^{-kt}$ $= -k(Ae^{-kt} + 35 - 35)$ $= -k(T - 35)$	1	
5 c(ii) when $t = 0$, $T = 1400$ $\Rightarrow$ $T = 35 + 1365e^{-kt}$ when $t = 15$, $T = 995$ $995 = 35 + 1365e^{-15k}$ $e^{-15k} = \frac{960}{1365}$ $-15k = \log\frac{64}{91}$ $k = -\frac{1}{15}\log\frac{64}{91}$ $= 0.02346509488\ldots$ $= 0.023$ (to two decimal places)	2	**2 marks** • Leaving answer as an exact value • Do not penalise for rounding error **1 mark** • Correctly finding a value for A
5 c)(iii) when $T = 200$; $200 = 35 + 1365e^{-kt}$ $e^{-kt} = \frac{165}{1365}$ $-kt = \log\frac{11}{91}$ $t = -\frac{1}{k}\log\frac{11}{91}$ $= 90.04712083\ldots$ $= 90$ minutes	2	**1 mark** • Significant progress towards a correct solution..

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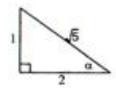
QUESTION 6		
6a)  Let $\alpha = \tan^{-1}\frac{1}{2}$ $\sin\left(2\tan^{-1}\frac{1}{2}\right) = \sin 2\alpha$ $= 2\sin\alpha\cos\alpha$ $= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}}$ $= \frac{4}{5}$	2	**1 mark** • Correct use of the $\sin 2\alpha$ identity
6 b)(i) Let X = the number of eggs with double yolks $P(X = 1) = {}^{12}C_1(0.95)^{11}(0.05)^1$ $= 0.341280054\ldots$ $= 0.3413$ (correct to 4 decimal places)	1	*Note: ignore rounding errors in all of 6 b)*
6 b)(ii) $P(X \geq 1) = 1 - P(X = 0)$ $= 1 - (0.95)^{12}$ $= 0.459639913\ldots$ $= 0.4596$ (correct to 4 decimal places)	2	**1 mark** • Solution involving the use of complementary events.
6 b)(iii) Let Y = the number of boxes containing at least one egg with a double yolk $P(Y = 2) = {}^3C_2(0.5404)^1(.4596)^2$ $= 0.342449578\ldots$ $= 0.342$ (correct to 3 decimal places)	2	**1 mark** • Progress towards a correct solution using their answer to part (ii)
6 c(i) $\frac{dV}{dt} = -\frac{3\pi}{16}$ $V = \frac{1}{3}\pi r^2 h$ $\frac{h}{r} = \frac{4}{3}$ $r = \frac{3}{4}h$ $\therefore V = \frac{1}{3}\pi\left(\frac{3h}{4}\right)^2 h$ $= \frac{3\pi}{16}h^3 \Rightarrow \frac{dV}{dh} = \frac{9\pi}{16}h^2$ $\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$ $= -\frac{3\pi}{16} \times \frac{16}{9\pi h^2}$ $= -\frac{1}{3h^2}$ when $h = 9$; $\frac{dh}{dt} = -\frac{1}{243}$ $\therefore$ the iceberg's height is decreasing at a rate of $\frac{1}{243}$ m/hour	3	**2 marks** • Correct expression for $\frac{dV}{dh}$ • Correct value for $\frac{dh}{dt}$ using their expression for $\frac{dV}{dh}$ **1 mark** • Finding an expression for r in terms of h
6 c)(ii) $A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$ $V = \frac{1}{3}\pi r^2 \times \frac{4}{3}r$ $= \frac{4}{9}\pi r^3 \Rightarrow \frac{dV}{dr} = \frac{4}{3}\pi r^2$ $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$ $= 2\pi r \times \frac{3}{4\pi r^2} \times -\frac{3\pi}{16}$ $= -\frac{9\pi}{32r}$ when $r = 12$; $\frac{dA}{dt} = -\frac{3\pi}{128}$ $\therefore$ the iceberg's base area is decreasing at a rate of $\frac{3\pi}{128}$ m/hour	2	**1 mark** • Solution involving $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$ or equivalent merit

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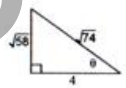
QUESTION 7		
7 a)(i) Consider the expansion of $(1+x)(1+x)^n$ $(1+x)(1+x)^n=(1+x)\left[\binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\ldots+\binom{n}{k-1}x^{k-1}+\binom{n}{k}x^k+\ldots\binom{n}{n}x^n\right]$ terms involving x^k are; $x\times\binom{n}{k-1}x^{k-1}+\binom{n}{k}x^k=\left[\binom{n}{k-1}+\binom{n}{k}\right]x^k$ Consider terms in the expansion of $(1+x)^{n+1}$ $T_{k+1}=\binom{n+1}{k}x^k$ $\therefore$ since $(1+x)(1+x)^n\equiv(1+x)^{n+1}$, equating coefficients gives $\binom{n}{k-1}+\binom{n}{k}=\binom{n+1}{k}$	2	**1 mark** • Considers terms in expansion or equivalent progress towards correct solution
7 a)(ii) When $n=3$; $LHS=\binom{5}{3}-\binom{3}{3}$ $=10-1$ $=9$ $RHS=3^2$ $=9$ $LHS=RHS$ Hence the result is true for $n=3$ Assume the result is true for $n=k$ i.e. $\binom{k+2}{3}-\binom{k}{3}=k^2$ Prove the result is true for $n=k+1$ i.e. Prove $\binom{k+3}{3}-\binom{k+1}{3}=(k+1)^2$ ***PROOF*** $\binom{k+3}{3}-\binom{k+1}{3}=\binom{k+2}{2}+\binom{k+2}{3}-\binom{k}{2}-\binom{k}{3}$ $=k^2+\binom{k+2}{2}-\binom{k}{2}$ $=k^2+\binom{k+1}{1}+\binom{k+1}{2}-\binom{k}{2}$ $=k^2+k+1+\binom{k}{1}+\binom{k}{2}-\binom{k}{2}$ $=k^2+k+1+k$ $=k^2+2k+1$ $=(k+1)^2$ Hence the result is true for $n=k+1$, if it is true for $n=k$ Since the result is true for $n=3$, then it is true for all $n\ge 3$ by induction.	4	There are 4 key parts of the induction; 1. Proving the result true for $n=3$ 2. Clearly stating the assumption and what is to be proven 3. Using the assumption in the proof 4. Correctly proving the required statement **4 marks** • Successfully does all of the 4 key parts **3 marks** • Successfully does 3 of the 4 key parts **2 marks** • Successfully does 2 of the 4 key parts **1 mark** • Successfully does 1 of the 4 key parts

Solution	Marks	Comments
QUESTION 7 … continued		
7 b)(i) $y=Vt\sin\theta-4.9t^2 \Rightarrow 250=Vt\sin\theta-4.9t^2$ $\dot{y}=V\sin\theta-9.8t \Rightarrow 30=V\sin\theta-9.8t$ $V\sin\theta=30+9.8t$ Solving simultaneously; $30t+9.8t^2-4.9t^2=250$ $4.9t^2+30t-250=0$ $t=\frac{-30+\sqrt{5800}}{9.8}$, $t>0$ $t=\frac{-30+10\sqrt{58}}{9.8}$ $=4.709972557\ldots$ $=4.71$ seconds (correct to 2 decimal places)	3	**2 marks** • Correctly finds a quadratic in terms of t, or equivalent merit **1 mark** • Finding an expression for the vertical velocity • Attempts to eliminate the parameters of V and θ in order to find a quadratic in terms of t
7 b)(ii) $x=Vt\cos\theta$ $\dot{x}=V\cos\theta \Rightarrow V\cos\theta=40$ $V\sin\theta=30+9.8t$ $=30-30+10\sqrt{58}$ $=10\sqrt{58}$ $\tan\theta=\frac{\sqrt{58}}{4}$ $\theta=62.29038921\ldots$ $=62.3^\circ$ (correct to 3 sig figs)  $V\cos\theta=40$ $V=\frac{40}{\cos\theta}$ $=40\times\frac{\sqrt{74}}{4}$ $=10\sqrt{74}$ $=86.02325267\ldots$ $=86.0$ m/s (correct to 3 sig figs)	3	**2 marks** • Correctly finds V • Correctly finds θ **1 mark** • Progress towards finding either V or θ using a correct method.